Introduction to Mobile Robotics

Wheeled Locomotion

Locomotion of Wheeled Robots

Locomotion (Oxford Dict.): Power of motion from place to place

- Differential drive (AmigoBot, Pioneer 2-DX)
- Car drive (Ackerman steering)
- Synchronous drive (B21)
- Mecanum wheels, XR4000

Instantaneous Center of Curvature

- For rolling motion to occur, each wheel has to move along its y-axis

Differential Drive

\[ \text{ICC} = [x - R \sin \theta, y + R \cos \theta] \]

\[ \omega(R + l/2) = v_r \]
\[ \omega(R - l/2) = v_l \]
\[ R = \frac{l (v_l + v_r)}{2 (v_r - v_l)} \]
\[ \omega = \frac{v_r - v_l}{l} \]
**Differential Drive: Forward Kinematics**

\[
\begin{bmatrix}
    x' \\
    y' \\
    \theta'
\end{bmatrix} =
\begin{bmatrix}
    \cos(\omega \delta t) & -\sin(\omega \delta t) & 0 \\
    \sin(\omega \delta t) & \cos(\omega \delta t) & 0 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    x - ICC_x \\
    y - ICC_y \\
    \theta
\end{bmatrix}
+ 
\begin{bmatrix}
    ICC_x \\
    ICC_y \\
    \omega \delta t
\end{bmatrix}
\]

\[
x(t) = \int_0^t v(t') \cos[\theta(t')] dt'
\]

\[
y(t) = \int_0^t v(t') \sin[\theta(t')] dt'
\]

\[
\theta(t) = \int_0^t \omega(t') dt'
\]

**Ackermann Drive**

\[
ICC = [x - R \sin \theta, y + R \cos \theta]
\]

\[
R = \frac{d}{\tan \varphi}
\]

\[
\omega(R + l/2) = v_r
\]

\[
\omega(R - l/2) = v_l
\]

\[
R = \frac{l (v_l + v_r)}{2 (v_r - v_l)}
\]

\[
\omega = \frac{v_r - v_l}{l}
\]

**Synchuous Drive**

\[
x(t) = \int_0^t v(t') \cos[\theta(t')] dt'
\]

\[
y(t) = \int_0^t v(t') \sin[\theta(t')] dt'
\]

\[
\theta(t) = \int_0^t \omega(t') dt'
\]
**XR4000 Drive**

\[
x(t) = \int_0^t v(t') \cos[\theta(t')] dt'
\]
\[
y(t) = \int_0^t v(t') \sin[\theta(t')] dt'
\]
\[
\theta(t) = \int_0^t \omega(t') dt'
\]

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**Mecanum Wheels**

\[
v_y = \frac{(v_0 + v_1 + v_2 + v_3)}{4}
\]
\[
v_x = \frac{(v_0 - v_1 + v_2 - v_3)}{4}
\]
\[
v_\theta = \frac{(v_0 + v_1 - v_2 - v_3)}{4}
\]
\[
v_{\text{error}} = \frac{(v_0 - v_1 - v_2 + v_3)}{4}
\]

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**Example: Priamos**

[Priamos, Karlsruhe]
Example

Tracked Vehicle: Urban Robot

Tracked Vehicle: OmniTread

Odometry
Non-Holonomic Constraints

- Non-holonomic constraints limit the possible incremental movements within the configuration space of the robot.
- Robots with differential drive or synchro-drive move on a circular trajectory and cannot move sideways.
- XR-4000 or Mecanum-wheeled robots can move sideways.

Holonomic vs. Non-Holonomic

- Non-holonomic constraints reduce the control space with respect to the current configuration (e.g., moving sideways is impossible).
- Holonomic constraints reduce the configuration space.

Non-Holonomic Drives

- Synchro-drive
- Differential drive
- Ackerman drive