Introduction to Mobile Robotics

Summary

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Probabilistic Robotics

Probabilistic Robotics

Key idea: Explicit representation of uncertainty

(using the calculus of probability theory)

- Perception = state estimation
- Action = utility optimization

Bayes Formula

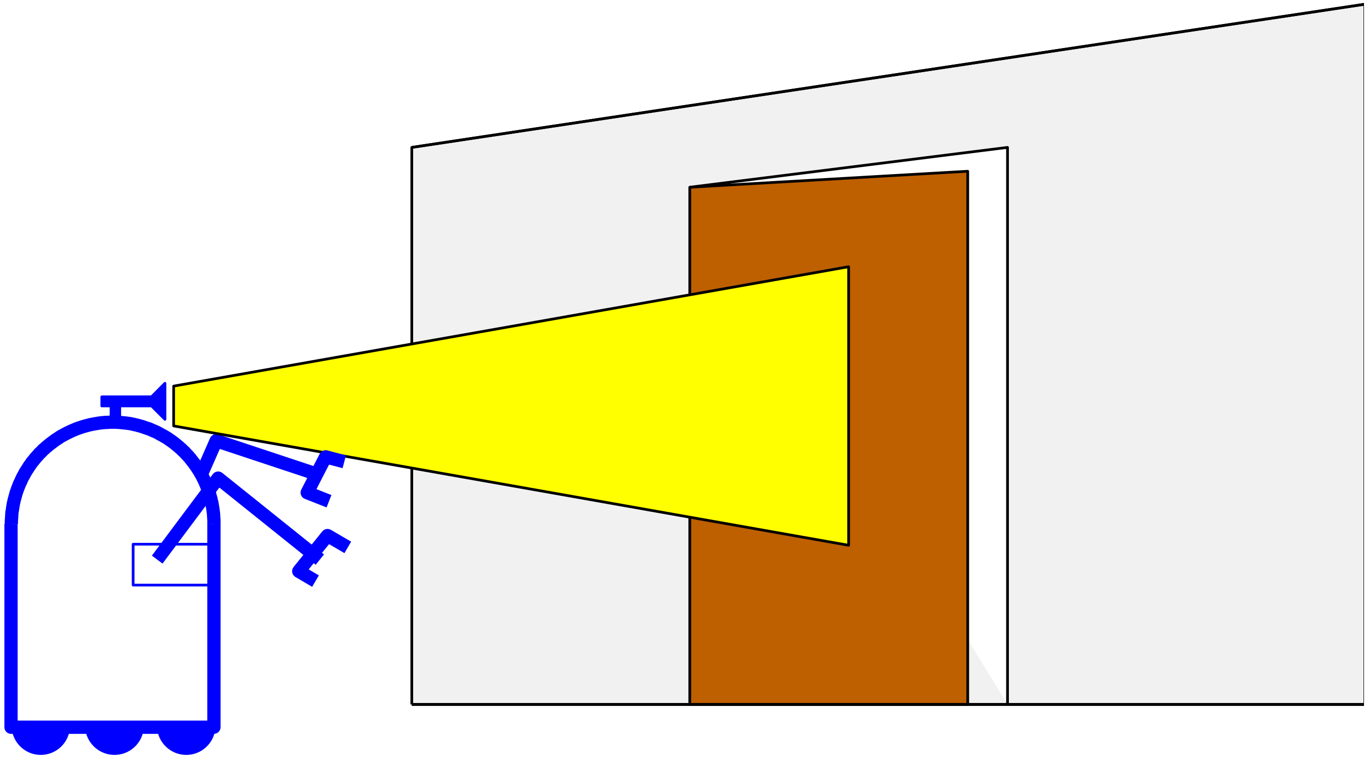
$$P(x, y) = P(x \mid y)P(y) = P(y \mid x)P(x)$$

$$\Rightarrow$$

$$P(x \mid y) = \frac{P(y \mid x)\ P(x)}{P(y)} = \frac{\text{likelihood} \cdot \text{prior}}{\text{evidence}}$$

Simple Example of State Estimation

- Suppose a robot obtains measurement z
- What is *P(open|z)?*



Causal vs. Diagnostic Reasoning

- $P(open|z)$ is diagnostic.
- $P(z|open)$ is causal.
- Often causal knowledge is easier to obtain.
- Bayes rule allows us to use causal knowledge:

count frequencies!

$$P(open \mid z) = \frac{P(z \mid open)P(open)}{P(z)}$$

Bayes Filters

z = observation
u = action
x = state

$$Bel(x_t) = P(x_t \mid u_1, z_1 \ldots, u_t, z_t)$$

Bayes $$= \eta \; P(z_t \mid x_t, u_1, z_1, \ldots, u_t) \; P(x_t \mid u_1, z_1, \ldots, u_t)$$

Markov $$= \eta \; P(z_t \mid x_t) \; P(x_t \mid u_1, z_1, \ldots, u_t)$$

Total prob. $$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_1, z_1, \ldots, u_t, x_{t-1}) \; P(x_{t-1} \mid u_1, z_1, \ldots, u_t) \; dx_{t-1}$$

Markov $$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; P(x_{t-1} \mid u_1, z_1, \ldots, u_t) \; dx_{t-1}$$

Markov $$= \eta P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; P(x_{t-1} \mid u_1, z_1, \ldots, z_{t-1}) \; dx_{t-1}$$

$$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}$$

Bayes Filters are Familiar!

$$Bel(x_t) = \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}$$

- Kalman filters
- Particle filters
- Hidden Markov models
- Dynamic Bayesian networks
- Partially Observable Markov Decision Processes (POMDPs)

Sensor and Motion Models

$$P(z \mid x, m) \qquad\qquad P(x \mid x', u)$$

Motion Models

- Robot motion is inherently uncertain.
- How can we model this uncertainty?

Probabilistic Motion Models

- To implement the Bayes Filter, we need the transition model $p(x \mid x', u)$.
- The term $p(x \mid x', u)$ specifies a posterior probability, that action *u* carries the robot from *x'* to *x*.

Typical Motion Models

- In practice, one often finds two types of motion models:
  - **Odometry-based**
  - **Velocity-based** (**dead reckoning**)
- Odometry-based models are used when systems are equipped with wheel encoders.
- Velocity-based models have to be applied when no wheel encoders are given.
- They calculate the new pose based on the velocities and the time elapsed.

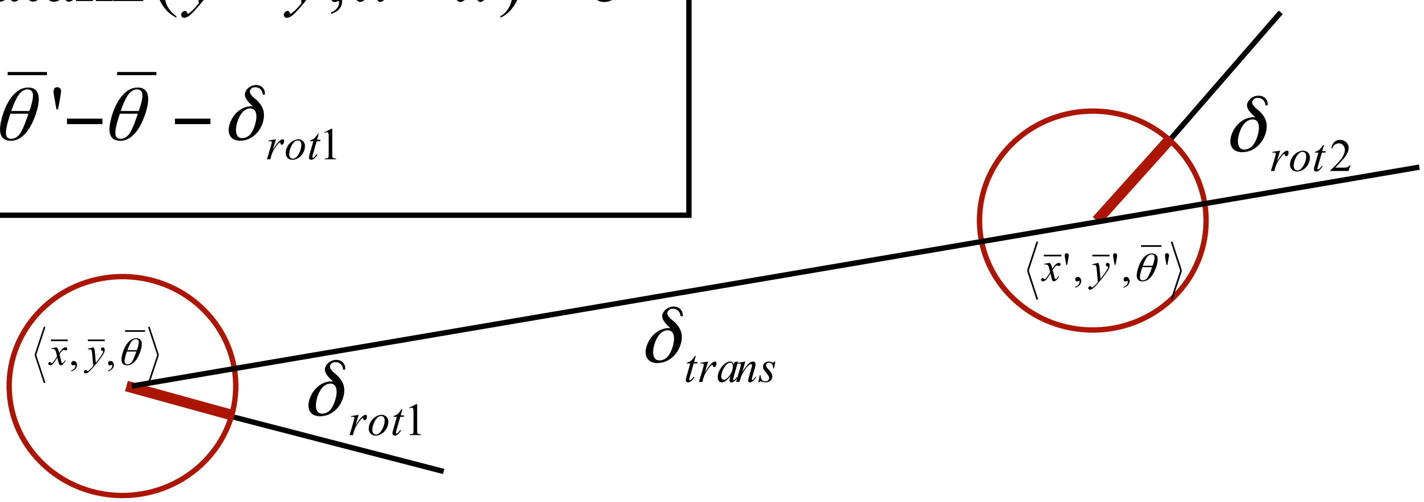
Odometry Model

- Robot moves from $\langle \bar{x}, \bar{y}, \bar{\theta} \rangle$ to $\langle \bar{x}', \bar{y}', \bar{\theta}' \rangle$.
- Odometry information $u = \langle \delta_{rot1}, \delta_{rot2}, \delta_{trans} \rangle$.

$$\delta_{trans} = \sqrt{(\bar{x}' - \bar{x})^2 + (\bar{y}' - \bar{y})^2}$$

$$\delta_{rot1} = \operatorname{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$$

$$\delta_{rot2} = \bar{\theta}' - \bar{\theta} - \delta_{rot1}$$



Sensors for Mobile Robots

- Contact sensors: Bumpers
- Internal sensors
  - Accelerometers (spring-mounted masses)
  - Gyroscopes (spinning mass, laser light)
  - Compasses, inclinometers (earth magnetic field, gravity)
- Proximity sensors
  - Sonar (time of flight)
  - Radar (phase and frequency)
  - Laser range-finders (triangulation, tof, phase)
  - Infrared (intensity)
- Visual sensors: Cameras
- Satellite-based sensors: GPS

Beam-based Sensor Model

- Scan z consists of K measurements.

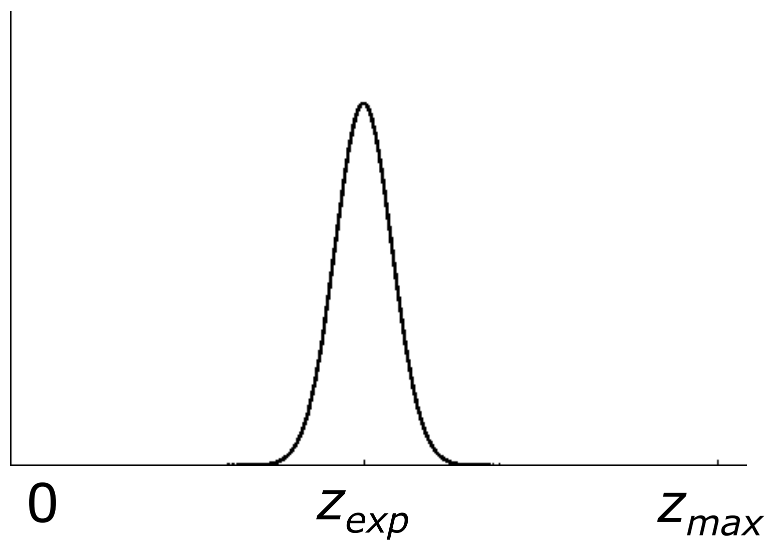
$$z = \{z_1, z_2, ..., z_K\}$$

- Individual measurements are independent given the robot position.

$$P(z \mid x, m) = \prod_{k=1}^{K} P(z_k \mid x, m)$$

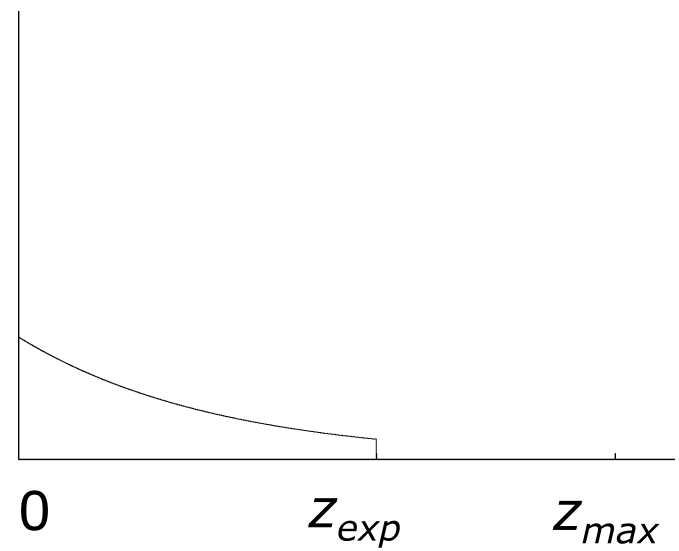
Beam-based Proximity Model

Measurement noise



$$P_{hit}(z \mid x, m) = \eta \frac{1}{\sqrt{2\pi b}} e^{-\frac{1}{2}\frac{(z - z_{\exp})^2}{b}}$$

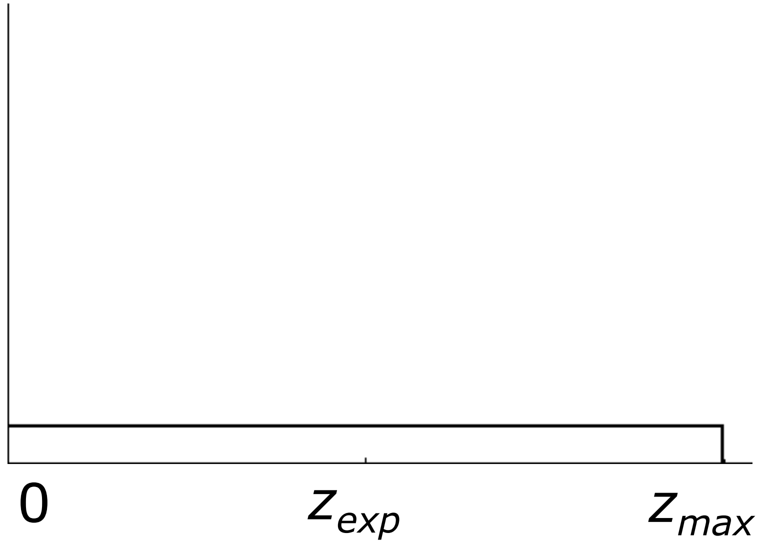
Unexpected obstacles



$$P_{\text{unexp}}(z \mid x, m) = \begin{cases} \eta \, \lambda \, e^{-\lambda z} & z < z_{\exp} \\ 0 & otherwise \end{cases}$$

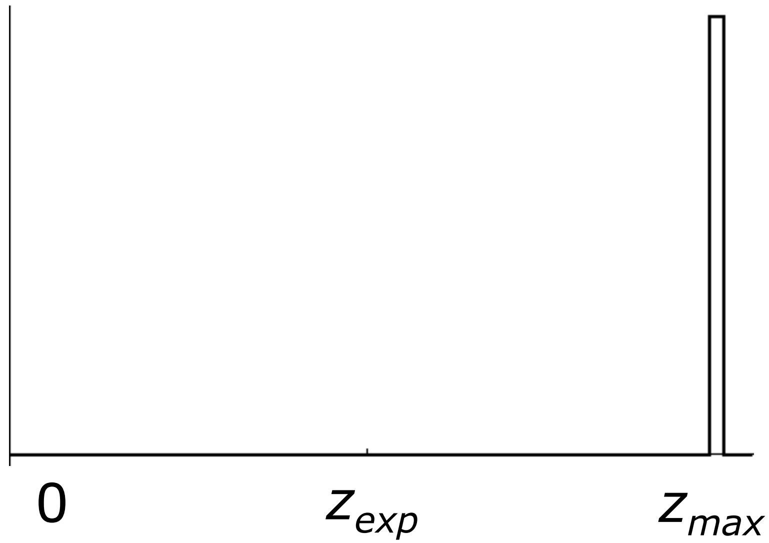
Beam-based Proximity Model

Random measurement



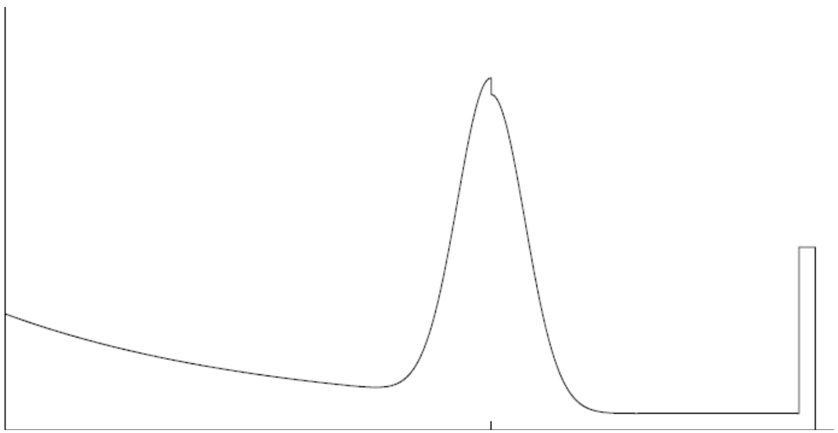
$$P_{rand}(z \mid x, m) = \eta \frac{1}{z_{\max}}$$

Max range



$$P_{\max}(z \mid x, m) = \eta \frac{1}{z_{small}}$$

Resulting Mixture Density



$$P(z \mid x,m) = \begin{pmatrix} \alpha_{\text{hit}} \\ \alpha_{\text{unexp}} \\ \alpha_{\max} \\ \alpha_{\text{rand}} \end{pmatrix}^T \cdot \begin{pmatrix} P_{\text{hit}}(z \mid x,m) \\ P_{\text{unexp}}(z \mid x,m) \\ P_{\max}(z \mid x,m) \\ P_{\text{rand}}(z \mid x,m) \end{pmatrix}$$

How can we determine the model parameters?

Bayes Filter in Robotics

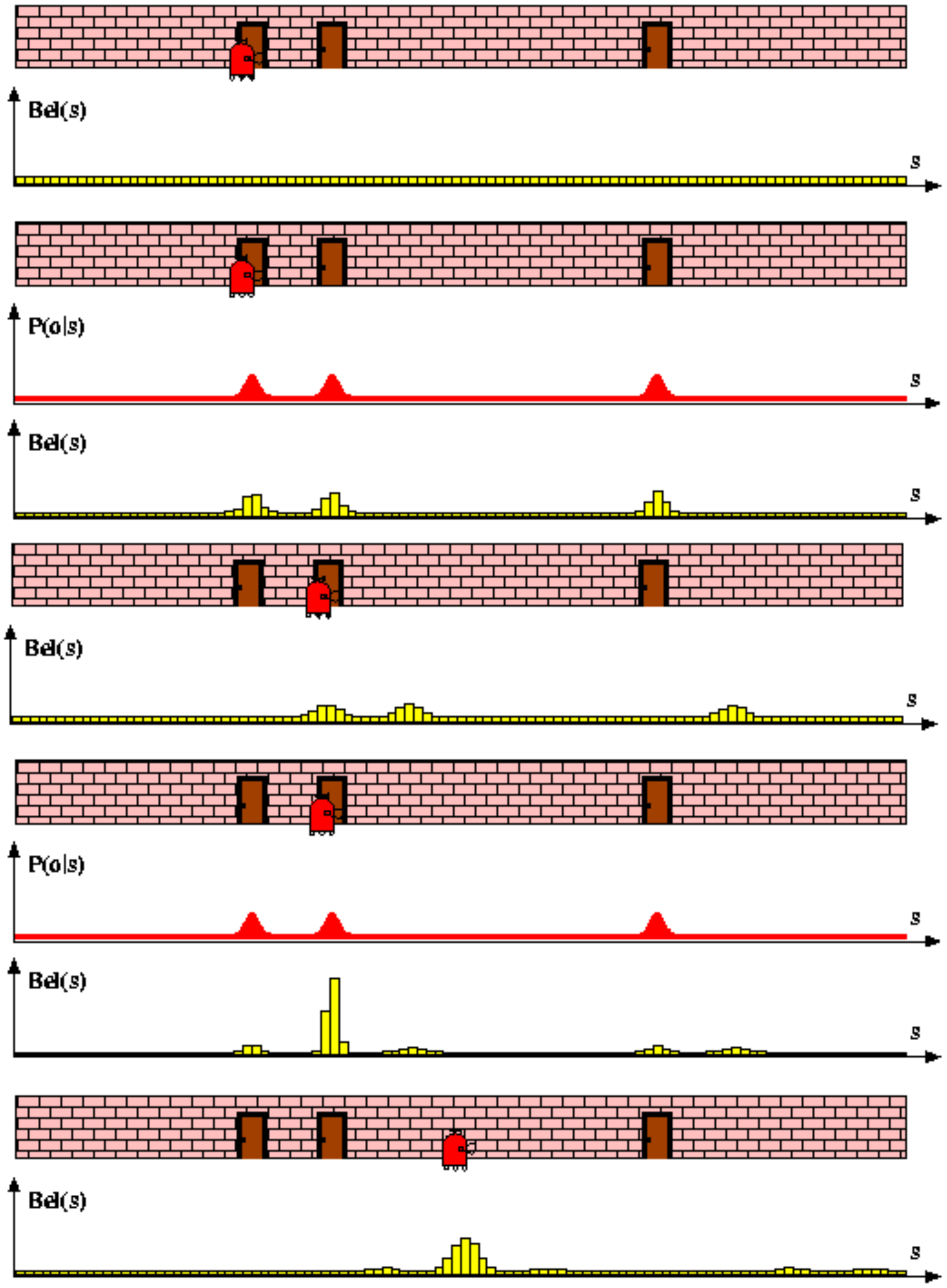
Bayes Filters in Action

- Discrete filters
- Kalman filters
- Particle filters

Discrete Filter

- The belief is typically stored in a histogram / grid representation
- To update the belief upon sensory input and to carry out the normalization one has to iterate over all cells of the grid

Piecewise Constant



Kalman Filter

- Optimal for linear Gaussian systems!
- Most robotics systems are nonlinear!
- Polynomial in measurement dimensionality k and state dimensionality n:

$$O(k^{2.376} + n^2)$$

Extended Kalman Filter

- Performs a linearization in each step
- Not optimal
- Can diverge if nonlinearities are large!
- Works surprisingly well even when all assumptions are violated!
- Same complexity than the KF

Particle Filter

- Basic principle
  - Set of state hypotheses ("particles")
  - Survival-of-the-fittest
- Particle filters are a way to efficiently represent non-Gaussian distribution

Mathematical Description

- Set of weighted samples

$$S = \left\{ \left\langle s^{[i]}, w^{[i]} \right\rangle \mid i = 1, \ldots, N \right\}$$

State hypothesis — Importance weight

- The samples represent the posterior

$$p(x) = \sum_{i=1}^{N} w_i \cdot \delta_{s^{[i]}}(x)$$

Particle Filter Algorithm in Brief

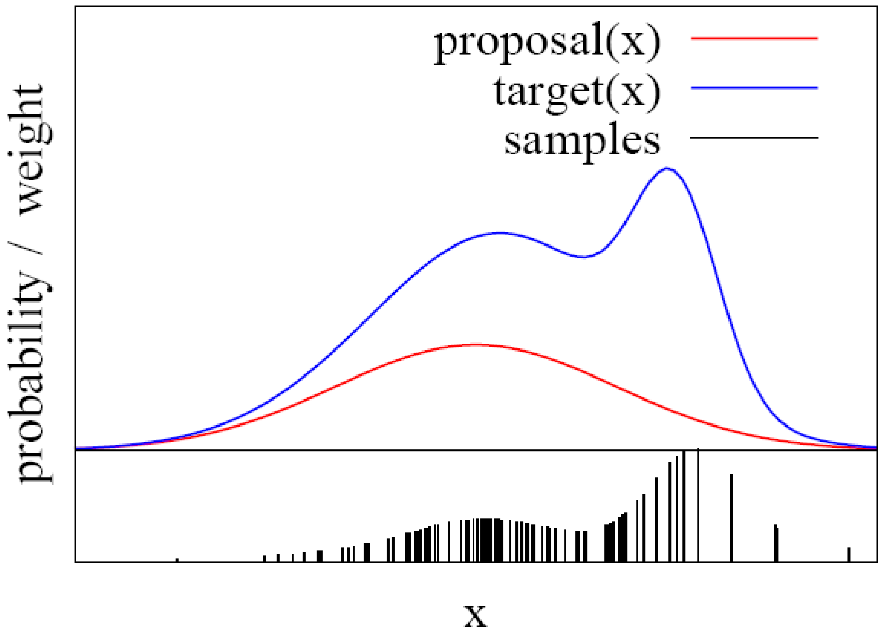
- Sample the next generation for particles using the proposal distribution

- Compute the importance weights :
  weight = target distribution / proposal distribution

- Resampling: “Replace unlikely samples by more likely ones”

Importance Sampling Principle

- We can even use a different distribution g to generate samples from f
- By introducing an importance weight w, we can account for the “differences between g and f ”
- $w = f / g$
- f is often called target
- g is often called proposal
- Pre-condition: $f(x)>0 \rightarrow g(x)>0$



Particle Filter Algorithm

1. Algorithm **particle_filter**(S_{t-1}, u_{t-1} z_t):
2. $S_t = \emptyset, \quad \eta = 0$
3. **For** $i = 1 \ldots n$ ***Generate new samples***
4. Sample index *j(i)* from the discrete distribution given by w_{t-1}
5. Sample x_t^i from $p(x_t \mid x_{t-1}, u_{t-1})$ using $x_{t-1}^{j(i)}$ and u_{t-1}
6. $w_t^i = p(z_t \mid x_t^i)$ ***Compute importance weight***
7. $\eta = \eta + w_t^i$ ***Update normalization factor***
8. $S_t = S_t \cup \{< x_t^i, w_t^i >\}$ ***Insert***
9. **For** $i = 1 \ldots n$
10. $w_t^i = w_t^i / \eta$ ***Normalize weights***

Particle Filter Algorithm

$$Bel(x_t) = \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_{t-1})\, Bel(x_{t-1})\, dx_{t-1}$$

draw x^i_{t-1} from $Bel(x_{t-1})$

draw x^i_t from $p(x_t \mid x^i_{t-1}, u_{t-1})$

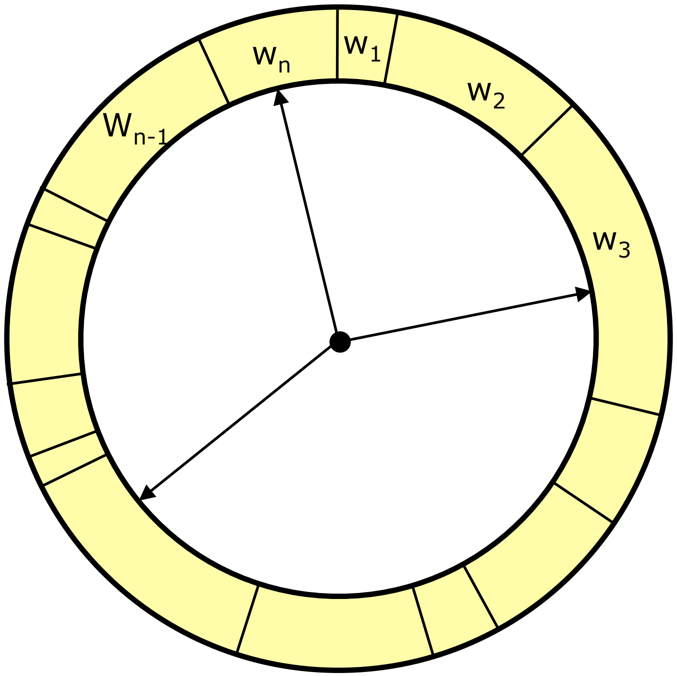
Importance factor for x^i_t:

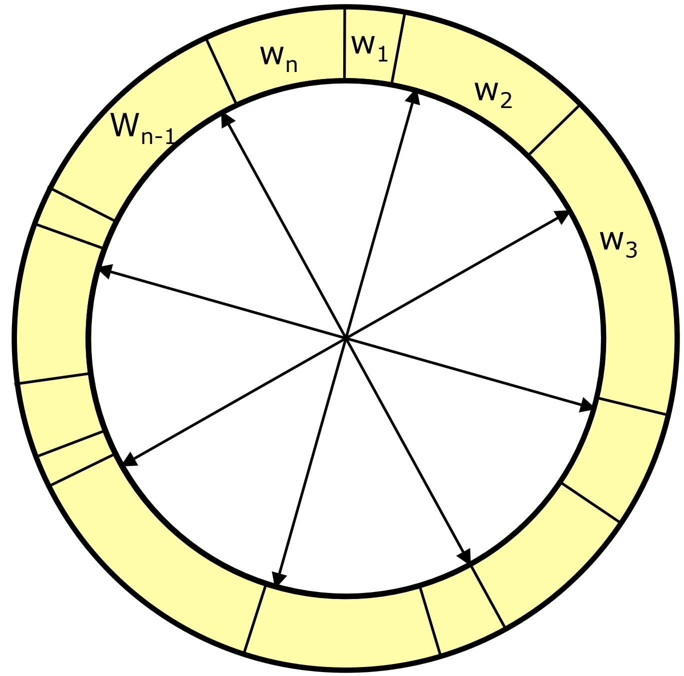
$$w^i_t = \frac{\text{target distribution}}{\text{proposal distribution}}$$

$$= \frac{\eta\, p(z_t \mid x_t)\; p(x_t \mid x_{t-1}, u_{t-1})\; Bel(x_{t-1})}{p(x_t \mid x_{t-1}, u_{t-1})\; Bel(x_{t-1})}$$

$$\propto p(z_t \mid x_t)$$

Resampling

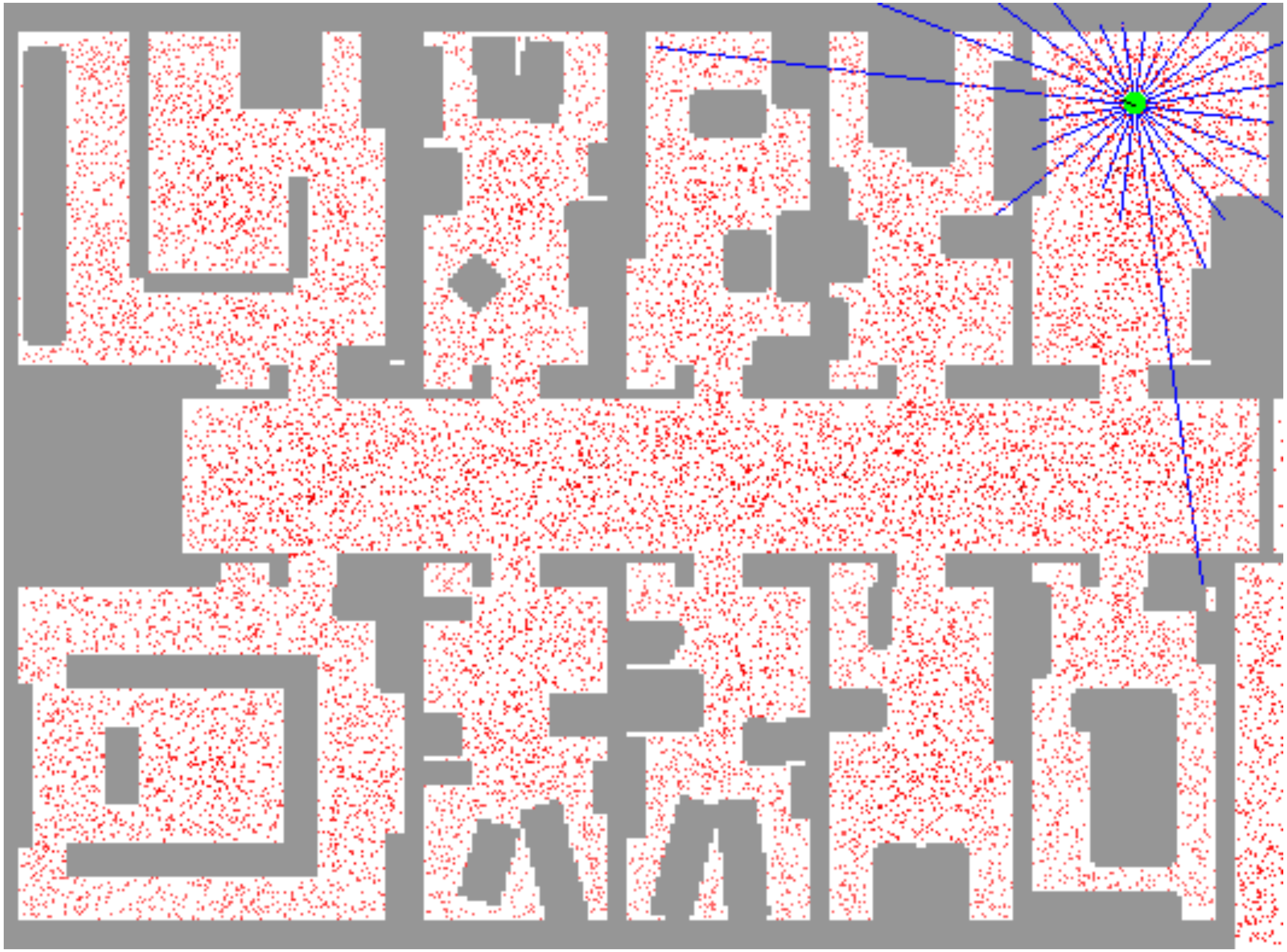




- Roulette wheel
- Binary search, n log n

- Stochastic universal sampling
- Systematic resampling
- Linear time complexity
- Easy to implement, low variance

MCL Example



Mapping

Why Mapping?

- Learning maps is one of the fundamental problems in mobile robotics
- Maps allow robots to efficiently carry out their tasks, allow localization …
- Successful robot systems rely on maps for localization, path planning, activity planning etc

Occupancy Grid Maps

- Discretize the world into equally spaced cells
- Each cells stores the probability that the corresponding area is occupied by an obstacle
- The cells are assumed to be conditionally independent
- If the pose of the robot is know, mapping is easy

Updating Occupancy Grid Maps

- Update the map cells using the inverse sensor model

$$Bel\left(m_t^{[xy]}\right) = 1 - \left(1 + \frac{P\left(m_t^{[xy]} \mid z_t, u_{t-1}\right)}{1 - P\left(m_t^{[xy]} \mid z_t, u_{t-1}\right)} \cdot \frac{1 - P\left(m_t^{[xy]}\right)}{P\left(m_t^{[xy]}\right)} \cdot \frac{Bel\left(m_{t-1}^{[xy]}\right)}{1 - Bel\left(m_{t-1}^{[xy]}\right)}\right)^{-1}$$

- Or use the log-odds representation

$$\begin{aligned}\bar{B}\left(m_t^{[xy]}\right) = & \log odds\left(m_t^{[xy]} \mid z_t, u_{t-1}\right) \\ & - \log odds\left(m_t^{[xy]}\right) \\ & + \bar{B}\left(m_{t-1}^{[xy]}\right)\end{aligned}$$

$$\bar{B}\left(m_t^{[xy]}\right) := \log odds(m_t^{[xy]})$$

$$odds(x) := \left(\frac{P(x)}{1 - P(x)}\right)$$

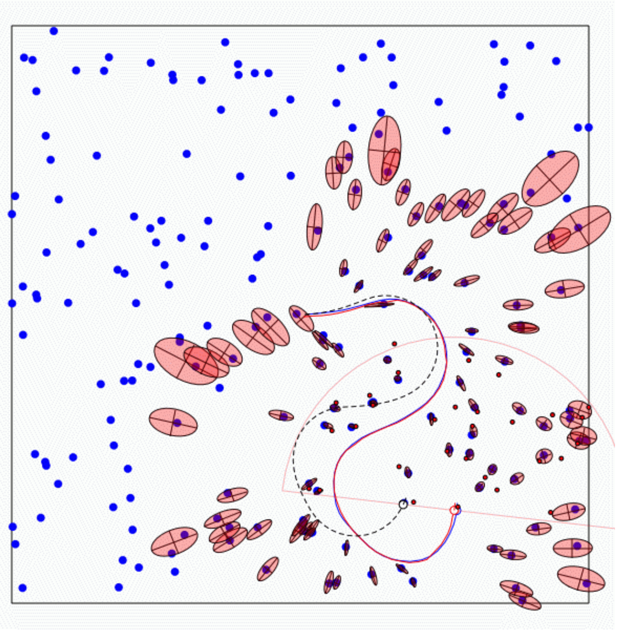
Reflection Probability Maps

- Value of interest: $P(reflects(x,y))$
- For every cell count
  - hits(x,y): number of cases where a beam ended at <x,y>
  - misses(x,y): number of cases where a beam passed through <x,y>

$$Bel(m^{[xy]}) = \frac{\text{hits}(x,y)}{\text{hits}(x,y) + \text{misses}(x,y)}$$

SLAM

The SLAM Problem



A robot is exploring an unknown, static environment.

Given:

- The robot's controls
- Observations of nearby features

Estimate:

- Map of features
- Path of the robot

Chicken-or-Egg

- SLAM is a chicken-or-egg problem
  - A map is needed for localizing a robot
  - A good pose estimate is needed to build a map
- Thus, SLAM is regarded as a hard problem in robotics
- A variety of different approaches to address the SLAM problem have been presented
- Probabilistic methods outperform most other techniques

SLAM:

Simultaneous Localization and Mapping

- Full SLAM: $p(x_{1:t}, m \mid z_{1:t}, u_{1:t})$

Estimates entire path and map!

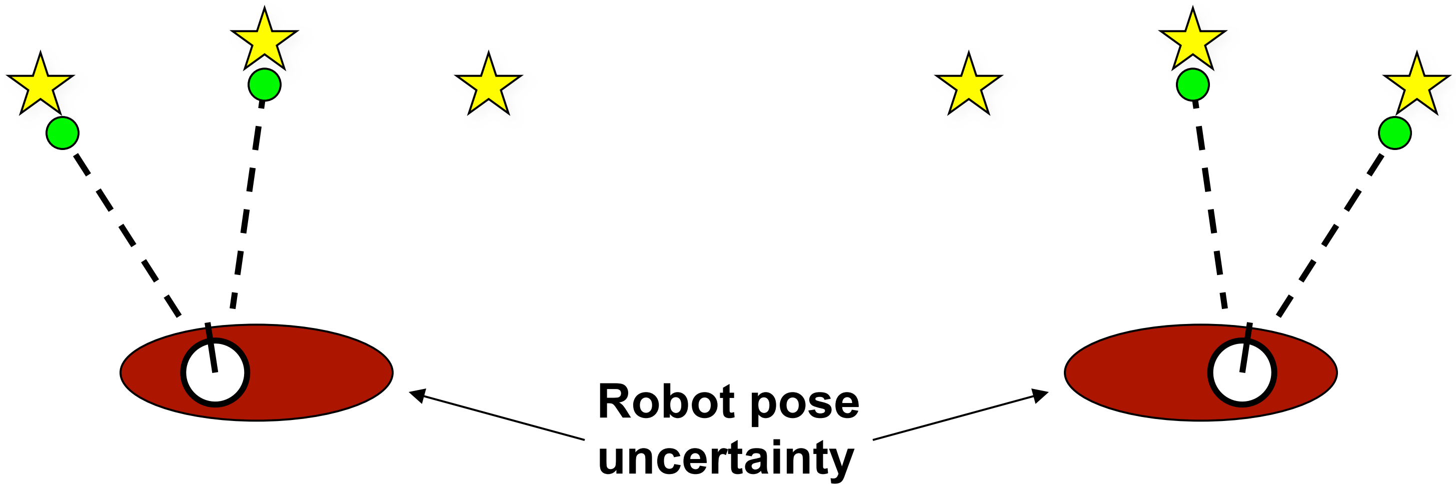
- Online SLAM:

$$p(x_t, m \mid z_{1:t}, u_{1:t}) = \int\int \ldots \int p(x_{1:t}, m \mid z_{1:t}, u_{1:t}) \, dx_1 dx_2 \ldots dx_{t-1}$$

Integrations typically done one at a time

Estimates most recent pose and map!

Why is SLAM a hard problem?



- In the real world, the mapping between observations and landmarks is unknown
- Picking wrong data associations can have catastrophic consequences
- Pose error correlates data associations

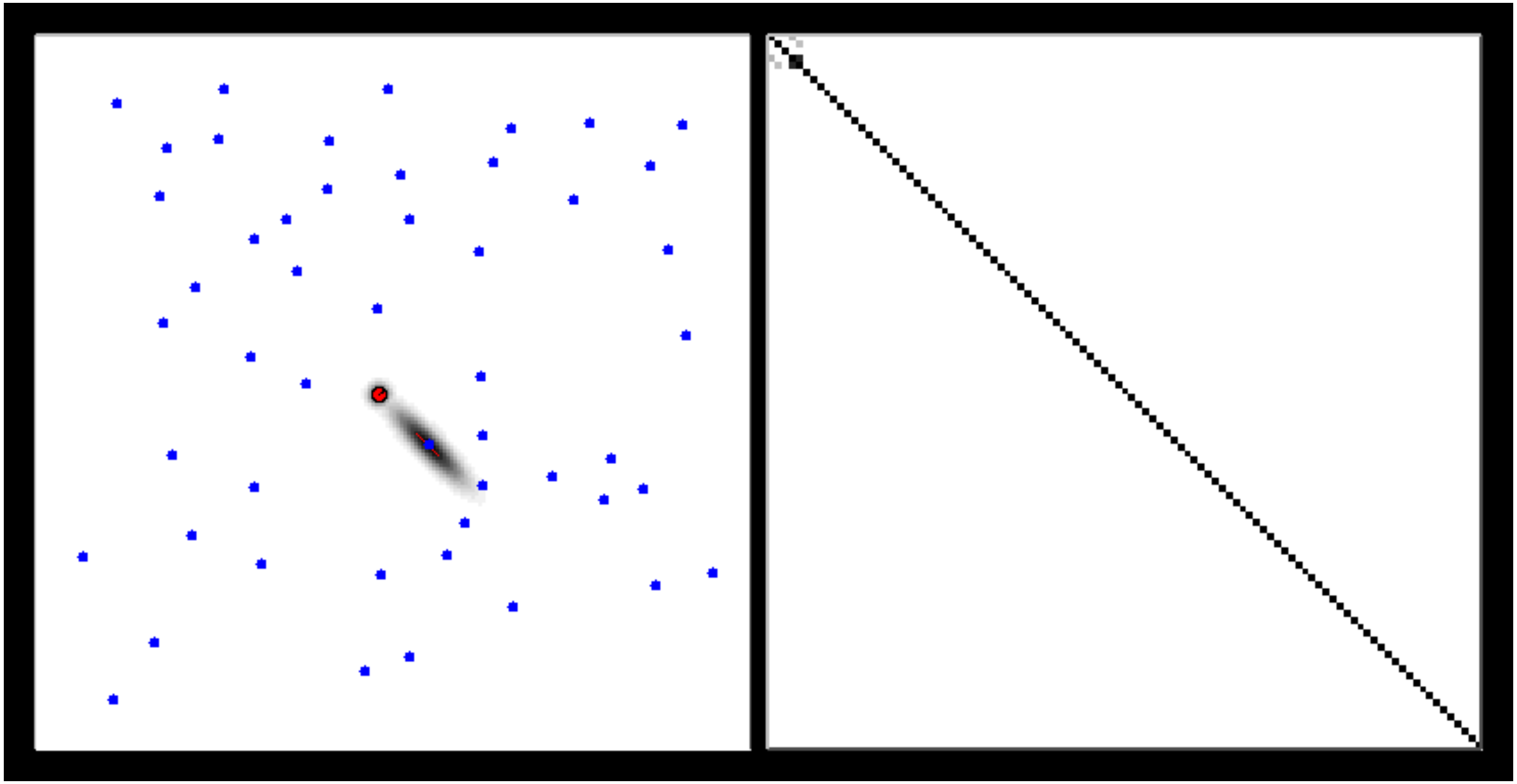
(E)KF-SLAM

- Map with N landmarks:(3+2N)-dimensional Gaussian

$$Bel(x_t, m_t) = \left\langle \begin{pmatrix} x \\ y \\ \theta \\ l_1 \\ l_2 \\ \vdots \\ l_N \end{pmatrix}, \begin{pmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xl_1} & \sigma_{xl_2} & \cdots & \sigma_{xl_N} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{y\theta} & \sigma_{yl_1} & \sigma_{yl_2} & \cdots & \sigma_{yl_N} \\ \sigma_{x\theta} & \sigma_{y\theta} & \sigma_\theta^2 & \sigma_{\theta l_1} & \sigma_{\theta l_2} & \cdots & \sigma_{\theta l_N} \\ \sigma_{xl_1} & \sigma_{yl_1} & \sigma_{\theta l_1} & \sigma_{l_1}^2 & \sigma_{l_1 l_2} & \cdots & \sigma_{l_1 l_N} \\ \sigma_{xl_2} & \sigma_{yl_2} & \sigma_{\theta l_2} & \sigma_{l_1 l_2} & \sigma_{l_2}^2 & \cdots & \sigma_{l_2 l_N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{xl_N} & \sigma_{yl_N} & \sigma_{\theta l_N} & \sigma_{l_1 l_N} & \sigma_{l_2 l_N} & \cdots & \sigma_{l_N}^2 \end{pmatrix} \right\rangle$$

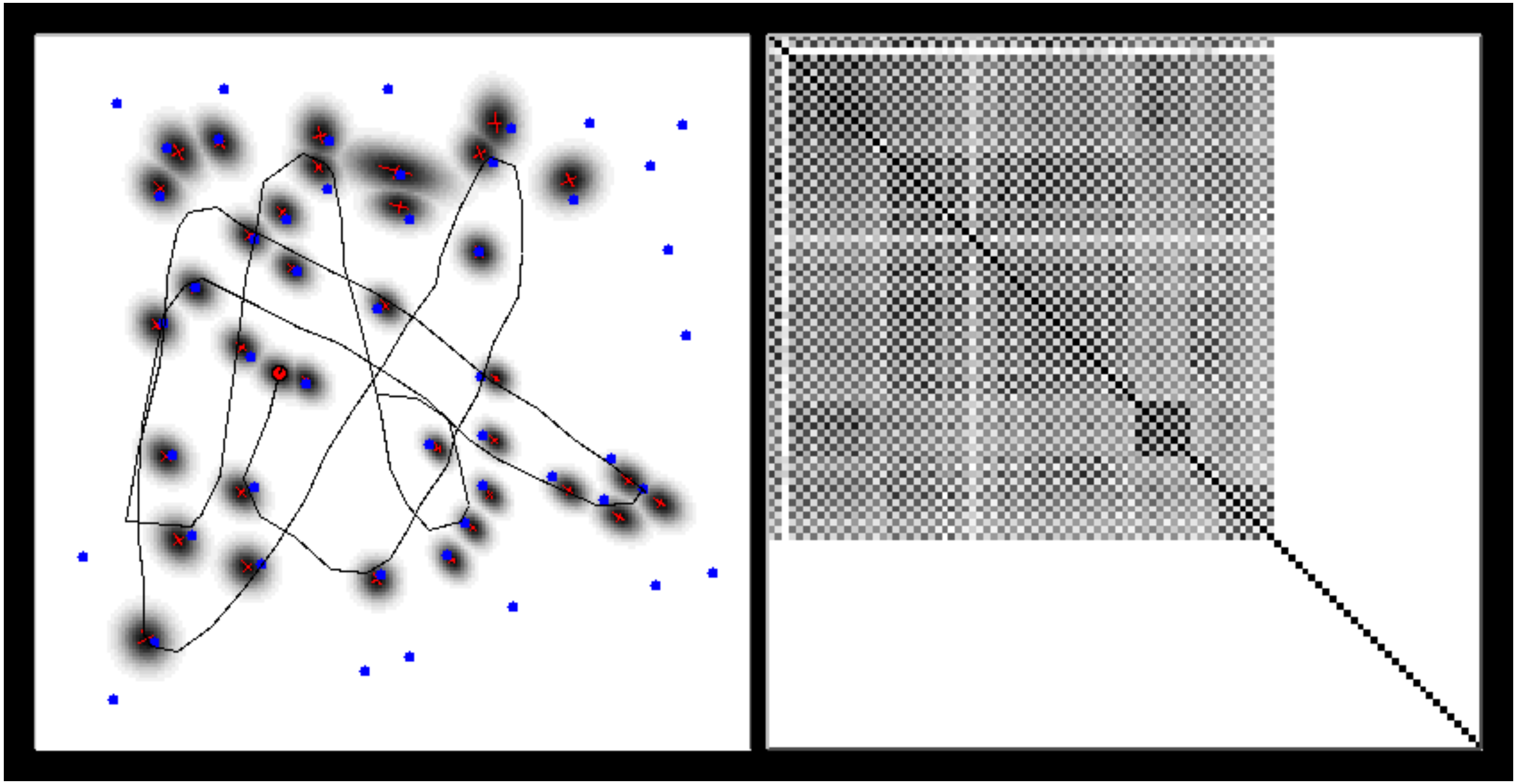
- Can handle hundreds of dimensions

EKF-SLAM



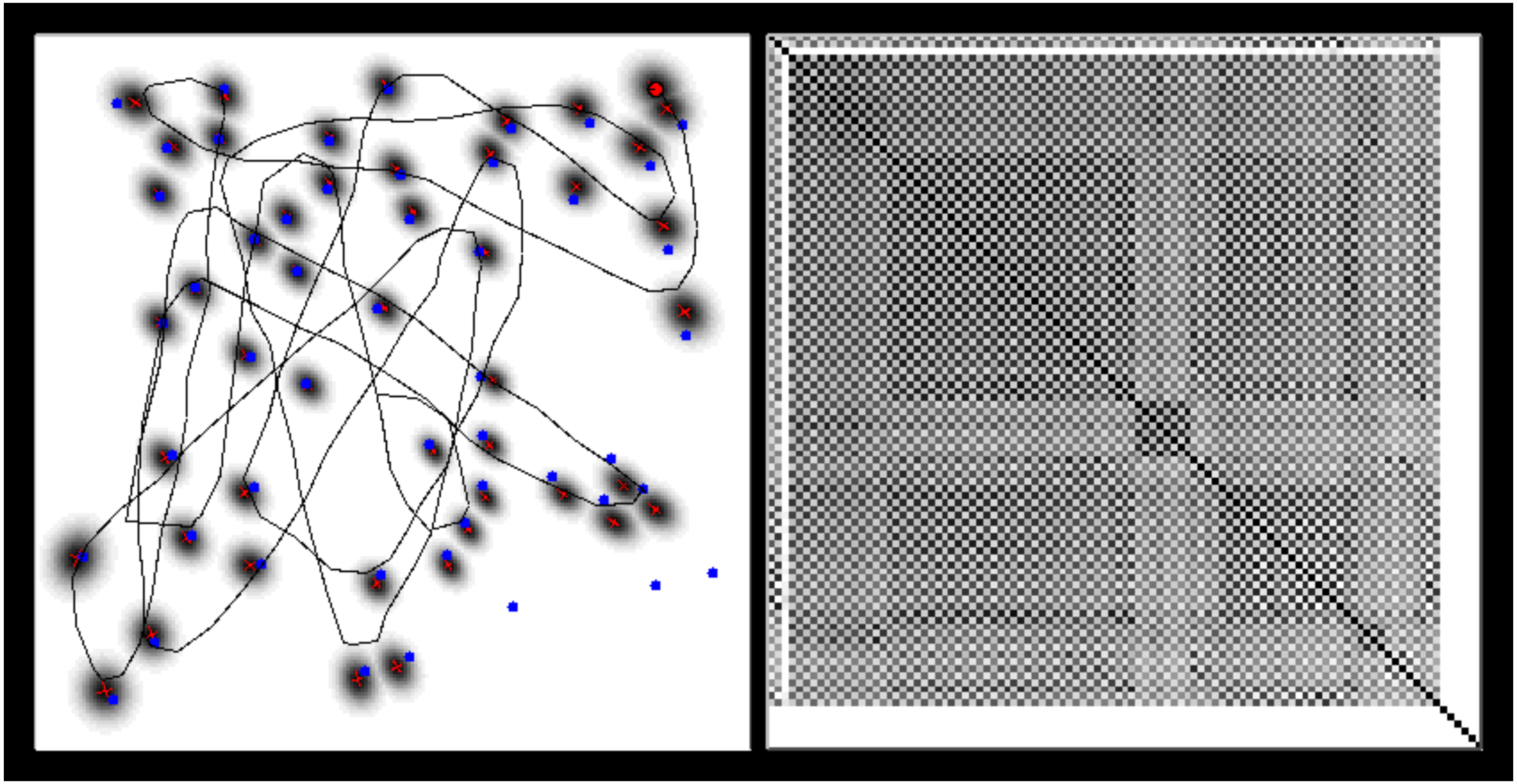
Map Correlation matrix

EKF-SLAM



Map Correlation matrix

EKF-SLAM



Map Correlation matrix

FastSLAM

- Use a particle filter for map learning
- Problem: the map is high-dimensional
- Solution: separate the estimation of the robot's trajectory from the one of the map of the environment
- This is done by means of a factorization in the SLAM posterior often called Rao-Blackwellization

Rao-Blackwellization

poses map observations & movements

$$p(x_{1:t}, m \mid z_{1:t}, u_{0:t-1}) =$$

$$p(x_{1:t} \mid z_{1:t}, u_{0:t-1}) \cdot p(m \mid x_{1:t}, z_{1:t})$$

SLAM posterior

Robot path posterior

Mapping with known poses

Factorization first introduced by Murphy in 1999

Rao-Blackwellized Mapping

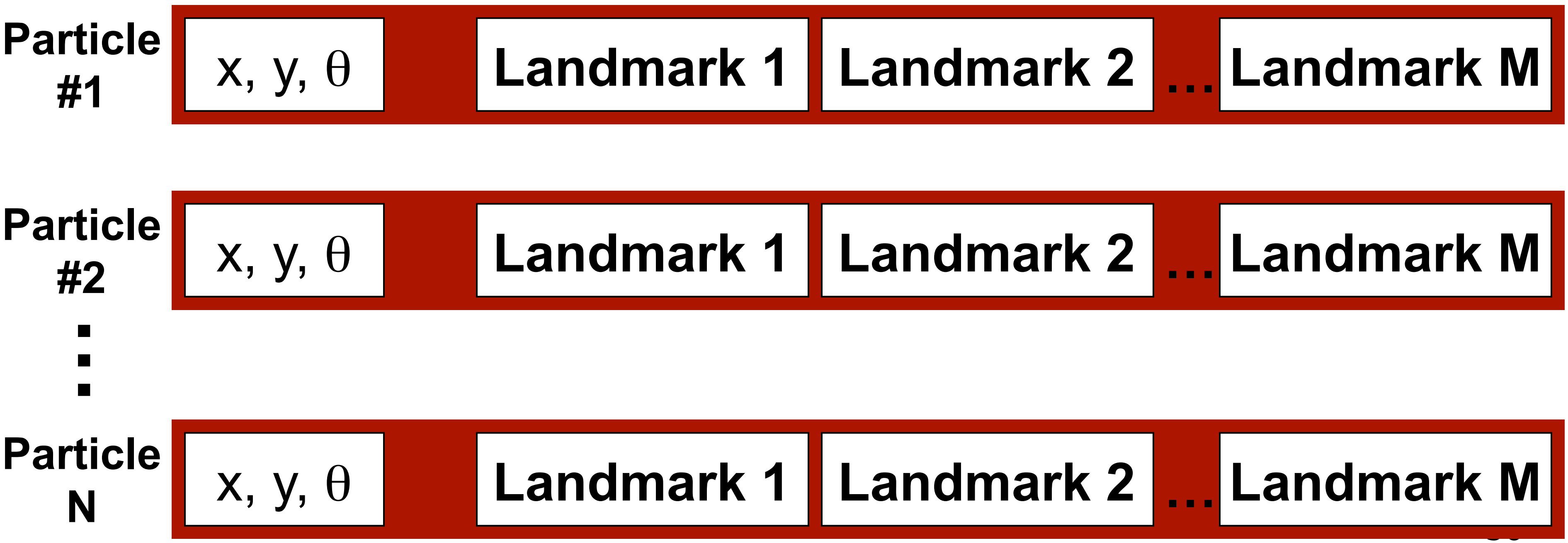
- Each particle represents a possible trajectory of the robot

- Each particle
  - maintains its own map and
  - updates it upon "mapping with known poses"

- Each particle survives with a probability proportional to the likelihood of the observations relative to its own map

FastSLAM

- Rao-Blackwellized particle filtering based on landmarks
- Each landmark is represented by a 2x2 Extended Kalman Filter (EKF)
- Each particle therefore has to maintain *M* EKFs



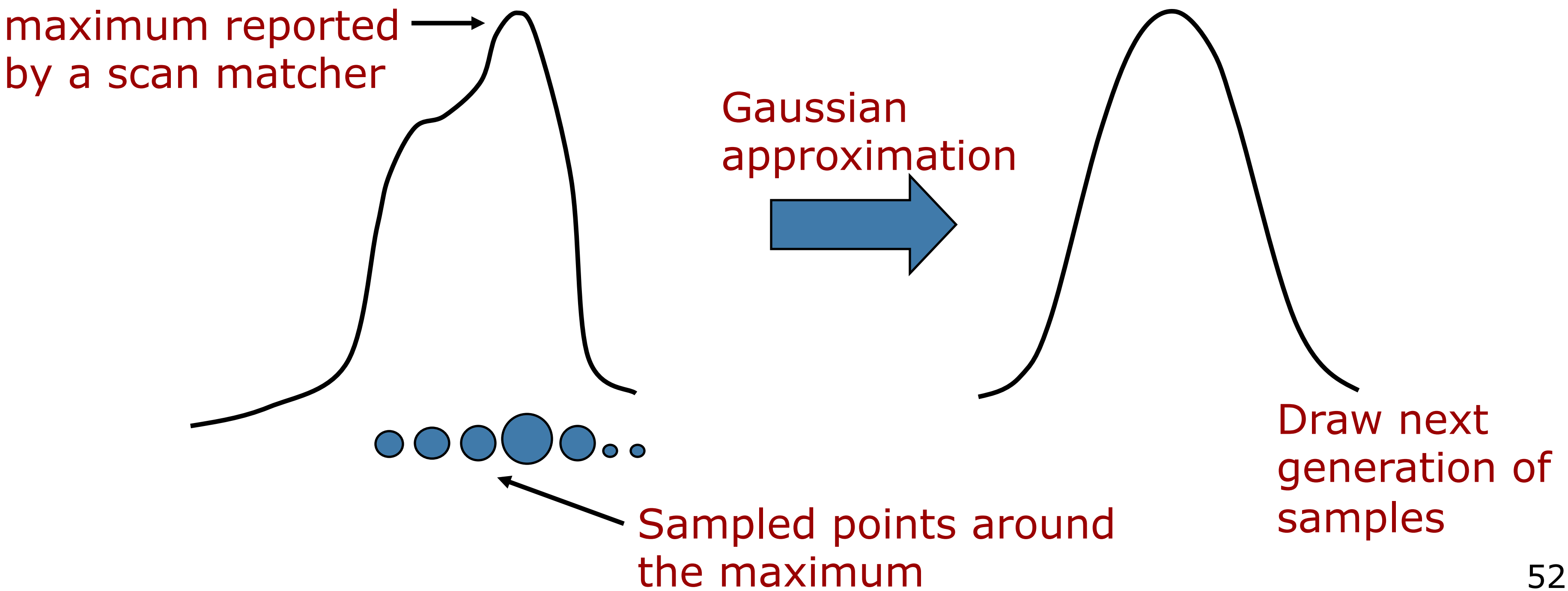
Grid-based FastSLAM

- Similar ideas can be used to learn grid maps
- To obtain a practical solution, an efficiently computable, informed proposal distribution is needed
- Idea: in the SLAM posterior, the observation model dominates the motion model (given an accurate sensor)

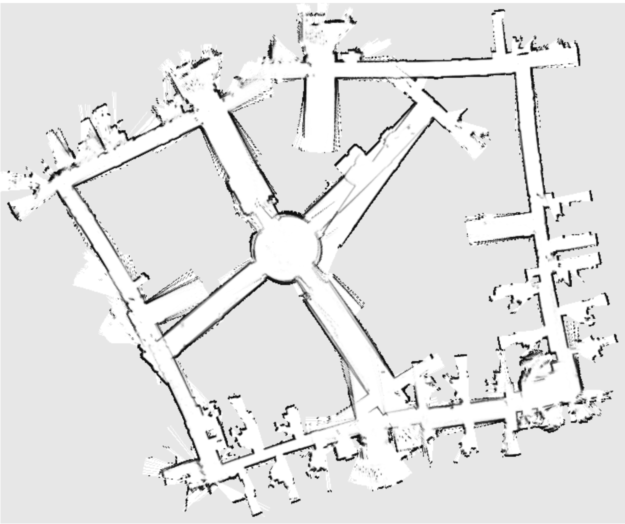
Proposal Distribution

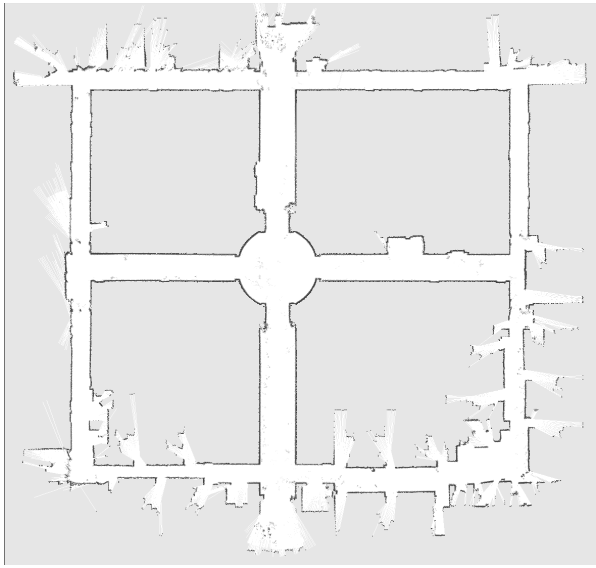
$$p(x_t|x_{t-1}^{(i)}, m^{(i)}, z_t, u_t) \simeq \frac{p(z_t|x_t, m^{(i)})}{\int_{x_t \in \{x|p(z_t|x,m^{(i)})>\epsilon\}} p(z_t|x_t, m^{(i)}) dx_t}$$

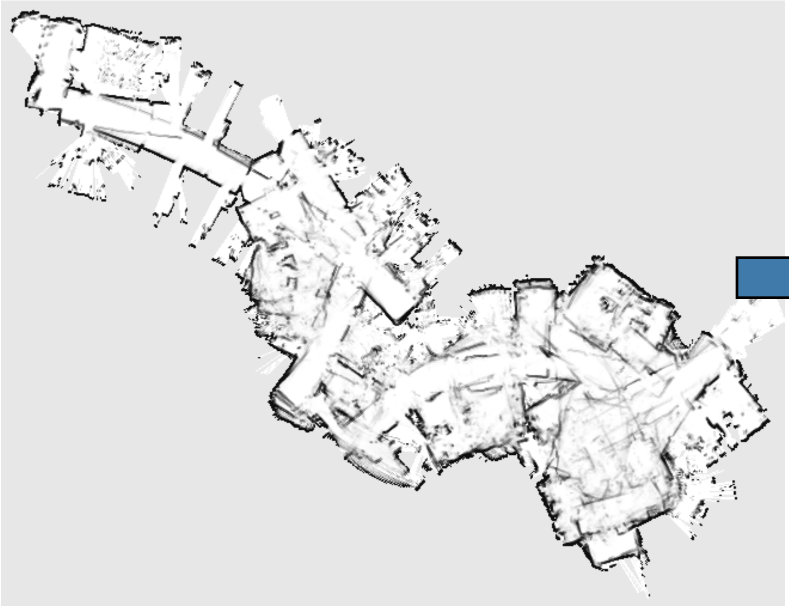
Approximate this equation by a Gaussian:



Typical Results







Robot Motion

Robot Motion Planning

Latombe (1991):

"…eminently necessary since, by definition, a robot accomplishes tasks by moving in the real world."

Goals:

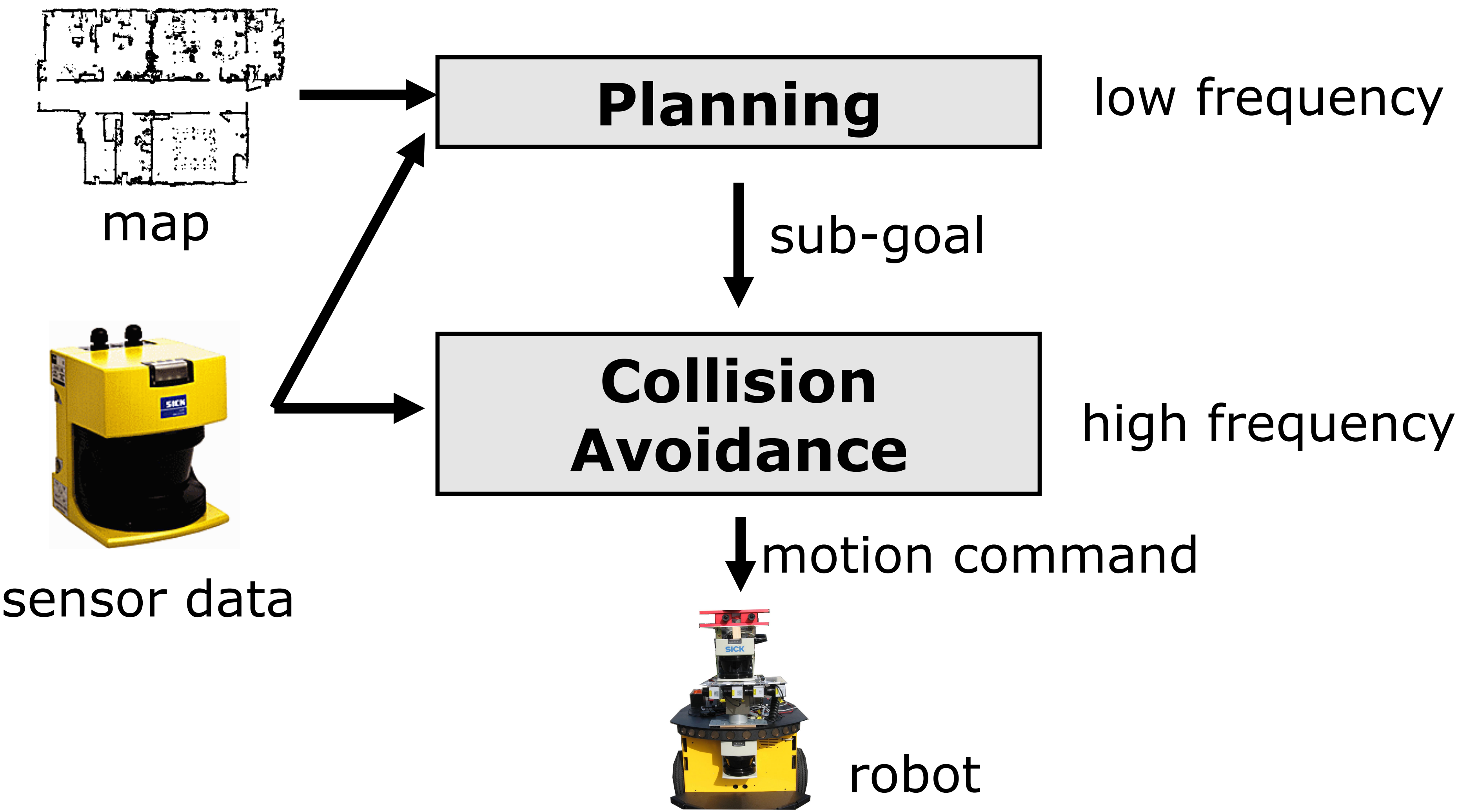
- Collision-free trajectories.
- Robot should reach the goal location as fast as possible.

Two Challenges

- Calculate the optimal path taking potential uncertainties in the actions into account

- Quickly generate actions in the case of unforeseen objects

Classic Two-layered Architecture



Multi-Robot Exploration

Given:

- Unknown environment
- Team of robots

Task:

- Coordinate the robots to efficiently learn a complete map of the environment

Complexity:

- NP-hard for single robots in known, graph-like environments
- Exponential in the number of robots

Levels of Coordination

- No exchange of information
- Implicit coordination: Sharing a joint map
  [Yamauchi et.al, 98]
  - Communication of the individual maps and poses
  - Central mapping system
- Explicit coordination: Determine better target locations to distribute the robots
  - Central planner for target point assignment

The Coordination Algorithm (informal)

1. Determine the frontier cells.
2. Compute for each robot the cost for reaching each frontier cell.
3. Choose the robot with the optimal overall evaluation and assign the corresponding target point to it.
4. Reduce the utility of the frontier cells visible from that target point.
5. If there is one robot left go to 3.

Information Gain-based Exploration

- SLAM is typically passive, because it consumes incoming sensor data
- Exploration actively guides the robot to cover the environment with its sensors
- Exploration in combination with SLAM: Acting under pose and map uncertainty
- Uncertainty should/needs to be taken into account when selecting an action
- Key question: **Where to move next?**

Mutual Information

- The mutual information I is given by the reduction of entropy in the belief

action to be carried out

$$I(X, M; Z^a) =$$ "uncertainty of the filter" – "uncertainty of the filter after carrying out action *a*"

Integrating Over Observations

- Computing the mutual information requires to integrate over potential observations

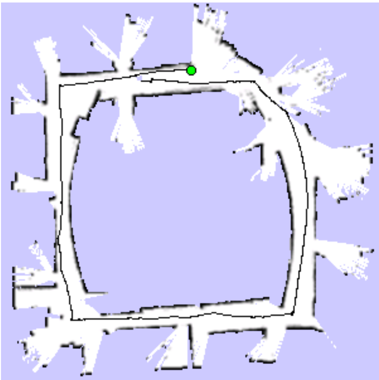
$$I(X,M;Z^a) = H(X,M) - H(X,M \mid Z^a)$$

$$H(X,M \mid Z^a) = \int_z p(z \mid a) H(X,M \mid Z^a = z) \, dz$$

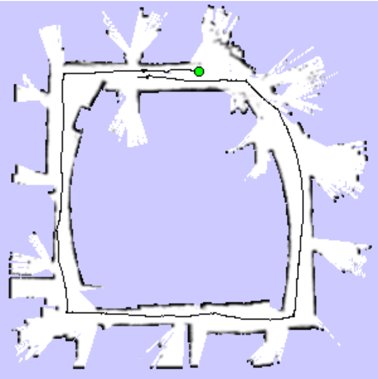
potential observation sequences

Integral Approximation

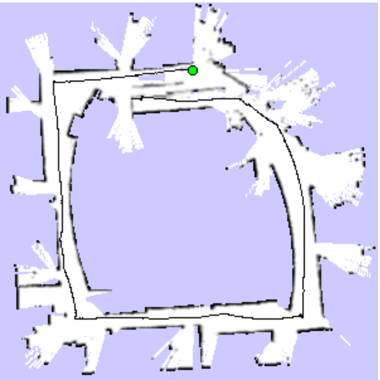
- The particle filter represents a posterior about possible maps



map of particle 1



map of particle 2



map of particle 3

...

Integral Approximation

- The particle filter represents a posterior about possible maps
- Simulate laser measurements in the maps of the particles

$$H(X,M \mid Z^a) = \sum_z p(z \mid a) H(X,M \mid Z^a = z)$$

measurement sequences simulated in the maps

likelihood (particle weight)

$$= \sum_i \omega^{[i]} H(X,M \mid Z^a = z_{sim_a}^{[i]})$$

Summary on Information Gain-based Exploration

- A decision-theoretic approach to exploration in the context of RBPF-SLAM
- The approach utilizes the factorization of the Rao-Blackwellization to efficiently calculate the expected information gain
- Reasons about measurements obtained along the path of the robot
- Considers a reduced action set consisting of exploration, loop-closing, and place-revisiting actions

The Exam is Approaching…

- This lecture gave a short overview over the most important topics addressed in this course
- For the exam, you need to know at least the basic formulas (e.g., Bayes filter, MCL eqs., Rao-Blackwellization, entropy, …)

Good luck for the exam!