Introduction to Mobile Robotics

Iterative Closest Point Algorithm
Motivation
The Problem

• Given: two corresponding point sets:
  \[ X = \{x_1, \ldots, x_n\} \]
  \[ P = \{p_1, \ldots, p_n\} \]

• Wanted: translation \( t \) and rotation \( R \) that minimizes the sum of the squared error:
  \[
  E(R, t) = \frac{1}{N_p} \sum_{i=1}^{N_p} \|x_i - Rp_i - t\|^2
  \]

  Where \( x_i \) and \( p_i \) are corresponding points.
Key Idea

• If the correct correspondences are known, the correct relative rotation/translation can be calculated in closed form.
Center of Mass

\[ \mu_x = \frac{1}{N_x} \sum_{i=1}^{N_x} x_i \quad \text{and} \quad \mu_p = \frac{1}{N_p} \sum_{i=1}^{N_p} p_i \]

are the centers of mass of the two point sets.

Idea:

- Subtract the corresponding center of mass from every point in the two point sets before calculating the transformation.
- The resulting point sets are:
  \[ X' = \{ x_i - \mu_x \} = \{ x_i' \} \quad \text{and} \quad P' = \{ p_i - \mu_p \} = \{ p_i' \} \]
Let \( W = \sum_{i=1}^{N_p} x_i^T p_i^T \) denote the singular value decomposition (SVD) of \( W \) by:

\[
W = U \begin{bmatrix}
\sigma_1 & 0 & 0 \\
0 & \sigma_2 & 0 \\
0 & 0 & \sigma_3 \\
\end{bmatrix} V^T
\]

where \( U, V \in \mathbb{R}^{3 \times 3} \) are unitary, and \( \sigma_1 \geq \sigma_2 \geq \sigma_3 \) are the singular values of \( W \).
SVD

**Theorem** (without proof):

If \( \text{rank}(W) = 3 \), the optimal solution of \( E(R,t) \) is unique and is given by:

\[
R = UV^T \\
t = \mu_x - R\mu_p
\]

The minimal value of error function at \((R,t)\) is:

\[
E(R, t) = \sum_{i=1}^{N_p} (\|x_i'\|^2 + \|y_i'\|^2) - 2(\sigma_1 + \sigma_2 + \sigma_3)
\]
ICP with Unknown Data Association

- If correct correspondences are not known, it is generally impossible to determine the optimal relative rotation/translation in one step.
ICP-Algorithm

- Idea: iterate to find alignment
- Iterated Closest Points (ICP) [Besl & McKay 92]
- Converges if starting positions are “close enough”
Iteration-Example
ICP-Variants

- Variants on the following stages of ICP have been proposed:
  1. Point subsets (from one or both point sets)
  2. Weighting the correspondences
  3. Data association
  4. Rejecting certain (outlier) point pairs
Performance of Variants

• Various aspects of performance:
  • Speed
  • Stability (local minima)
  • Tolerance wrt. noise and/or outliers
  • Basin of convergence
    (maximum initial misalignment)

• Here: properties of these variants
ICP Variants

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
3. Data association
4. Rejecting certain (outlier) point pairs
Selecting Source Points

- Use all points
- Uniform sub-sampling
- Random sampling
- Feature based Sampling
- Normal-space sampling
  - Ensure that samples have normals distributed as uniformly as possible
Normal-Space Sampling

uniform sampling  normal-space sampling
Comparison

- Normal-space sampling better for mostly-smooth areas with sparse features [Rusinkiewicz et al.]
Feature-Based Sampling

- try to find “important” points
- decrease the number of correspondences
- higher efficiency and higher accuracy
- requires preprocessing
Application

[Nuechter et al., 04]
ICP Variants

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
3. Data association
4. Rejecting certain (outlier) point pairs
Selection vs. Weighting

• Could achieve same effect with weighting
• Hard to guarantee that enough samples of important features except at high sampling rates
• Weighting strategies turned out to be dependent on the data.
• Preprocessing / run-time cost tradeoff (how to find the correct weights?)
ICP Variants

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
3. Data association
4. Rejecting certain (outlier) point pairs
Data Association

• has greatest effect on convergence and speed
• Closest point
• Normal shooting
• Closest compatible point
• Projection
• Using kd-trees or oc-trees
Closest-Point Matching

- Find closest point in other the point set

Closest-point matching generally stable, but slow and requires preprocessing
Normal Shooting

• Project along normal, intersect other point set

Slightly better than closest point for smooth structures, worse for noisy or complex structures
Point-to-Plane Error Metric

- Using point-to-plane distance instead of point-to-point lets flat regions slide along each other [Chen & Medioni 91]
Projection

- Finding the closest point is the most expensive stage of the ICP algorithm
- Idea: simplified nearest neighbor search
- For range images, one can project the points according to the viewpoint [Blais 95]
Projection-Based Matching

• Slightly worse alignments per iteration
• Each iteration is one to two orders of magnitude faster than closest-point
• Requires point-to-plane error metric
Closest Compatible Point

• Improves the previous two variants by considering the compatibility of the points

• Compatibility can be based on normals, colors, etc.

• In the limit, degenerates to feature matching
ICP Variants

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
3. Nearest neighbor search
4. Rejecting certain (outlier) point pairs
Rejecting (outlier) point pairs

- sorting all correspondences with respect to their error and deleting the worst t%, Trimmed ICP (TrICP) [Chetverikov et al. 2002]
- t is to Estimate with respect to the Overlap

Problem: Knowledge about the overlap is necessary or has to be estimated
ICP-Summary

• ICP is a powerful algorithm for calculating the displacement between scans.
• The major problem is to determine the correct data associations.
• Given the correct data associations, the transformation can be computed efficiently using SVD.