Albert-Ludwigs-Universität Freiburg
Lecture: Introduction to Mobile Robotics
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Sheet 3

Submission deadline: Wed 17.05.2006, 9:00 (before class)

Exercise 1: Odometry Motion Model

Let a robot be equipped with wheel encoders and on-board software that transforms the physical measuring data into time-discrete odometry measurements $\langle \delta_{rot_1}, \delta_{trans}, \delta_{rot_2} \rangle$.

(a) Derive equations for the calculation of the end-pose $\langle x', y', \theta' \rangle$ after a performed motion $\langle \delta_{rot_1}, \delta_{trans}, \delta_{rot_2} \rangle$ from a starting pose $\langle x, y, \theta \rangle$.

(b) Let the robot start at pose $\langle x, y, \theta \rangle = \langle 0m, 0m, 0^\circ \rangle$ and obtain the following subsequent odometry measurements:

$$\begin{aligned} \delta^1_{rot_1} &= 10^\circ \\ \delta^1_{trans} &= 3m \\ \delta^1_{rot_2} &= 10^\circ \end{aligned}$$

$$\begin{aligned} \delta^2_{rot_1} &= -20^\circ \\ \delta^2_{trans} &= 10m \\ \delta^2_{rot_2} &= -10^\circ \end{aligned}$$

Assuming perfect measurements calculate the final pose of the robot.

(c) How would your pose estimate look like under the following simple error model? Please draw both movements and pose estimates in one diagram.

$$\begin{aligned} \hat{\delta}_{rot_1} &= \delta_{rot_1} \pm \varepsilon_{rot_1}, & \varepsilon_{rot_1} &= 5^\circ \\ \hat{\delta}_{trans} &= \delta_{trans} \pm \varepsilon_{trans}, & \varepsilon_{trans} &= 0.5m \\ \hat{\delta}_{rot_2} &= \delta_{rot_2} \pm \varepsilon_{rot_2}, & \varepsilon_{rot_2} &= 10^\circ \end{aligned}$$

Exercise 2: Velocity Motion Model

The robot in Figure 1 moves on a noise-free error with constant velocities v w starting at $\langle x, y\rangle$ and ending at $\langle x', y'\rangle$

(a) Using trigonometry, derive the following expression for the center $\langle x_c, y_c\rangle$:

$$\begin{pmatrix} x_c \\ y_c \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -\frac{v}{w}\sin\theta \\ \frac{v}{w}\cos\theta \end{pmatrix}$$

(b) Derive the following second expression for the center $\langle x_c, y_c\rangle$:

$$\begin{pmatrix} x_c \\ y_c \end{pmatrix} = \begin{pmatrix} \frac{x+x'}{2} + \mu(y-y') \\ \frac{y+y'}{2} + \mu(x'-x) \end{pmatrix}$$

HINT: the second expression is a straightforward constraint that the center of the circle lies on a ray that lies on the half-way point between $\langle x, y\rangle$ and $\langle x', y'\rangle$ and is orthogonal to the line between these coordinates. You can use the parametric equation for a line to represent this ray.

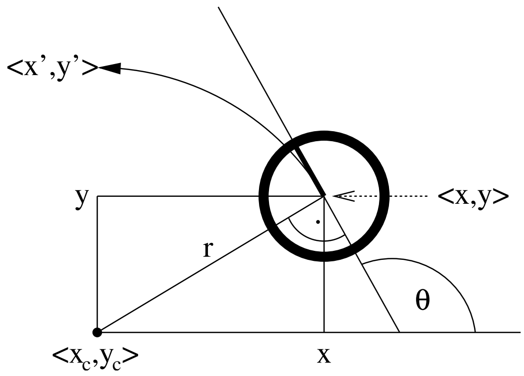

Figure 1: Free-noise movement.

Exercise 3: Velocity Motion Model

A robot starting a pose $x_{t-1} = \langle x, y, \theta\rangle = \langle 0, 0, 90^o\rangle$ moves with control command $u_t = \langle v, \omega\rangle = \langle 0.1 \text{ m/s}, 0.1 \text{ rad/s}\rangle$ during $t = 3$ seconds.

(a) Assuming an error-free movement, calculate the final pose $x_t = \langle x', y', \theta'\rangle$

(b) Calculate the probability $p(x_t|u_t, x_{t-1})$ if we assume a non error-free movement. Use the algorithm **motion_model_velocity** with parameters $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 2.5$.