Foundations of AI

13. Planning

Planning in the Situation Calculus, STRIPS Formalism, Non-Linear Planning, Graphplan, Heuristic Search Planning

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Planning

- Given an *logical description* of the *initial situation*,
- a *logical description* of the *goal conditions*, and
- a *logical description* of a set of *possible actions*,

→ find a *sequence of actions* (a *plan of action*) that brings us from the initial situation to a situation in which the goal conditions hold.
Planning vs. Problem-Solving

Basic difference: *Explicit, logic-based representation*

- **States/Situations**: Through descriptions of the world by logical formula vs. data structures
  This way, the agent can explicitly think about and communicate

- **Goal conditions** as logical formulae vs. goal test (black box)
  The agent can also reflect on its goals.

- **Operators**: Axioms or transformation on formulae vs. modification of data structures by programs
  The agent can gain information about the effects of actions by inspecting the operators.
Planning vs. Automatic Programming

Difference between planning and automatic programming:

- Logic-based description of the world.

- Plans are usually only linear programs (no control structures).
Planning vs. MDP Policy Determination

- Simpler model: deterministic vs. probabilistic and goals vs. utilities
- More abstract, implicit state space description: logical description instead of explicit enumeration of states
- Can handle much larger state spaces!
Planning as Logical Inference (1)

Planning can be elegantly formalized with the help of the situation calculus.

Initial state:

\[ \text{At(Home,} s_0\text{)} \land \neg \text{Have(milk,} s_0\text{)} \land \neg \text{Have(banana,} s_0\text{)} \land \neg \text{Have(drill,} s_0\text{)} \]

Operators (successor-state axioms):

\[ \forall a, s \ \text{Have(milk, } do(a, s)\text{)} \iff \{ a = \text{buy(milk)} \land \text{Poss(buy(milk), s)} \land \text{Have(milk, s)} \land a \neq \text{drop(milk)}\} \]

Goal conditions (query):

\[ \exists s \text{ At(} \text{home, } s\text{)} \land \text{Have(milk, } s\text{)} \land \text{Have(banana,} s\text{)} \land \text{Have(drill,} s\text{)} \]

When the initial state, all prerequisites and all successor-state axioms are given, the constructive proof of the existential query delivers a plan that does what is desired.
Planning as Logical Inference (2)

The variable bindings for $s$ could be as follows:

$$\text{do(go(home), do(buy(drill), do(go(hardware_store), do(buy(banana), do(buy(milk), do(go(supermarket), s0))))})$$

I.e. the plan (term) would be

$$\langle \text{go(super\_market}, \text{buy(milk)}, \ldots \rangle$$

However, the following plan is also correct:

$$\langle \text{go(super\_market}, \text{buy(milk)}, \text{drop(milk)}, \text{buy(milk)}, \ldots \rangle$$

In general, planning by theorem proving is very inefficient

**Specialized inference system** for limited representation.

→ **Planning algorithm**
The STRIPS Formalism

STRIPS: STanford Research Institute Problem Solver (early 70s)

The system is obsolete, but the formalism is still used. Usually simplified version is used:

**World state** (including initial state): Set of ground atoms (called fluents), no function symbols except for constants, interpreted under closed world assumption (**CWA**). Sometimes also standard interpretation, i.e. negative facts must be explicitly given

**Goal conditions**: Set of ground atoms

Note: No explicit state variables as in sitation calculus. Only the current world state is accessible.
STRIPS Operators

Operators are triples, consisting of

Action Description: Function name with parameters (as in situation calculus)

Preconditions: Conjunction of positive literals; must be true before the operator can be applied (after instantiation)

Effects: Conjunction of positive and negative literals; positive literals are added (ADD list), negative literals deleted (DEL list) (no frame problem!).

\[
\text{Op}(
\begin{align*}
\text{Action: } & \text{Go}(\text{here, there}), \\
\text{Precond: } & \text{At}(\text{here}) \land \text{Path}(\text{here, there}), \\
\text{Effect: } & \text{At}(\text{there}) \land \neg \text{At}(\text{here})
\end{align*}
)\
\]
**Actions and Executions**

- An **action** is an operator, where all variables have been *instantiated*:

  \[ \text{Op}( \text{Action: } \text{Go(Home, SuperMarket)}, \text{Precond: } \text{At(Home)} \land \text{Path(Home, SuperMarket)}, \text{Effect: } \text{At(Supermarket)} \land \neg \text{At(Home)}) \]

- An action can be **executed** in a state, if its precondition is satisfied. It will then bring about its effects
Linear Plans

- A sequence of actions is a plan
- For a given initial state $I$ and goal conditions $G$, such a plan $P$ can be successfully executed in $I$ iff there exists a sequence of states $s_0, s_1, ..., s_n$ such that
  - the $i$th action in $P$ can be executed in $s_{i-1}$ and results in $s_i$
  - $s_0 = I$ and $s_n$ satisfies $G$
- $P$ is called a solution to the planning problem specified by the operators, $I$ and $G$
Searching in the State Space

We can now search through the state space (the set of all states formed by truth assignments to fluents) – and in this way reduce planning to searching.

We can search forwards (progression planning):

Or alternatively, we can start at the goal and work backwards (regression planning).

Possible since the operators provide enough information
Searching in the Plan Space

Instead of searching in the state space, we can search in the **space of all plans**.

The initial state is a **partial plan** containing only start and goal states:

Operators in the plan space:

**Refinement operators** make the plan more complete (more steps etc.)

**Modification operators** modify the plan (in the following, we use only refinement operators)
Plan = Sequence of Actions?

Often, however, it is neither meaningful nor possible to commit to a specific order early-on (put on socks and shoes).

→ Non-linear or partially-ordered plans (least-commitment planning)
Representation of Non-Linear Plans

A plan step = STRIPS operator

A plan consists of

- A set of plan steps with partial ordering (<), where $S_i < S_j$ implies $S_i$ must be executed before $S_j$.
- A set of variable assignments $x = t$, where $x$ is a variable and $t$ is a constant or a variable.
- A set of causal relationships $S_i \rightarrow S_j$ means “$S_i$ produces the precondition c for $S_j$” (implies $S_i < S_j$).

Solutions to planning problems must be complete and consistent.
Completeness and Consistency

Complete Plan:
Every precondition of a step is fulfilled:

\[ \forall S_j \forall c \in \text{Precond}(S_j) : \]

\[ \exists S_i \text{ with } S_i < S_j \text{ and } c \in \text{Effects}(S_i) \text{ and} \]

for every linearization of the plan:
\[ \forall S_k \text{ with } S_i < S_k < S_j, \neg c \notin \text{Effect}(S_k). \]

Consistent Plan:

if \( S_i < S_j \), then \( S_j \not\sqsubseteq S_i \) and
if \( x = A \), then \( x \neq B \) for distinct \( A \) and \( B \) for a variable \( x \).
(\text{unique name assumption } = \text{UNA})

A complete, consistent plan is called a solution to a planning problem (all linearizations are executable linear plans).
Example

Actions:

Op(Action: Go(here, there),
   Precond: At(here) \implies Path(here, there),
   Effect: At(there) \implies \neg At(here))

Op(Action: Buy(store, x),
   Precond: At(store) \implies Sells(store, x),
   Effect: Have(x))

Note: there, here, x, store are variables.

Note: In figures, we may just write Buy(Banana) instead of Buy(SM, Banana)
Plan Refinement (1)

Regression Planning: Fulfils the Have predicates:

... after instantiation of the variables:

Thin arrow = <, thick arrow = causal relationship + <
Plan Refinement (2)

Shop at the right store...
Plan Refinement (3)

First, you have to go there...

Note: So far no searching, only simple backward chaining.

Now: Conflict! If we have done go(HWS), we are no longer At(home). Likewise for go(SM).
Protection of Causal Links

(a) Conflict: $S_3$ threatens the causal relationship between $S_1$ and $S_2$.

Conflict solutions:

(b) **Demotion**: Place the threatening step before the causal relationship.

(c) **Promotion**: Place the threatening step after the causal relationship.
A Different Plan Refinement...

- We cannot resolve the conflict by “protection”.
  - It was a mistake to choose to refine the plan.
- **Alternative:** When instantiating $\alpha(t(x))$ in $go(SM)$, choose $x = HWS$ (with causal relationship)
- **Note:** This threatens the purchase of the drill $\rightarrow$ promotion of $go(SM)$. 

![Plan Diagram]
The Complete Solution
The POP Algorithm

function POP(initial, goal, operators) returns plan

plan ← Make-Minimal-Plan(initial, goal)
loop do
  if SOLUTION?(plan) then return plan
  Sneed, c ← SELECT-SUBGOAL(plan)
  CHOOSE-OPERATOR(plan, operators, Sneed, c)
  RESOLVE-THREATS(plan)
end

function SELECT-SUBGOAL(plan) returns Sneed, c

pick a plan step Sneed from STEPS(plan)
  with a precondition c that has not been achieved
return Sneed, c

procedure CHOOSE-OPERATOR(plan, operators, Sneed, c)

choose a step Sadd from operators or STEPS(plan) that has c as an effect
if there is no such step then fail
add the causal link Sadd \rightarrow Sneed to LINKS(plan)
add the ordering constraint Sadd \rightarrow Sneed to ORDERINGS(plan)
if Sneed is a newly added step from operators then
  add Sneed to STEPS(plan)
  add Start \rightarrow Sadd \rightarrow Finish to ORDERINGS(plan)

procedure RESOLVE-THREATS(plan)

for each Sthreat that threatens a link S_i \rightarrow S_j in LINKS(plan) do
  choose either
  Promotion: Add Sthreat \rightarrow S_i to ORDERINGS(plan)
  Demotion: Add S_j \rightarrow Sthreat to ORDERINGS(plan)
if not CONSISTENT(plan) then fail
end
Properties of the POP Algorithm

Correctness:
Every result of the POP algorithm is a complete, correct plan.

Completeness:
If breadth-first-search is used, the algorithm finds a solution, given one exists.

Systematicity:
Two distinct partial plans do not have the same total ordered plans as a refinement provided the partial plans are not refinements of one another (and totally ordered plans contain causal relationships).

Problems:
Informed choices are difficult to make & data structure is expensive

→ Instantiation of variables is not addressed.
New Approaches

Since 1995, a number of new algorithmic approaches have been developed, which are much faster than the POP algorithm:

- Planning based on planning graphs
- Satisfiability based planning
- BDD-based approaches (good for multi-state problems – which we ignore here)
- Heuristic-search based planning
Planning Graphs

- **Parallel** execution of actions possible
- Assumption: Only **positive preconditions**
- Describe possible developments in a layered graph (fact level/action level)
  - links from (positive) facts to **preconditions**
  - **positive effects** generate (positive) facts
  - **negative effects** are used to mark **conflicts**
- **Extract plan** by choosing only non-conflicting parts of graph
Generate a Planning Graph

- Add all applicable actions
- In order to propagate unchanged property $p$, use special action $noop_p$
- Generate all positive effects on next fact level
- Mark conflicts (between actions that cannot be executed in parallel)
- Expand planning graph as long as not all atoms in fact level
Extract a Plan

- Start at last fact level with goal facts
- Select minimal set of non-conflicting actions generating the goals
- Use preconditions of these actions as goals on next lower level
- Backtrack if no non-conflicting choice is possible
Conflict Information

- Two actions interfere (cannot be executed in parallel):
  - one action deletes or asserts the precondition of the other action
  - they have opposite effects on one atomic fact
- They are marked as such
  - and this information is propagated to prune the search early on
Mutex Pairs

- No pair of facts is *mutex* at fact level 0
- A pair of facts is *mutex* at fact level $i > 0$ if all ways of making them true involve actions that are *mutex* at the action level $i-1$
- A pair of actions is *mutex* at action level $i$ if
  - they interfere or
  - one precondition of one action is *mutex* to a precondition of the other action at fact level $i-1$
- *Mutex* pairs cannot be true/executed at the same time
- Note that we do not found all pairs that cannot be true/executed at the same time, but only the easy to spot pairs
Planning Graphs: General Method

- Expand planning graph until all goal atoms are in fact level and they are not mutex
- If not possible, terminate with failure
- Iterate:
  - Try to extract plan and terminate with plan if successful
  - Expand by another action and fact level
- Termination for unsolvable planning problems can be guaranteed – but is complex
Properties of the Planning Graph Approach

- Finds an **optimal solution** (for parallel plans)
- Terminates on **unsolvable** planning instances
- Is **much** faster than POP planning
- Has problems with **symmetries**:
  - Example: Transport *n* objects from room A to room B using one gripper
  - If shortest plan has *k* steps, it proves that there is no *k-1* step plans
Planning as Satisfiability

- Based on planning graphs of depth $k$, one can generated a set of propositional CNF formulae
  - such that each model of these formulae correspond to a $k$-step plan
  - basically, one performs a different kind of search in the planning graph (middle out instead of regression search)
  - Can be considerable faster, sometimes ...
Heuristic Search Planning

- Forward state-space search is often considered as too inefficient because of the high branching factor.
- Why not use a heuristic estimator to guide the search?
- Could that be automatically derived from the representation of the planning instance?
- Yes, since the actions are not “black boxes” as in search!
Ignoring Negative Effects

- Ignore all **negative effects** (assuming again we have only positive preconditions)
  - **monotone planning**

- Example for the buyer’s domain:
  - Only *Go* and *Drop* have negative effects (perhaps also *Buy*).
  - Minimal length plan: `<Go(HWS), Buy(Drill), Go(SM), Buy(Bananas), Buy(Milk), Go(Home)>`
  - Ignoring negative effects: `<Go(HWS), Buy(Drill), Go(SM), Buy(Bananas), Buy(Milk)>`

- Usually plans with simplified ops. are **shorter**.
Monotone Planning

- Monotone planning is easy, i.e., can be solved in \textit{polynomial time}:
  - While we have not made all goal atoms true:
    - Pick any action that
      - is applicable and
      - has not been applied yet
    - and apply it
    - If there is no such action, return failure
    - otherwise continue
Monotone Optimal Planning

- Finding the *shortest plan* is what we need to get an *admissible heuristic*, though!
- This is NP-hard, even if there are no preconditions!
  
  *Minimum Set Cover*, which is NP-complete, can be reduced to this problem
Minimum Set Cover

- **Given**: A set $S$, a collection of subsets $C = \{C_1, \ldots, C_n\}$, $C_i \subseteq S$, and a natural number $k$.
- **Question**: Does there exist a subset of $C$ of size $k$ covering $S$?
  - Problem is **NP-complete**
  - and obviously a special case of the **monotone planning optimization** problem
Simplifying it Further ...

- Since the monotone planning heuristic is computationally too expensive, simplify it further:
  - compute heuristic distance for each atom (recursively) by assuming independence of sub-goals
  - solve the problem with any planner (i.e. the planning graph approach) and use this as an approximative solution
  - both approaches may over-estimate, i.e., it is not an admissible heuristic any longer
The Fast-Forward (FF) System

- **Heuristic**: Solve the monotone planning problem resulting from the relaxation using a planning graph approach
- **Search**: Hill-climbing extended by breadth-first search on plateaus and with
- **Pruning**: Only those successors are considered that are part of a relaxed solution
- **Fall-back strategy**: complete best-first search
Relative Performance of FF

- FF performs very well on the planning benchmarks that are used for the planning competitions
- Examples:
  - Blocks world
  - Logistics
  - Freecell
- Meanwhile refined and also new planners such as Malte’s FDD
Freecell (Domain)
Freecell (Performance)

CPU time

Solution size
Search Space Topology

- Why works the FF heuristic so well?
- Look for search space properties such as
  - local minima
  - size of plateaus
  - dead ends (detected & undetected)
- Estimate by
  - exploring small instances
  - sampling large instance
- Try to prove conjectures found this way
  - Goes some way in understanding problem structure
Summary

- Planning differs from problem-solving in that the representation is more flexible.
- We can search in the plan space instead of the state space.
- The POP algorithm realizes non-linear planning and is complete and correct, but it is difficult to design good heuristics.
- Recent approaches to planning have boosted the efficiency of planning methods significantly.
- Heuristic search planning appears to be one of the fastest (non-optimal) methods.
- Currently, search technology is transferred into the area of formal verification and synthesis (and vice versa).
- While it may still be long time before we can afford to use these techniques instead of domain-specific methods, the progress looks promising.