Bayes' Rule Example

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1   What does the test t for an illness i tell me?

Given we have:

- 1% of the population is ill: $p(i) = 0.01$

- Given an ill person, the test is positive in 90% of the cases: $p(t \mid i) = 0.9$

- Given a person that is not ill, the test is positive in 20% of the cases: $p(t \mid \neg i) = 0.2$

What is the probability of being ill given a positive test?

$$
\begin{align}
p(i \mid t) &= \frac{p(t \mid i)p(i)}{p(t)} \tag{1} \\
&= \frac{p(t \mid i)p(i)}{\sum_i p(t \mid i)p(i)} \tag{2} \\
&= \frac{p(t \mid i)p(i)}{p(t \mid i)p(i) + p(t \mid \neg i)p(\neg i)} \tag{3} \\
&= \frac{p(t \mid i)p(i)}{p(t \mid i)p(i) + p(t \mid \neg i)(1 - p(i))} \tag{4} \\
&= \frac{0.9 \times 0.01}{0.9 \times 0.01 + 0.2 \times 0.99} \tag{5} \\
&= \frac{0.009}{0.207} \tag{6} \\
&\approx 0.043 \tag{7} \\
&\approx 4\% \tag{8}
\end{align}
$$

The probability of being ill is only 4%!