Sheet 1
Submission deadline: Fri 29.04.2005, 11:15 a.m. (before class)

General Notice:
To be admitted to the final exam, every student has to submit solutions for at least 50% of the exercises and present one of the solutions in class. The exercises should be done in groups of two students.
One bonus point for the final exam is granted for every reasonable solution of a complete exercise sheet.

Exercise 1:
A robot equipped with a differential drive starts at the position $x = 0$, $y = 0$ and with heading $\alpha = \frac{\pi}{4}$ ($\frac{\pi}{2}$ is the direction of the $y$-axis). It has to move to the position $x = 300\text{cm}$, $y = 0$, $\alpha = \frac{\pi}{2}$. The movement of the vehicle is described by steering commands ($v_l =$ speed of left wheel, $v_r =$ speed of right wheel, $t =$ driving time).

(a) What is the minimal number of steering commands ($v_l, v_r, t$) needed to guide the vehicle to the desired target location?

(b) What is the length of the shortest trajectory under this constraint?

(c) Which sequence of steering commands guides the robot on the shortest trajectory to the desired location if an arbitrary number of steering commands can be used?

(d) What is the length of this trajectory?

Exercise 2:
The robot introduced in Exercise 1 starts at the position $x = 0$, $y = 0$ and with heading $\alpha = 0$ (0 is the direction of the $x$-axis). The distance between both wheels is $w = 50\text{cm}$. It executes the following sequence of steering commands:

$c_1 = (v_l = 30\text{cm/s}, v_r = 20\text{cm/s}, t = 3s),$
$c_2 = (v_l = 30\text{cm/s}, v_r = 30\text{cm/s}, t = 2s),$
$c_3 = (v_l = 10\text{cm/s}, v_r = -10\text{cm/s}, t = 1s).$

What is the position $(x, y, \alpha)$ of the robot after executing these commands?
Exercise 3:
Show that a robot equipped with a synchro-drive moves on a sequence of circular arcs, if for the rotational velocity $\omega$ holds $\omega \neq 0$. Then the circular arcs have the radius $r = \frac{v}{\omega}$.

Exercise 4:
The following equations define the motion model for a robot equipped with Mecanum wheels:

\begin{align*}
    v_y &= \frac{(v_0 + v_1 + v_2 + v_3)}{4} \\
    v_x &= \frac{(v_0 - v_1 + v_2 - v_3)}{4} \\
    v_y &= \frac{(v_0 + v_1 - v_2 - v_3)}{4} \\
    v_{error} &= \frac{(v_0 - v_1 - v_2 + v_3)}{4}
\end{align*}

(a) Derive the mapping of the four wheel velocities $v_1, v_2, v_3, v_4$ to the four individual wheels.

(b) For which values of the four wheel velocities $v_1, v_2, v_3, v_4$ does the robot rotate on the spot?