Introduction to Mobile Robotics

Information Gain-Based Exploration Using Rao-Blackwellized Particle Filters
Tasks of Mobile Robots

mapping

localization

integrated approaches

exploration

path planning

SLAM

active localization
Exploration and SLAM

- SLAM is typically *passive*, because it consumes incoming sensor data
- Exploration *actively guides the robot* to cover the environment with its sensors
- Exploration in combination with SLAM: *Acting under pose and map uncertainty*
- The uncertainty needs to be taken into account when selecting an action
Factorization Underlying Rao-Blackwellized Mapping

\[ p(x, m \mid z, u) \]

\[ = p(m \mid x, z, u) p(x \mid z, u) \]

poses map observations & odometry

Mapping with known poses

Particle filter representing trajectory hypotheses
Example

map of particle 1

3 particles

map of particle 2

map of particle 3
Combining Rao-Blackwellized Mapping with Exploration

- Mapping
- Localization
- Path Planning
- Integrated Approaches
- Exploration
- Active Localization
- SLAM
Exploration

- The approaches seen so far are purely passive

- By reasoning about control, the mapping process can be made much more effective

- Question: Where to move next?
Where to Move Next?
Decision-Theoretic Approach

- Learn the map using a Rao-Blackwellized particle filter
- Consider a set of potential actions
- Apply an exploration approach that minimizes the overall uncertainty

\[ \text{Utility} = \text{uncertainty reduction} - \text{cost} \]
The Uncertainty of a Posterior

- Entropy is a general measure for the uncertainty of a posterior

\[ H(p(x)) = - \int_x p(x) \log p(x) \, dx \]
\[ = E_x[-\log(p(x))] \]

- Information Gain = Uncertainty Reduction

\[ I(t + 1 \mid t) = H(p(x_t)) - H(p(x_{t+1})) \]
Entropy Computation

\[ H(p(x, y)) \]
\[ = E_{x,y}[ - \log p(x, y)] \]
\[ = E_{x,y}[ - \log(p(x) \cdot p(y | x))] \]
\[ = E_{x,y}[ - \log p(x)] + E_{x,y}[ - \log p(y | x)] \]
\[ = H(p(x)) + \int_{x,y} -p(x, y) \log p(y | x) \, dx \, dy \]
\[ = H(p(x)) + \int_{x,y} -p(y | x)p(x) \log p(y | x) \, dx \, dy \]
\[ = H(p(x)) + \int_x p(x) \int_y -p(y | x) \log p(y | x) \, dy \, dx \]
\[ = H(p(x)) + \int_x p(x) H(p(y | x)) \, dx \]
Computing the Map and Pose Uncertainty

\[ H(p(x, m \mid d)) = H(p(x \mid d)) + \int_x p(x \mid d) H(p(m \mid x, d)) \, dx \]

\[ \approx H(p(x \mid d)) + \sum_{i=1}^{\#\text{particles}} \omega[i] H(p(m[i] \mid x[i], d)) \]
Computing the Entropy of the Map Posterior

Occupancy Grid map $m$:

$$H(p(m)) = - \sum_{c \in m} p(c) \log p(c) + (1 - p(c)) \log (1 - p(c))$$

- map uncertainty
- grid cells
- probability that the cell is occupied
Computing the Entropy of the Trajectory Posterior

1. High-dimensional Gaussian

\[ H(\mathcal{G}(\mu, \Sigma)) = \log((2\pi e)^{n/2} |\Sigma|) \]

reduced rank for sparse particle sets

2. Grid-based approximation

\[ H(p(x \mid d)) \sim const. \]

for sparse particle clouds
Approximation of the Trajectory Posterior Entropy

Average pose entropy over time:

\[
H(p(x_{1:t} \mid d)) \approx \frac{1}{t} \sum_{t'=1}^{t} H(p(x_{t'} \mid d))
\]
Information Gain

- The reduction of entropy in the model

\[ I(\tilde{z}, a) = H(p(m, x \mid d)) - H(p(m, x, \tilde{x} \mid d, a, \tilde{z})) \]
Computing the Expected Information Gain

- To compute the information gain one needs to know the observations obtained when carrying out an action.

- Since this quantity is unknown, we have to integrate over all potential measurements.

\[
E[I(a)] = \int_{\tilde{z}} p(\tilde{z} \mid a, d) \cdot I(\tilde{z}, a) \, d\tilde{z}
\]
Reasoning about Measurements

- The filter represents a posterior about possible maps
- Use these maps to reason about possible observation
- Simulate laser measurements in the maps of the particles

\[
E[I(a)] = \int_{\mathbf{\hat{z}}} p(\mathbf{\hat{z}} | a, d) \cdot I(\mathbf{\hat{z}}, a) \, d\mathbf{\hat{z}}
\]
Reasoning about Measurements

- Ray-casting in the map of each particle to generate observation sequences

map of particle i

planned trajectory (action)

pose of particle i while carrying out the action

simulated scan
The Utility

- To take into account the cost of an action, we compute a utility

\[ U(a) = I(a) - \alpha \cdot \text{cost}(a) \]

- Select the action with the highest expected utility

\[ a^* = \underset{a}{\text{argmax}} \{ E[U(a)] \} \]
Focusing on Specific Actions

To efficiently sample actions we consider

- exploratory actions (1-3)
- loop closing actions (4) and
- place revisiting actions (5)
Dual Representation for Loop Detection

- **Trajectory graph** ("topological map") stores the path traversed by the robot
- **Occupancy grid** map represents the space covered by the sensors

- **Loops** correspond to **long paths in the trajectory graph** and **short paths in the grid map**
Example: Trajectory Graph
Application Example

high pose uncertainty
Example: Possible Targets

timestep 35
Example: Evaluate Targets
Example: Move Robot to Target

![Decision at timestep 35](chart)

- **Expected Utility**
- **Target Location**

![Robot Path](diagram)

- **Robot**
- **Start**
Example: Evaluate Targets

timestep 70

expected utility

target location

decision at timestep 70

robot
Example: Move Robot

![Diagram showing expected utility vs target location and a robot path from start to decision point at timestep 70.]

... continue ...
Example: Entropy Evolution
Comparison

Map uncertainty only:

After loop closing action:
Real Exploration Example
Corridor Exploration
Summary

- A decision-theoretic approach to exploration in the context of RBPF-SLAM
- The approach utilizes the factorization of the Rao-Blackwellization to efficiently calculate the expected information gain
- Reasons about measurements obtained along the path of the robot
- Considers a reduced action set consisting of exploration, loop-closing, and place-revisiting actions
- Experimental results demonstrate the usefulness of the overall approach