Traversability Analysis for Mobile Robots in Outdoor Environments: A Semi-Supervised Learning Approach Based on 3D-Lidar Data

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Abstract— **The ability to safely navigate is a crucial pre-requisite for truly autonomous systems. A robot has to distinguish obstacles from traversable ground. Failing on this task can cause great damage or restrict the robots movement unnecessarily. Due to the security relevance of this problem, great effort is typically spent to design models for individual robots and sensors, and the complexity of such models is correlated to the complexity of the environment and the capabilities of the robot. We present a semi supervised learning approach, where the robot learns its traversability capabilities from a human operating it. From this partially and only positive labeled training data, our approach infers a model for the traversability analysis, thereby requiring very little manual effort for the human. In practical experiments we show that our method can be used for robots that need to reliably navigate on dirt roads as well as for robots that have very restricted traversability capabilities.**

I. Introduction

The focus of research for robotic applications evolved during the last decade from well structured indoor environments over urban outdoor environments to unstructured outdoor environments. With this expansion of interest, it is an important pre-requisite to reliably classify traversable ground in the environment. This topic is typically referred to as traversability analysis or obstacle detection. The quality of the traversability analysis for a mobile robot affects the free movement of the platform, as well as the safety, and therefore much attention has to be put into this task. In well structured indoor environments people often choose to employ only a horizontal 2d-laser scanner and the traversability is simply classified based on the observed obstacles in the scans. Yet, in environments, where the ground is not flat or contains obstacles that are not purely vertical, this basic approach can not be safely used anymore. In unstructured environments, we need a sensor setup that perceives a dense model of the world. In these cases, 3d range data is necessary, as provided, *e.g.*, by stereo-cameras, radar or 3d-laser scanners, or a fusion of different sensors. Our approach works on 3d-lidar data, not only using the purely spatial information but also including remission values to add a visual component to the process. The definition of traversability highly depends on the individual mobile robot that is used in the application, since they can provide quite different capabilities regarding ground clearance, motor power, stability, and, *e.g.*, if it is equipped with wheels or tracks. Much effort can be put into designing the model for the traversability analysis for a specific platform, with a specific sensor setup. This is often a time consuming and costly process. It would be much easier if one could just steer the robot manually in the environment to train the traversability analysis. But to make use of this data, the problem is that the learning algorithm only gets the information of the traversed part during the training. This means that the training set contains only incomplete positive labels and no negative information. In our approach we adapt learning algorithms for that kind of problem to a frame work that makes it possible to infer the traversability analysis for the mobile robot.

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Fig. 1. Different mobile robot outdoor platforms with different capabilities and different fields of applications.

II. Related Work

Recently, Papadakis published a survey of traversability analysis methods for unmanned ground vehicles [14]. It states that the predominant approaches to measure traversability are based on occupancy grid maps, which are accredited to Moravec and Elfes [11]. More concrete, they are based on the analysis of 2d elevation maps, where 3d information is represented in 2d maps. Pfaff et al. presented an approach, where the 2d elevation maps were used for traversability analysis as well as for mapping and localization purposes [16]. A more general representation is a 2d grid map, where each cell stores features that provide enhanced information from the senors. Papadakis identified this as the preferred choice when dense 3d point clouds are available [6, 7, 8]. Our approach also uses this kind of 2d grid map, where each cell is associated with at least one feature vector that is computed from the 3d points that are mapped to the respective cell. Many methods to perform traversability analysis are based on heuristics that represent the capabilities of the robot, in combination with measurement models that describe the sensor noise [2, 3, 7]. These methods to classify traversability typically work well for many environments, but they are limited in their generality, since they often do not explicitly distinguish different types of obstacles, like rocks or grass. Moreover, a specific heuristic has to be developed for every robot and also for different combinations of sensors and terrain. Murphy and Newman use a probabilistic costmap, which is generated from the combined output of a multiclass gaussian process classifier and gaussian process regressors. It models the spatial variation in cost for the terrain type, to perform fast path planning, taking the uncertainty of the terrain cost into account [12].

Another problem which is hard to tackle are the so called negative obstacles, like holes in the ground or downwards leading steps. The sensor is not necessarily able to perceive the lower part of the structure and therefore the robot has to reason about the cause for missing data, which might result from untraversable gaps or simple visibility issues [10, 18]. This is of special interest in search and rescue scenarios after disasters, where the environment is very complex due to irregularities. This kind of analysis is especially critical when the sensors that are used only provide a sparse representation of the environment, like rigidly mounted, downwards facing 2d-laser scanners.

A natural thing would be to let the robot learn about the traversability of the environment. This has the advantage that there is no need for a heuristic to interpret the senor data. But in supervised scenarios one has to provide labeled data to the approach to learn from. Lalonde et al. [9] proposed to segment 3d point clouds into the categories *scatter*, to represent porous volumes, *linear*, to capture, *e.g.*, wires and tree branches, and *surface* to capture solid objects like ground surfaces. The authors achieve this by training a Gaussian Mixture Model with an expectation maximization algorithm on hand labeled data. A different way to perceive the environment is to use proprioceptive measures, like bumper hits, measuring slip, or vibration. Those can be combined with geometric measures and used, *e.g.*, to project the current measurements into the environment [1, 6, 17]. Yet, the use of proprioceptive measures requires an adequately robust robot that is physically able to traverse the terrain in question. Even though such methods allow the robot to autonomously learn a model of the environment, the trial and error part of this methodology involves a high risk to damage the robot.

In contrast to this, our approach uses data collected from a human operator that drove a safe trajectory and therefore provided partially labeled training data. This is a very convenient, safe, and time efficient way to train a classifier. An approach that follows a similar idea was presented by Ollis et al. [13]. Their system uses data from a stereo camera, radar, as well as 2d- and 3d-lidar sensors. Features are computed as multidimensional histograms and a distribution is learned for the traversed cells. The approach makes use of a monotonicity assumption that states that cells with higher values of the features would be expected to be less traversable and the inferred probabilities were enforced to meet this assumption. The resulting values are mapped to a cost function that is then used for planning.

In our approach there are no heuristic assumptions about the features and their relation to the traversability. To solve this special learning problem, we adapted the work of Denis et al. [4] and Elkan and Noto [5] to learn the probabilities from the available data.

III. Basic Structure

For our approach, we interpret traversability as a local and static characteristic of the environment. We use a mobile robot equipped with a 3d-lidar sensor that also provides remission values of the measurements, which we assume to be calibrated. Since we interpret the characteristic of traversability to be static, we further assume that dynamic objects are detected and removed in advance. The map representation we use is a 2d occupancy grid $\mathcal{G}$ with resolution $r \in \mathbb{R}_{>0}$, where each cell can hold one or more feature vectors. The perceived 3d-points are mapped to the grid cells by projection and after a covered distance of $d_P \in \mathbb{R}_{>0}$ we compute and add a feature vector from the points in the cell. This feature vector is composed of geometrical measures and the remission values. By discretization of the feature vectors, using fixed size increments, we create a Vocabulary $\mathcal{V}$ of discrete features. We expect those features to be multinomial distributed given the traversability state of a cell, $P(. \mid \mathit{state}) \sim \mathit{Multinomial}$ with $\mathit{state} \in \{\mathit{trav}, \neg\mathit{trav}\}$. Therefore the goal of our approach is to learn the parameters for that distribution in order to calculate $P(\mathit{trav} \mid f_1, \ldots, f_n)$ for $f_i \in \mathcal{V}$. To avoid accumulated registrations errors, we use only local maps that are used for a distance of $d_M \in \mathbb{R}_{>0}$, which is a typical workaround to avoid global inconsistencies of the registration process.

IV. The Learning Problem

One of the goals of our approach is that it can be used by humans that are not especially educated to design traversability models for mobile robots. We achieve this by designing a naive method for generation of training data. More concrete, the training data is generated by a human that operates the robot in an environment that is similar to the environment where the robot should later be able to reliable operate in. From this training trajectory, the cells that intersect with the projection of the footprint of the robot are labeled as positive examples. Using this process for training data generation has the advantage that it is fairly easy to execute but also has the drawback that we get only scarce positive examples and tons of unlabeled data to learn from. Fortunately this kind of data is a very common problem in, *e.g.*, text classification and biological approaches like protein categorization and we can adapt existing methods for our approach. We use and compare two strategies to learn a classifier from this kind of training data, one called

Fig. 2. Trajectory of Robot1 on the forest track that was used for the experiments on an aerial image (© Google)

Fig. 3. Aerial image (© Google) with the training (blue) and evaluation (red) trajectory of Robot2 on the Campus.

Positive Naive Bayes (PNB) introduced by Denis et al. [4] and *Learning Classifiers from Only Positive and Unlabeled Data* (POS) by Elkan and Noto [5]. The former one was developed for text classification, where this kind of data is very common, and also assumes the words to be multinomial distributed. The latter one is a more general approach that can deal with a variety of distributions.

A. Feature Design

In feature based approaches, it is important that the features are designed to capture the world for the intended task. For our approach this means that they should be able to distinguish obstacles from traversable ground for different platforms. They have to distinguish flat solid ground from moderate steps, between many kinds of vegetation and from solid obstacles of certain heights, since it depends strongly on the robot, if these are traversable or not. For our approach we use feature vectors that contain the following measures.

- The absolute maximum difference in the z-coordinate
- The mean of the remission values
- The variance of the remission values
- The roughness of the cell
- The slope

Each dimension should help to distinguish different types of terrain as well as traversability constraints of the robot. The maximum height difference and the slope reflect the ground-clearance of the robot as well as the motor power. The remission values and the roughness help to distinguish concrete and vegetation types. Since the calculation of the first three dimensions is straight forward, we shortly explain the calculation of the latter two, which are based on the eigenvalues and the respective eigenvectors of the covariance matrix of the points in the cell. The smallest eigenvalue is used as a roughness parameter. The eigenvector that belongs to the smallest eigenvalue is used as the normal vector of the cell, and the slope is the angle between the normal and the vector of gravity. To ensure that these values are well defined, we ignore cells that contain less than five points.

B. Positive Naive Bayes

The *Positive Naive Bayes Classifier*, as introduced by Denis et al. [4], estimates the frequencies of observed features in the classical way. Since the data is only incompletely labeled and contains no negative labeled samples, it calculates an estimate for the negative frequencies from the previous estimate of the positive frequencies and the prior. In particular, let the set PD contain all the positive labeled cells including their observations and UD be the set of the unlabeled cells. Let $C : \mathcal{V} \times 2^{\mathcal{G}} \to \mathbb{N}$ be the counting function, *i.e.*, for $S \subset \mathcal{G}$, $f \in \mathcal{V}$ we define $C(f,S) := \sum_{c \in S} \sum_{f_c \in c} \mathbb{1}_f(f_c)$, whereat $\mathbb{1}$ is the indicator function. Further we define $C(S) := \sum_{f \in \mathcal{V}} C(f,S)$ as the number of observations, including multiplicity, in the set S. The probability given the positive class, which means in our case the traversable class, is estimated by:

$$P(f \mid trav) = \frac{\alpha_p + C(f, PD)}{\alpha_p |\mathcal{V}| + C(PD)}$$

Where $\alpha_p \in [0,1]$ is the additional smoothing parameter, which was in our case set to $\alpha_p = 1/|\mathcal{V}|$. To estimate the probability given the negative class is a little bit more complicated, due the problem that no negative examples are available. Therefore we substitute the counting function with $C_N(f) := \max\{C(f,UD) - P(f|trav)P(trav)C(UD); 0\}$ With this approximate counting function we estimate the probability for a feature given the negative class.

$$P(f \mid \neg trav) = \frac{\alpha_n + C_N(f)\eta}{\alpha_n |\mathcal{V}| + (1 - P(trav))C(UD)}$$

Where η is the normalizer for the not smoothed probability using $C_N(f)$. For the negative class we used the smoothing factor $\alpha_n = 1$. The reason for using different values for α_p and α_n is that if we observe a feature that was never observed before, we would get $P(f \mid trav) > P(f \mid \neg trav)$ since in our case $C(PD) \ll (1 - P(trav))C(UD)$. This would result in a positive classification for a cell that contains only unseen features, which is incompatible with the safety requirements for traversability analysis. We finally compute $P(trav \mid f_1, \ldots, f_n)$ using *Naive Bayes*.

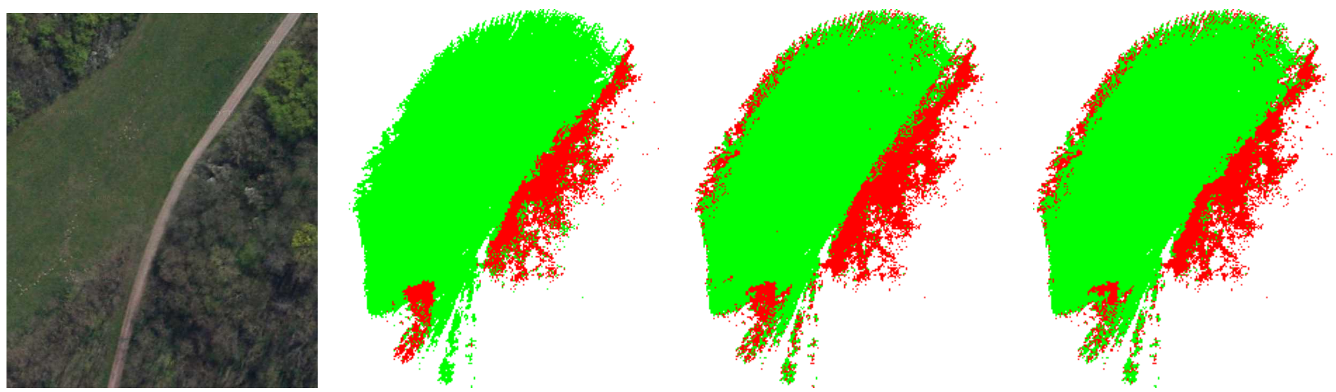


Fig. 4. Traversability map from the forest run with Robot1 using our approach. From left to right: Aerial image of the scene, ground truth labeled map which was used for the evaluation, our approach using the PNB-based classifier and our approach using the POS-based classifier. The grass on the mid-upper left side is correctly labeled as traversable (green) while the parts of the forest are labeled as obstacles (red). The POS-based classifier has false positives in the lower left and mid right. Both classifiers have problems with the measurements at the border of the map.

C. Learning from Positive Only

The classifier that is proposed in the work of Elkan and Noto [5] follows a different strategy. In their work they use the sets PD and UD to estimate the distribution for a feature f to get a label (always positive) during the training, $P(label \mid f)$, $f \in \mathcal{V}$. This is now a classical learning problem with full labeled data, since we know for each feature whether it got a label or not. Once the distribution of $P(label \mid f)$ is estimated, Elkan and Noto elaborated a way to transfer this to $P(trav \mid f)$. Elkan and Noto proofed, that given the *selected completely at random assumption* there exists $c > 0$ such that $P(trav \mid f) = P(label \mid f)/c$. While they provide different ways to estimate c using a validation set, we use the maximum estimate for c, since it should be the most conservative one. Nevertheless, since we have only incomplete data, it is still possible that for some features $P(label \mid f) > c$. To cope with such cases we set $P(trav \mid f) = \min\{P(label \mid f)/c \;; (1-\epsilon)\}$. In our approach we train the distribution $P(label \mid f)$ using standard Naive Bayes using the same smoothing parameters as described in Sec. IV-B. We use the efficient log-odds update to integrate multiple measurements within one cell, utilizing the static map assumption.

$$\begin{aligned} logodds(trav \mid f_1,\ldots,f_n) = {} & logodds(trav|f_n) \\ & + logodds(trav \mid f_1,\ldots,f_{n-1}) \\ & + logodds(trav) \end{aligned}$$

D. Terrain Models

Since the learning algorithms only get the positive data of the trajectory the unlabeled data may also contain features of a different type of terrain that is traversable. The learning algorithms may get confused if we merge all the data within one distribution. For example if during the training we traverse most of the time the street and only a short time grass, then the ratio of labeled grass data is very small and therefore the learning algorithms can not adept grass to be traversable. This kind of problem will occur whenever the training set of the terrain types is not balanced. For the method described in Sec. IV-C it will also violate the *selected completely at random* assumption. To overcome this problem we use a set of different terrain models $\mathcal{M}$. The positive examples of a local map are compared to the existing terrain models using Pearson's χ^2-test, [15], with a significance level of $\alpha = 0.05$. If the test cannot discard the null hypothesis, we merge the data of the local map with the respective model. Otherwise, if the test discards the null hypothesis for all existing models, a new model is added to $\mathcal{M}$. For the method described in Sec. IV-B we use a one-vs-all strategy for the final classifier.

$$P(trav \mid f_1 \ldots, f_n) = \max_{m \in \mathcal{M}} P_m(trav \mid f_1, \ldots, f_n)$$

For the method described in Sec. IV-C we need to specify how to compute $P(trav \mid f)$ in the context of terrain models. We use a featurewise one-vs-all strategy here.

$$P(trav \mid f) = \max_{m \in \mathcal{M}} P_m(trav \mid f)$$

E. Training

The training phase is fairly easy for the user. The robot is operated by a human over all kinds of terrain it can traverse. During this phase the local maps are given to the learning algorithm. Then the statistic test is computed for the terrain models. Afterwards the selected model, it may be an existing one or a new one, is merged with the data from the local map and the current distribution of the models are computed. More formal, for a selected model $m \in \mathcal{M}$ the set of labeled data becomes $PD_m = PD_m \cup PD_l$ and the set of unlabeled data becomes $UD_m = UD_m \cup UD_l$. This sequential structure of our learning strategy also allows to retrain the robot at any point in time. This might be interesting for scenarios where the robot acts mainly autonomous but is connected to a command center where it can put requests if for example it can not find a path to the mission goal.

V. Experiments

In the experiments, we used two mobile robots with different capabilities, like in Fig. 1. One robot is capable of urban as well as outer urban environments, providing good motor power, high ground clearance and good stability (Robot1). The other is only capable of urban environments, with small ground clearance and weak stability (Robot2). On both platforms we evaluate the quality of the classification using hand labeled ground truth on suitable test tracks, *e.g.*, Fig. 2 and Fig. 3. Furthermore we compare the quality of the classifier when we omit the remission values (NoRe) and when we omit the roughness and slope values (NoRS) of the feature vector, see Sec. IV-A. For the experiments we used $d_M = 20m$, $d_P = 0.5m$ and limited the maximum range of our 3d-lidar sensor to $20m$. We used the same parameters to discretize the feature vectors for both robots. Consecutive local maps were used to train and evaluate the classifiers. Cells were classified as traversable if and only if $P(trav \mid f_1, \ldots, f_n) > 0.5$. Dynamic Obstacles were removed from the scans using an online dynamic obstacle detection approach based on scan differencing. The point clouds are registered using an *Applanix* Navigation System. The robots are equipped with 3d-lidar sensors from *Velodyne*, providing $360°$ horizontal and $\sim 30°$ vertical field of view.

A. Evaluation with Robot1

We trained Robot1 on the Campus, by driving over grass of different heights and with different flowers, dirt, walkways and streets. For the evaluation of our classifier we use a test track containing dirt roads, Fig. 2, and on the campus where we traversed walkways as well as grass areas. For the quality measures of the classifiers we labeled 30 local maps from the forest track and 5 from the campus track, which is about 10% of the local maps that were created during the run. The ground truth was labeled rather conservative, *i.e.*, especially in forest environment the cluttered areas between the trees were hard to classify for each and every cell, in doubt they were classified as not traversable, since the measure of false positives is more important for traversability analysis. Nevertheless, a false positive was counted if and only if the inspected cell and all eight adjacent cells were classified as positive (traversable). In this experiment our approach shows better results, in terms of precision and specificity, when we use the PNB-based classifier than with the POS-based classifier. On the combined data set, with the full feature vector, the PNB-based classifier reaches 0.992 while the POS-based classifier has 0.945. The POS-based classifier has problems especially with the forest data, 0.990 vs. 0.934, while this difference is not that substantial for the campus data set, 0.998 vs. 0.990. It is interesting to notice that for the PNB-based classifier the remission values (0.990 for NoRe) seem not to be as important as the roughness and slope (0.953 for NoRS) values. This role changes for the POS-based classifier where the precision without remission is worse than without roughness and slope. For both classifiers the full feature vector is superior to the pruned feature vectors. As expected the performance for the recall is antithetic to the precision.

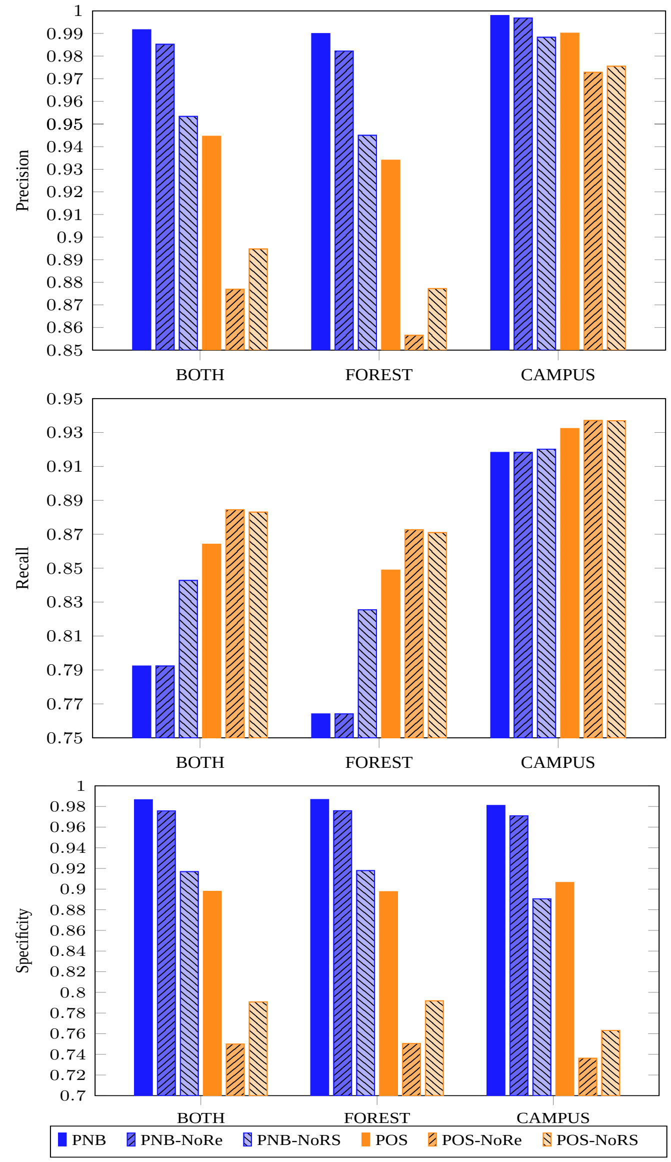
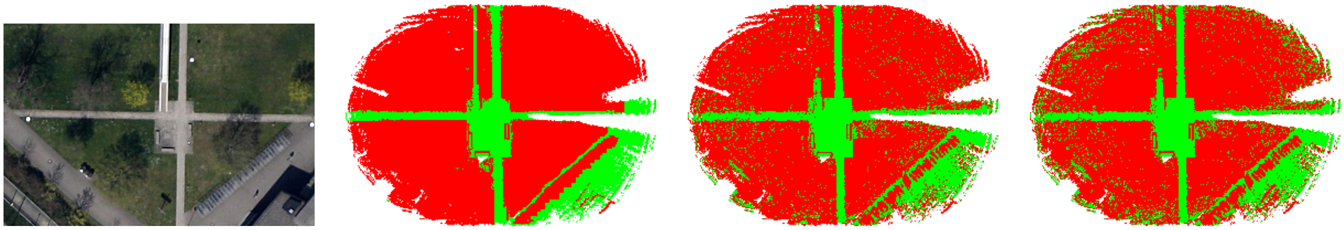


Fig. 6. Precision, Recall and Specificity for the test trajectories of Robot1. We compare the performance of the full feature vector, without remission values (NoRe) and without roughness and slope values (NoRS). The classifier based on PNB is shown in blue and the one based on POS in orange. For both methods using the full feature vector improves precision and specificity, the role of remission values and roughness and slope values behave different for the two methods.

In this measure the POS-based classifier, 0.87, is superior to the PNB-based classifier, 0.79. Like for precision this difference is larger on the forest data set than on the campus data set. The last quality measure we used in our evaluation is the specificity, see Fig. 6 bottom. This measure is of great importance, since it measures the rate of the true negative classifications. A *Type I Error* means wrong classification of a negative sample. In the case of traversability analysis this means missing an obstacle. Here again, already indicated by the precision measure, the PNB-based classifier, 0.987, is superior to the POS-based classifier, 0.898. While we observed different gaps between the classifiers for precision and recall on the forest and campus data set, measuring the



Aerial image | Ground truth | PNB-based classifier | POS-based classifier

Fig. 5. Traversability map from the campus run with Robot2 using our approach. From left to right: Aerial image of the scene, ground truth labeled map which was used for the evaluation, our approach using the PNB-based classifier and our approach using the POS-based classifier. Both classifiers produce similar results.

TABLE I

EVALUATION FOR ROBOT2 ON THE CAMPUS TRAJECTORY.

Method	Measure	Full	NoRe	NoRS
PNB	Precision	0.978	0.924	0.958
	Recall	0.947	0.868	0.954
	Specificity	0.984	0.945	0.967
POS	Precision	0.975	0.637	0.840
	Recall	0.947	0.940	0.961
	Specificity	0.982	0.589	0.859

Results for Robot2 on the campus trajectory (Fig. 3). Both learning methods behave quite similar in this scenario when using the full feature vector. The absence of roughness and slope measures (NoRS) gives a better performance in this scenario than the absence of remission values (NoRe).

specificity the difference is roughly the same for both data sets. Note that the results of this experiment do not prove that the PNB method is in general superior to the POS method, but for this data set and the way we use it.

B. Evaluation with Robot2

For Robot2 we used only a short training trajectory on the campus, see the blue part of Fig. 3, since the complexity of the environment is much lower than for the forest data set. In this environment both classifiers perform almost identical with the full feature vector, see Tab. I. Both the classifiers reach the precision of 0.98, recall of 0.95 and specificity of 0.98. In this scenario the remission values are more important than the roughness and slope parameters. Using the POS-based classifier the precision without remission values is 0.63 and without roughness and slope it is 0.84. Especially for the POS-based classifier the combination of both improves the performance substantially, while for the PNB-based classifier the performance is similar.

VI. CONCLUSION

We presented an easy to use approach to learn traversability for mobile robots. In the experiments we showed that our approach can be applied to different robots with different traversability characteristics. Moreover, our approach is usable in outdoor urban environments as well as in unstructured non-urban environments like forest roads and grassland.

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