**Introduction to Mobile Robotics**

**Wheeled Locomotion**

**Locomotion of Wheeled Robots**

Locomotion (Oxford Dict.):
Power of motion from place to place

- Differential drive (AmigoBot, Pioneer 2-DX)
- Car drive (Ackerman steering)
- Synchronous drive (B21)
- Mecanum wheels, XR4000

**Instantaneous Center of Curvature**

- For rolling motion to occur, each wheel has to move along its y-axis

**Differential Drive**

\[
\text{ICC} = [x - R \sin \theta, y + R \cos \theta]
\]

\[
\begin{align*}
\omega(R + l/2) &= v_r \\
\omega(R - l/2) &= v_l \\
R &= \frac{l}{2} \frac{(v_r + v_l)}{2} \\
\omega &= \frac{v_r - v_l}{l}
\end{align*}
\]
Differential Drive: Forward Kinematics

\[
\begin{bmatrix}
  x' \\
  y' \\
  \theta'
\end{bmatrix} = \begin{bmatrix}
  \cos(\alpha \delta t) & -\sin(\alpha \delta t) & 0 \\
  \sin(\alpha \delta t) & \cos(\alpha \delta t) & 0 \\
  0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
  x - ICC \\
  y - ICC \\
  \theta
\end{bmatrix}
\]

\[
x(t) = \int_0^t v(t') \cos(\theta(t')) dt'
\]

\[
y(t) = \int_0^t v(t') \sin(\theta(t')) dt'
\]

\[
\theta(t) = \int_0^t \omega(t') dt'
\]

Ackermann Drive

\[
\text{ICC} = [x - R \sin \theta, y + R \cos \theta]
\]

\[
R = \frac{d}{\tan \phi}
\]

\[
\omega(R + l/2) = v_r
\]

\[
\omega(R - l/2) = v_l
\]

\[
R = \frac{l}{2} \frac{(v_r + v_l)}{v_r - v_l}
\]

\[
\omega = \frac{v_r - v_l}{l}
\]

Synchronous Drive

\[
x(t) = \int_0^t v(t') \cos(\theta(t')) dt'
\]

\[
y(t) = \int_0^t v(t') \sin(\theta(t')) dt'
\]

\[
\theta(t) = \int_0^t \omega(t') dt'
\]
**XR4000 Drive**

\[
x(t) = \int_0^t v(t') \cos[\theta(t')] \, dt' \\
y(t) = \int_0^t v(t') \sin[\theta(t')] \, dt' \\
\theta(t) = \int_0^t \omega(t') \, dt'
\]

**Mecanum Wheels**

\[
\begin{align*}
v_x &= \frac{v_0 + v_1 + v_2 + v_3}{4} \\
v_y &= \frac{v_0 - v_1 + v_2 - v_3}{4} \\
v_b &= \frac{v_0 + v_1 - v_2 - v_3}{4} \\
v_{\text{error}} &= \frac{v_0 - v_1 - v_2 + v_3}{4}
\end{align*}
\]
Example

Tracked Vehicle: Urban Robot

Odometry

Tracked Vehicle: OmniTread

[courtesy by Johann Borenstein]
Non-Holonomic Constraints

- Non-holonomic constraints limit the possible incremental movements within the configuration space of the robot.
- Robots with differential drive or synchro-drive move on a circular trajectory and cannot move sideways.
- XR-4000 or Mecanum-wheeled robots can move sideways.

Holonomic vs. Non-Holonomic

- Non-holonomic constraints reduce the control space with respect to the current configuration (e.g., moving sideways is impossible).
- Holonomic constraints reduce the configuration space.

Non-Holonomic Drives

- Synchro-drive
- Differential drive
- Ackerman drive