Introduction to Mobile Robotics

SLAM Mapping
SLAM Applications

Indoors

Undersea

Space

Underground
Mapping with Raw Odometry
Representations

- **Grid maps or scans**

  [Lu & Milios, 97; Gutmann, 98; Thrun 98; Burgard, 99; Konolige & Gutmann, 00; Thrun, 00; Arras, 99; Haehnel, 01;...]

- **Landmark-based**

  [Leonard et al., 98; Castelanos et al., 99; Dissanayake et al., 2001; Montemerlo et al., 2002;...]
The SLAM Problem

A robot is exploring an unknown, static environment.

**Given:**

- The robot’s controls
- Observations of nearby features

**Estimate:**

- Map of features
- Path of the robot
Structure of the Landmark-based SLAM-Problem
Why is SLAM a hard problem?

**SLAM**: robot path and map are both unknown

Robot path error correlates errors in the map
Why is SLAM a hard problem?

- In the real world, the mapping between observations and landmarks is unknown
- Picking wrong data associations can have catastrophic consequences
- Pose error correlates data associations
SLAM: Simultaneous Localization and Mapping

- **Full SLAM:**
  \[ p(x_{1:t}, m \mid z_{1:t}, u_{1:t}) \]
  Estimates entire path and map!

- **Online SLAM:**
  \[ p(x_t, m \mid z_{1:t}, u_{1:t}) = \int \int \ldots \int p(x_{1:t}, m \mid z_{1:t}, u_{1:t}) \, dx_1 \, dx_2 \ldots dx_{t-1} \]
  Integrations typically done one at a time
  Estimates most recent pose and map!

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Graphical Model of Online SLAM:

\[
p(x_t, m \mid z_{1:t}, u_{1:t}) = \int \int \ldots \int p(x_{1:t}, m \mid z_{1:t}, u_{1:t}) \, dx_1 \, dx_2 \ldots dx_{t-1}
\]
Graphical Model of Full SLAM:

\[ p(x_{1:t}, m \mid z_{1:t}, u_{1:t}) \]
Techniques for Generating Consistent Maps

• Scan matching
• EKF SLAM
• Fast-SLAM
• Probabilistic mapping with a single map and a posterior about poses Mapping + Localization
• Graph-SLAM, SEIF
Scan Matching

Maximize the likelihood of the i-th pose and map relative to the (i-1)-th pose and map.

\[
\hat{x}_t = \arg\max_{x_t} \left\{ p(z_t | x_t, \hat{m}^{[t-1]}_t) \cdot p(x_t | u_{t-1}, \hat{x}_{t-1}) \right\}
\]

Calculate the map \( \hat{m}^{[t]} \) according to “mapping with known poses” based on the poses and observations.
Scan Matching Example
Kalman Filter Algorithm

1. Algorithm `Kalman_filter( \mu_{t-1}, \Sigma_{t-1}, u_t, z_t)`: 

2. Prediction:
3. $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$
4. $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$

5. Correction:
6. $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$
7. $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$
8. $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$

9. Return $\mu_t, \Sigma_t$
**(E)KF-SLAM**

- Map with $N$ landmarks: $(3+2N)$-dimensional Gaussian

$$
\text{Bel}(x_t, m_t) = \begin{pmatrix}
  x \\
  y \\
  \theta \\
  l_1 \\
  l_2 \\
  \vdots \\
  l_N
\end{pmatrix},
\begin{pmatrix}
  \sigma^2_x & \sigma_{xy} & \sigma_{x\theta} \\
  \sigma_{xy} & \sigma^2_y & \sigma_{y\theta} \\
  \sigma_{x\theta} & \sigma_{y\theta} & \sigma^2_{\theta} \\
  \sigma_{x_l1} & \sigma_{x_l2} & \cdots & \sigma_{x_lN} \\
  \sigma_{y_l1} & \sigma_{y_l2} & \cdots & \sigma_{y_lN} \\
  \sigma_{\theta_l1} & \sigma_{\theta_l2} & \cdots & \sigma_{\theta_lN} \\
  \sigma^2_{l1} & \sigma^2_{l2} & \cdots & \sigma^2_{LN}
\end{pmatrix}
$$

- Can handle hundreds of dimensions
Classical Solution – The EKF

- **Blue path** = true path
- **Red path** = estimated path
- **Black path** = odometry

- Approximate the SLAM posterior with a high-dimensional Gaussian [Smith & Cheesman, 1986] ...
- **Known data association**
EKF-SLAM

Map

Correlation matrix
EKF-SLAM

Map

Correlation matrix
EKF-SLAM

Map

Correlation matrix
Properties of KF-SLAM (Linear Case) [Dissanayake et al., 2001]

*Theorem*: The determinant of any sub-matrix of the map covariance matrix decreases monotonically as successive observations are made.

*Theorem*: In the limit the landmark estimates become fully correlated
Victoria Park Data Set

[courtesy by E. Nebot]
Victoria Park Data Set Vehicle

[courtesy by E. Nebot]
Data Acquisition

[courtesy by E. Nebot]
SLAM

[courtesy by E. Nebot]
Map and Trajectory

[courtesy by E. Nebot]
Landmark Covariance

[courtesy by E. Nebot]
Estimated Trajectory

[courtesy by E. Nebot]
EKF SLAM Application

[courtesy by J. Leonard]
EKF SLAM Application

odometry

estimated trajectory

[courtesy by John Leonard]
Approximations for SLAM

• Local submaps
  [Leonard et al. 99, Bosse et al. 02, Newman et al. 03]

• Sparse links (correlations)
  [Lu & Milios 97, Guivant & Nebot 01]

• Sparse extended information filters
  [Frese et al. 01, Thrun et al. 02]

• Thin junction tree filters
  [Paskin 03]

• Rao-Blackwellisation (FastSLAM)
  [Murphy 99, Montemerlo et al. 02, Eliazar et al. 03, Haehnel et al. 03]
Sub-maps for EKF SLAM
EKF-SLAM Summary

- Quadratic in the number of landmarks: $O(n^2)$
- Convergence results for the linear case.
- Can diverge if nonlinearities are large!
- Have been applied successfully in large-scale environments.
- Approximations reduce the computational complexity.