Introduction to Mobile Robotics

Information Gain-Based Exploration Using Rao-Blackwellized Particle Filters

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Tasks of Mobile Robots

mapping

localization

integrated approaches

exploration

path planning

SLAM

active localization
Exploration and SLAM

- SLAM is typically \textit{passive}, because it consumes incoming sensor data
- Exploration \textit{actively guides the robot} to cover the environment with its sensors
- Exploration in combination with SLAM: Acting under pose and map uncertainty
- The uncertainty needs to be taken into account when selecting an action
Factorization Underlying Rao-Blackwellized Mapping

\[ p(x, m \mid z, u) \]

\[ = p(m \mid x, z, u)p(x \mid z, u) \]

Mapping with known poses

Particle filter representing trajectory hypotheses
Example

map of particle 1

map of particle 2

map of particle 3

3 particles
Combining Rao-Blackwellized Mapping with Exploration

- mapping
- localization
- integrated approaches
- exploration
- path planning
- active localization

SLAM
Exploration

- The approaches seen so far are purely passive

- By reasoning about control, the mapping process can be made much more effective

- Question: **Where to move next?**
Where to Move Next?
Decision-Theoretic Approach

- Learn the map using a Rao-Blackwellized particle filter
- Consider a set of potential actions
- Apply an exploration approach that minimizes the overall uncertainty

Utility = uncertainty reduction - cost
The Uncertainty of a Posterior

- Entropy is a general measure for the uncertainty of a posterior

\[
H(p(x)) = - \int x p(x) \log p(x) \, dx = Ex[- \log (p(x))] 
\]

- Information Gain = Uncertainty Reduction

\[
I(t + 1 \mid t) = H(p(x_t)) - H(p(x_{t+1}))
\]
Entropy Computation

\[ H(p(x, y)) \]
\[ = E_{x,y}[ - \log p(x, y)] \]
\[ = E_{x,y}[ - \log (p(x) \ p(y \mid x))] \]
\[ = E_{x,y}[ - \log p(x)] + E_{x,y}[ - \log p(y \mid x)] \]
\[ = H(p(x)) + \int_{x,y} -p(x, y) \log p(y \mid x) \ dx \ dy \]
\[ = H(p(x)) + \int_{x,y} -p(y \mid x)p(x) \log p(y \mid x) \ dx \ dy \]
\[ = H(p(x)) + \int_{x} p(x) \int_{y} -p(y \mid x) \log p(y \mid x) \ dy \ dx \]
\[ = H(p(x)) + \int_{x} p(x)H(p(y \mid x)) \ dx \]
Computing the Map and Pose Uncertainty

\[ H(p(x, m | d)) \]

\[ = H(p(x | d)) + \int_x p(x | d) H(p(m | x, d)) \, dx \]

\[ \approx H(p(x | d)) + \sum_{i=1}^{\#\text{particles}} \omega[i] H(p(m[i] | x[i], d)) \]

trajectory uncertainty  
particle weight  
map uncertainty
Computing the Entropy of the Map Posterior

Occupancy Grid map \( m \):

\[
H(p(m)) = - \sum_{c \in m} p(c) \log p(c) + (1 - p(c)) \log(1 - p(c))
\]

- map uncertainty
- grid cells
- probability that the cell is occupied
Computing the Entropy of the Trajectory Posterior

1. High-dimensional Gaussian

\[ H(\mathcal{G}(\mu, \Sigma)) = \log((2\pi e)^{(n/2)}|\Sigma|) \]

reduced rank for sparse particle sets

2. Grid-based approximation

\[ H(p(x|d)) \sim \text{const.} \]

for sparse particle clouds
Approximation of the Trajectory Posterior Entropy

Average pose entropy over time:

\[
H(p(x_{1:t} \mid d)) \approx \frac{1}{t} \sum_{t'=1}^{t} H(p(x_{t'} \mid d))
\]
**Information Gain**

- The reduction of entropy in the model

\[
I(\hat{z}, a) = H(p(m, x | d)) - H(p(m, x, \hat{x} | d, a, \hat{z}))
\]
Computing the Expected Information Gain

- To compute the information gain one needs to know the observations obtained when carrying out an action.

- This quantity is not known! Reason about potential measurements.

\[
E[I(a)] = \int_{\tilde{z}} p(\tilde{z} \mid a, d) \cdot I(\tilde{z}, a) \, d\tilde{z}
\]
Reasoning about Measurements

- The filter represents a posterior about possible maps
- Use these maps to reason about possible observation
- Simulate laser measurements in the maps of the particles

\[
E[I(a)] = \int \hat{z} p(\hat{z} \mid a, d) \cdot I(\hat{z}, a) \, d\hat{z}
\]

measurement sequences simulated in the maps
likelihood (particle weight)
Reasoning about Measurements

- Ray-casting in the map of each particle to generate observation sequences

map of particle i

planned trajectory (action)

pose of particle i while carrying out the action

simulated scan
The Utility

- To take into account the cost of an action, we compute a utility

\[ U(a) = I(a) - \alpha \cdot \text{cost}(a) \]

- Select the action with the highest expected utility

\[ a^* = \arg\max_a \{ E[U(a)] \} \]
Focusing on Specific Actions

To efficiently sample actions we consider

- exploratory actions (1-3)
- loop closing actions (4) and
- place revisiting actions (5)
Dual Representation for Loop Detection

- **Trajectory graph** ("topological map") stores the path traversed by the robot
- **Occupancy grid** map represents the space covered by the sensors

- **Loops** correspond to long paths in the trajectory graph and short paths in the grid map
Example: Trajectory Graph
Application Example

high pose uncertainty
Example: Possible Targets

timestep 35
Example: Evaluate Targets
Example: Move Robot to Target

![Graph showing expected utility at different target locations]

- **Expected Utility**
  - Decision at timestep 35
  - Target location
  - Graph bars indicating utility levels

- **Diagram**
  - Robot
  - Start point
  - Path taken by the robot

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27
Example: Evaluate Targets

timestep 70

robot

expected utility

decision at timestep 70

target location
Example: Move Robot

... continue ...
Example: Entropy Evolution
Comparison

Map uncertainty only:

After loop closing action:
Real Exploration Example
Corridor Exploration
Summary

- A decision-theoretic approach to exploration in the context of RBPF-SLAM
- The approach utilizes the factorization of the Rao-Blackwellization to efficiently calculate the expected information gain
- Reasons about measurements obtained along the path of the robot
- Considers a reduced action set consisting of exploration, loop-closing, and place-revisiting actions
- Experimental results demonstrate the usefulness of the overall approach