Exercise 1: Use A Kalman Filter To Observe Persons
Consider a laser range finder observing a walking person in an environment. The laser range finder provides only informations about the position of the observed person. A Kalman Filter is used to track the person. It should estimate the person’s position \((x, y)\) and velocity \((\dot{x}, \dot{y})\) (no action can be executed by the system, it just observes and estimates). Consider that a new measurements can be incorporated every \(\Delta t = 0.5s\).

(a) Specify the dimensions of the state vector.

(b) Specify the matrix \(A\).

(c) Specify the matrix \(C\).

Exercise 2: Kalman Filter Example
Consider the situation in Exercise 1 and use your definitions for \(A\) and \(C\). The initial state \(\hat{x}_0\) of the system is given by: \((x = 0.8, y = 0, \dot{x} = 0.4, \dot{y} = 0)^T\). The estimate error covariance matrix \(\Sigma\) and the measurement error covariance matrix \(Q\) is given by:

\[
\Sigma = \begin{pmatrix}
0.5 & 0.2 & 0.3 & 0.3 \\
0.2 & 0.5 & 0.3 & 0.3 \\
0.3 & 0.3 & 0.5 & 0.2 \\
0.3 & 0.3 & 0.2 & 0.5 \\
\end{pmatrix}, \quad Q = \begin{pmatrix}
0.05 & 0.0 \\
0.0 & 0.05 \\
\end{pmatrix}
\]  

(1)

The Kalman Gain \(K\) is computed by: \(K = \Sigma C^T (C \Sigma C^T + Q)^{-1}\). Since not everyone of you has access to MatLab, the result of this computation is:

\[
K = \begin{pmatrix}
0.8952 & 0.0381 \\
0.0381 & 0.8952 \\
0.4000 & 0.4000 \\
0.4000 & 0.4000 \\
\end{pmatrix}
\]  

(2)

Consider that in this example the matrixes \(\Sigma\) and \(Q\) stay constant over time and are not updated. The first measurement, which is taken after time \(\Delta t\) is \((x = 1, y = 0.1)\). Compute the next state of the system \(\hat{x}_1\).