Exercise 1:

(a) Prove the conditionalized version of the general product rule:

\[
P(A, B | E) = P(A | B, E) \cdot P(B | E)
\]

(b) Prove the conditionalized version of Bayes’ rule:

\[
P(A | B, C) = \frac{P(B | A, C) \cdot P(A | C)}{P(B | C)}
\]

Exercise 2:
Suppose you are a witness to a nighttime hit-and-run accident involving a taxi in Athens. All taxi cars in Athens are blue or green. You swear, under oath, that the taxi was blue. Extensive testing shows that, under the dim lighting conditions, discrimination between blue and green is 75% reliable. Is it possible to calculate the most likely color for the taxi? (Hint: distinguish carefully between the proposition that the taxi is blue and the proposition that the taxi appears blue.) What is your resulting estimate, given that 9 out of 10 Athenian taxis are green?

Exercise 3:
Show that the Statement

\[
P(A, B | C) = P(A | C) \cdot P(B | C)
\]

is equivalent to either of the statements

\[
P(A | B, C) = P(A | C)
\]

and
\[ P(B \mid A, C) = P(B \mid C) \]

**Exercise 4:**

Consider the following every-day situation: You help your grandma to buy some groceries. Unfortunately, her car is rather old and the speed indicator is not working any more. Since you cannot afford another speeding ticket, you have to reason about your speed using just the public speed indicators on the side of the street (see the picture below). You guess, that your current speed \( v_0 \) is distributed as follows:

\[
\begin{array}{c|ccc|c|c|c|c|c}
 & v = 20 & 30 & 40 & 50 & 60 & \text{p}(v) \\
\hline
v_0 & 0.6 & 0.2 & 0.2 & & & \\
\end{array}
\]

Of course, the acceleration is not perfect for such an old car. For each possible action, \( a = -10 \) (slowing down 10 km/h), \( a = +10 \) (accelerating by 10 km/h), \( a = 0 \) (keeping the speed), the transition probabilities for the speed \( v \) of your car are given in the following table.

\[
\begin{array}{c|c|c|c}
 & v_{i+1} = v_i - 10 & v_{i+1} = v_i & v_{i+1} = v_i + 10 \\
\hline
a_i = -10 & 0.6 & 0.4 & 0 \\
a_i = 0 & 0 & 1 & 0 \\
a_i = +10 & 0 & 0.2 & 0.8 \\
\end{array}
\]

The public speed indicators that provide you with speed measurements \( m_i \) have the following measurement accuracy:

\[
\begin{array}{c|c|c|c|c}
 & m_i < v_i - 10 & m_i = v_i - 10 & m_i = v_i & m_i = v_i + 10 & m_i > v_i + 10 \\
\hline
\text{probability} & 0 & 0.1 & 0.7 & 0.2 & 0 \\
\end{array}
\]

On the ride to the supermarket, you perform the following actions and obtain the following measurements. Each measurement \( m_i \) is obtained after the according action \( a_i \) has had its effect on the speed.

<table>
<thead>
<tr>
<th>time ( i )</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>action ( a_i )</td>
<td>+10</td>
<td>0</td>
<td>-10</td>
</tr>
<tr>
<td>measurement ( m_i )</td>
<td>60</td>
<td>50</td>
<td>40</td>
</tr>
</tbody>
</table>
Please use the Bayesian Filtering technique to calculate your belief about the car speed after each time step $i$. Is it likely that you have exceeded the speed limit of 50 km/h at some point?