Exercise 1: Use A Kalman Filter To Observe Persons
Consider a laser range finder observing a walking person in an environment. The laser range finder provides only informations about the position of the observed person. A Kalman Filter is used to track the person. It should estimate the person’s position \((x, y)\) and velocity \((\dot{x}, \dot{y})\) (no action can be executed by the system, it just observes and estimates). Consider that a new measurements can be incorporated every \(\Delta t = 0.5s\).

Consider the notation given in “An Introduction to the Kalman Filter” by Welch and Bishop (see homepage).

(a) Specify the dimensions of the state vector.

(b) Specify the matrix A.

(c) Specify the matrix H.

Exercise 2: Kalman Filter Example
Consider the situation in Exercise 1 and use your definitions for \(A\) and \(H\). The initial state \(\hat{x}_0\) of the system is given by: \((x = 0.8, y = 0, \dot{x} = 0.4, \dot{y} = 0)^T\). The estimate error covariance matrix \(P^-\) and the measurement error covariance matrix \(R\) is given by:

\[
P^- = \begin{pmatrix} 0.5 & 0.2 & 0.3 & 0.3 \\ 0.2 & 0.5 & 0.3 & 0.3 \\ 0.3 & 0.3 & 0.5 & 0.2 \\ 0.3 & 0.3 & 0.2 & 0.5 \end{pmatrix}, \quad R = \begin{pmatrix} 0.05 & 0.0 \\ 0.0 & 0.05 \end{pmatrix}
\]

(1)

The Kalman Gain \(K\) is computed by: \(K = P^-H^T(HP^-H^T + R)^{-1}\). Since not everyone of you has access to MatLab, the result of this computation is:

\[
K = \begin{pmatrix} 0.8952 & 0.0381 \\ 0.0381 & 0.8952 \\ 0.4000 & 0.4000 \\ 0.4000 & 0.4000 \end{pmatrix}
\]

(2)

Consider that in this example the matrixes \(P^-\) and \(R\) stay constant over time and are not updated. The first measurement, which is taken after time \(\Delta t\) is \((x = 1, y = 0.1)\). Compute the next state of the system \(\hat{x}_1\).