Exercise 1: Implement the Potential Field Method

Consider a point-size robot driven by the Potential Field Method in an unbounded environment. The robots starts at location \((x = 0m, y = 0m)\) and should reach the goal location at \((x = 10m, y = 10m)\). The robot has a maximum velocity of \(0.5m/s\) and a mass \(m = 1kg\). The robot can adapt it’s velocities twice a second \((\Delta t = 0.5s)\). In the environment are a set of point-size obstacles introducing repulsive potentials. The obstacles are located at \((x_0 = 5m, y_0 = 5m)\), \((x_1 = 5m, y_1 = 7m)\), \((x_2 = 5m, y_2 = 3m)\) at \((x_3 = 8m, y_3 = 3m)\). Every repulsive potential has a radius of influence of \(3m\). The repulsive potential is given by:

\[
U_{\text{rep},i}(x) = \begin{cases} 
    m \cdot \left( \frac{1}{\rho_i(x)} - \frac{1}{\rho_0(x)} \right)^2, & \rho_i(x) \leq \rho_0 \\
    0, & \text{otherwise}
\end{cases}
\]

The attractive Potential around the goal location is defined as:

\[
U_{\text{att}}(x) = \begin{cases} 
    \frac{m}{2} \cdot \rho_{\text{goal}}^2(x), & \rho_{\text{goal}}(x) \leq 1m \\
    m \cdot \rho_{\text{goal}}(x), & \text{otherwise}
\end{cases}
\]

Here, the function \(\rho_i(x)\) is the euclidian distance from \(x\) to the obstacle \(i\). The update of the robot’s position can be computed using:

\[
x(t + 1) = x(t) + v(t) \cdot \Delta t + \frac{1}{2} \cdot a(x(t)) \cdot \Delta t^2.
\]  

(1)

where the acceleration is computed by the derivation of the sum over all potentials and a frictional force given by:

\[
F(v(t)) = -\frac{m}{\Delta t} \cdot v(t).
\]  

(2)

Write a C/C++ program that guides the robot from it’s initial location to the goal location.

Exercise 2: Kalman Filter

Consider the update-rule in one dimensional case. Proof that the variance converges to zero if the mean stays constant over time.