Introduction to Mobile Robotics

Information Gain-Based Exploration Using Rao-Blackwellized Particle Filters

Exploration and SLAM

- SLAM is typically passive, because it consumes incoming sensor data
- Exploration actively guides the robot to cover the environment with its sensors
- Exploration in combination with SLAM: Acting under pose and map uncertainty
- The uncertainty needs to be taken into account when selecting an action

Tasks of Mobile Robots

Factorization Underlying Rao-Blackwellized Mapping

\[ p(x, m \mid z, u) = p(m \mid x, z, u)p(x \mid z, u) \]

Mapping with known poses

Particle filter representing trajectory hypotheses
Example

3 particles

map of particle 1

map of particle 2

map of particle 3

Combining Rao-Blackwellized Mapping with Exploration

mapping

localization

integrated approaches

exploration

path planning

SLAM

active localization

Exploration

- The approaches seen so far are purely passive

- By reasoning about control, the mapping process can be made much more effective

- Question: Where to move next?

Where to Move Next?
Decision-Theoretic Approach

- Learn the map using a Rao-Blackwellized particle filter
- Consider a set of potential actions
- Apply an exploration approach that minimizes the overall uncertainty

**Utility = uncertainty reduction - cost**

The Uncertainty of a Posterior

- Entropy is a general measure for the uncertainty of a posterior
  \[ H(p(x)) = - \int p(x) \log p(x) \, dx \]
  \[ = E_x[- \log(p(x))] \]

- Information Gain = Uncertainty Reduction
  \[ I(t + 1 \mid t) = H(p(x_t)) - H(p(x_{t+1})) \]

Entropy Computation

\[
H(p(x, y)) = E_{x,y}[- \log p(x, y)] \\
= E_{x,y}[- \log(p(x) \, p(y \mid x))] \\
= E_{x,y}[- \log p(x)] + E_{x,y}[- \log p(y \mid x)] \\
= H(p(x)) + \int_{x,y} -p(x, y) \log p(y \mid x) \, dx \, dy \\
= H(p(x)) + \int_{x,y} -p(y \mid x)p(x) \log p(y \mid x) \, dx \, dy \\
= H(p(x)) + \int_x p(x) \int_y -p(y \mid x) \log p(y \mid x) \, dy \, dx \\
= H(p(x)) + \int_x p(x)H(p(y \mid x)) \, dx
\]

Computing the Map and Pose Uncertainty

\[
H(p(x, m \mid d)) \\
= H(p(x \mid d)) + \int_x p(x \mid d)H(p(m \mid x, d)) \, dx \\
\approx H(p(x \mid d)) + \sum_{i=1}^{\text{#particles}} \omega[i]H(p(m[i] \mid x[i], d))
\]

- data (laser and odometry)
- #particles
- trajectory uncertainty
- particle weight
- map uncertainty
Computing the Entropy of the Map Posterior

Occancy Grid map \( m \):

\[
H(p(m)) = - \sum_{c \in m} p(c) \log p(c) + (1 - p(c)) \log(1 - p(c))
\]

- map uncertainty
- grid cells
- probability that the cell is occupied

Computing the Entropy of the Trajectory Posterior

1. High-dimensional Gaussian

\[
H(\mathcal{G}(\mu, \Sigma)) = \log((2\pi e)^{n/2} |\Sigma|)
\]

reduced rank for sparse particle sets

2. Grid-based approximation

\[
H(p(x | d)) \sim \text{const.}
\]

for sparse particle clouds

Approximation of the Trajectory Posterior Entropy

Average pose entropy over time:

\[
H(p(x_{1:t} | d)) \approx \frac{1}{t} \sum_{t'=1}^{t} H(p(x_{t'} | d))
\]

Information Gain

- The reduction of entropy in the model

\[
I(\tilde{z}, a) = H(p(m, x | d)) - H(p(m, x, \tilde{x} | d, a, \tilde{z}))
\]

observations to be obtained

new poses introduced by action

action

H before action is carried out

H after action is carried out
Computing the Expected Information Gain

- To compute the information gain one needs to know the observations obtained when carrying out an action.
- Since this quantity is unknown, we have to integrate over all potential measurements.

\[ E[I(a)] = \int_{\hat{z}} p(\hat{z} \mid a, d) \cdot I(\hat{z}, a) \, d\hat{z} \]

Reasoning about Measurements

- Ray-casting in the map of each particle to generate observation sequences.
- Simulate laser measurements in the maps of the particles.

\[ E[I(a)] = \int_{\hat{z}} p(\hat{z} \mid a, d) \cdot I(\hat{z}, a) \, d\hat{z} \]

The Utility

- To take into account the cost of an action, we compute a utility:

\[ U(a) = I(a) - \alpha \cdot \text{cost}(a) \]

- Select the action with the highest expected utility:

\[ a^* = \arg\max_a \{ E[U(a)] \} \]
**Focusing on Specific Actions**

To efficiently sample actions we consider:
- exploratory actions (1-3)
- loop closing actions (4) and
- place revisiting actions (5)

**Example: Trajectory Graph**

**Dual Representation for Loop Detection**

- **Trajectory graph** ("topological map") stores the path traversed by the robot
- **Occupancy grid** map represents the space covered by the sensors
- **Loops** correspond to **long paths in the trajectory graph and short paths in the grid map**

**Application Example**

high pose uncertainty
Example: Move Robot

Comparison
Map uncertainty only:

After loop closing action:

Example: Entropy Evolution

Real Exploration Example
Corridor Exploration

Summary

- A decision-theoretic approach to exploration in the context of RBPF-SLAM
- The approach utilizes the factorization of the Rao-Blackwellization to efficiently calculate the expected information gain
- Reasons about measurements obtained along the path of the robot
- Considers a reduced action set consisting of exploration, loop-closing, and place-revisiting actions
- Experimental results demonstrate the usefulness of the overall approach