Foundations of AI

4. Informed Search Methods

Heuristics, Local Search Methods, Genetic Algorithms

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Contents

- Best-First Search
- A* and IDA*
- Local Search Methods
- Genetic Algorithms
Best-First Search

Search procedures differ in the way they determine the next node to expand.

**Uninformed Search:** Rigid procedure with no knowledge of the cost of a given node to the goal.

**Informed Search:** Knowledge of the cost of a given node to the goal is in the form of an *evaluation function* \( f \) or \( h \), which assigns a real number to each node.

**Best-First Search:** Search procedure that expands the node with the “best” \( f \)- or \( h \)-value.
General Algorithm

```plaintext
function BEST-FIRST-SEARCH(problem, EVAL-FN) returns a solution sequence

inputs: problem, a problem
        Eval-Fn, an evaluation function

    Queueing-Fn ← a function that orders nodes by EVAL-FN

return GENERAL-SEARCH(problem, Queueing-Fn)
```

When $h$ is always correct, we do not need to search!
Greedy Search

A possible way to judge the “worth” of a node is to estimate its distance to the goal.

\[ h(n) = \text{estimated distance from } n \text{ to the goal} \]

The only real condition is that \( h(n) = 0 \) if \( n \) is a goal.

A best-first search with this function is called a greedy search.

Route-finding problem: \( h = \text{straight-line distance between two locations} \).
Greedy Search Example
Greedy Search from Arad to Bucharest
Heuristics

The evaluation function $h$ in greedy searches is also called a *heuristic* function or simply a *heuristic*.

- The word *heuristic* is derived from the Greek word ευρισκειν (note also: ευρηκα!)
- The mathematician Polya introduced the word in the context of problem solving techniques.
- In AI it has two meanings:
  - Heuristics are fast but in certain situations incomplete methods for problem-solving [Newell, Shaw, Simon 1963] (The greedy search is actually generally incomplete).
  - Heuristics are methods that improve the search in the average-case.

→ In all cases, the heuristic is *problem-specific* and *focuses* the search!
A*: Minimization of the estimated path costs

A* combines the greedy search with the uniform-search strategy.

\[ g(n) = \text{actual cost from the initial state to } n. \]

\[ h(n) = \text{estimated cost from } n \text{ to the next goal.} \]

\[ f(n) = g(n) + h(n), \text{ the estimated cost of the cheapest solution through } n. \]

Let \( h^*(n) \) be the actual cost of the optimal path from \( n \) to the next goal.

\( h \) is admissible if the following holds for all \( n \):

\[ h(n) \leq h^*(n) \]

We require that for A*, \( h \) is admissible (straight-line distance is admissible).
A* Search Example
A* Search from Arad to Bucharest
Contours in A*

Within the search space, contours arise in which for the given $f$-value all nodes are expanded.

Contours at $f = 380, 400, 420$
Example: Path Planning for Robots in a Grid-World
Optimality of A*

**Claim:** The first solution found has the minimum path cost.

**Proof:** Suppose there exists a goal node $G$ with optimal path cost $f^*$, but A* has found another node $G_2$ with $g(G_2) > f^*$. 
Let $n$ be a node on the path from the start to $G$ that has not yet been expanded. Since $h$ is admissible, we have

$$f(n) \leq f^*.$$  

Since $n$ was not expanded before $G_2$, the following must hold:

$$f(G_2) \leq f(n)$$

and

$$f(G_2) \leq f^*.$$  

It follows from $h(G_2) = 0$ that

$$g(G_2) \leq f^*.$$  

→ Contradicts the assumption!
Completeness and Complexity

Completeness:

If a solution exists, A* will find it provided that (1) every node has a finite number of successor nodes, and (2) there exists a positive constant $\delta$ such that every operator has at least cost $\delta$.

$\rightarrow$ Only a finite number of nodes $n$ with $f(n) \leq f^*$.

Complexity:

In the case where $|h^*(n) - h(n)| \leq O(\log(h^*(n)))$, only a sub-exponential number of nodes will be expanded.

Normally, growth is exponential because the error is proportional to the path costs.
Heuristic Function Example

\[ h_1 = \text{the number of tiles in the wrong position} \]
\[ h_2 = \text{the sum of the distances of the tiles from their goal positions (Manhatten distance)} \]
**Empirical Evaluation**

- $d = \text{distance from goal}$
- Average over 100 instances

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Iterative Deepening A* Search (IDA*)

Idea: A combination of IDS and A*. All nodes inside a contour are searched.

```plaintext
function IDA*(problem) returns a solution sequence
    inputs: problem, a problem
    static: f-limit, the current f- COST limit
            root, a node

    root ← MAKE-NODE(INITIAL-STATE[problem])
    f-limit ← f- COST(root)
    loop do
        solution, f-limit ← DFS-COUREN(root, f-limit)
        if solution is non-null then return solution
        if f-limit = ∞ then return failure; end

function DFS-COUREN(node, f-limit) returns a solution sequence and a new f- COST limit
    inputs: node, a node
            f-limit, the current f- COST limit
    static: next-f, the f- COST limit for the next contour, initially ∞

    if f- COST[node] > f-limit then return null, f- COST[node]
    if GOAL-TEST[problem](STATE[node]) then return node, f-limit
    for each node s in SUCCESSORS(node) do
        solution, new-f ← DFS-COUREN(s, f-limit)
        if solution is non-null then return solution, f-limit
        next-f ← MIN(next-f, new-f); end
    return null, next-f
```
Local **Search Methods**

In many problems, it is unimportant how the goal is reached – only the goal itself matters (8-queens problem, VLSI Layout, TSP).

If in addition a quality measure for states is given, a **local search** can be used to find solutions.

Idea: Begin with a randomly-chosen configuration and improve on it stepwise \(\rightarrow\) **Hill Climbing**.
Hill Climbing

function HILL-CLIMBING(problem) returns a solution state
  inputs: problem, a problem
  static: current, a node
           next, a node

  current ← MAKE-NODE(INITIAL-STATE[problem])
  loop do
    next ← a highest-valued successor of current
    if VALUE[next] < VALUE[current] then return current
    current ← next
  end
Example: 8-Queens Problem

Selects a column and moves the queen to the square with the fewest conflicts.
Problems with Local Search Methods

- **Local maxima**: The algorithm finds a sub-optimal solution.
- **Plateaus**: Here, the algorithm can only explore at random.
- **Ridges**: Similar to plateaus.

**Solutions:**
- *Start over* when no progress is being made.
- “Inject smoke” → random walk
- Tabu search: Do not apply the last $n$ operators.

Which strategies (with which parameters) are successful (within a problem class) can usually only empirically be determined.
Simulated Annealing

In the simulated annealing algorithm, “smoke” is injected systematically: first a lot, then gradually less.

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
    inputs: problem, a problem
             schedule, a mapping from time to “temperature”
    static: current, a node
             next, a node
             T, a “temperature” controlling the probability of downward steps
    current ← MAKE-NODE(INITIAL-STATE[problem])
    for t ← 1 to ∞ do
        T ← schedule[t]
        if T=0 then return current
        next ← a randomly selected successor of current
        ΔE ← VALUE[next] – VALUE[current]
        if ΔE > 0 then current ← next
        else current ← next only with probability e^{ΔE/T}
```

Has been used since the early 80’s for VSLI layout and other optimization problems.
Genetic Algorithms

Evolution appears to be very successful at finding good solutions.

**Idea:** Similar to evolution, we search for solutions by “crossing”, “mutating”, and “selecting” successful solutions.

**Ingredients:**

- Coding of a solution into a string of symbols or bit-string
- A fitness function to judge the worth of configurations
- A population of configurations

**Example:** 8-queens problem as a chain of 8 numbers. Fitness is judged by the number of non-attacks. The population consists of a set of arrangements of queens.
Selection, Mutation, and Crossing

Many variations:
how selection will be applied, what type of cross-overs will be used, etc.

Selektion: Selection of individuals and pairing. Determination where to break and reassemble. With a certain small probability, something in the string will be changed.

Kreuzen: Crossing is determined where to break and reassemble. With a certain small probability, something in the string will be changed.

Mutation: With a certain small probability, something in the string will be changed.
Summary

- **Heuristics** focus the search
- **Best-first search** expands the node with the highest worth (defined by any measure) first.
- With the minimization of the evaluated costs to the goal $h$ we obtain a **greedy search**.
- The minimization of $f(n) = g(n) + h(n)$ combines uniform and greedy searches. When $h(n)$ is admissible, i.e. $h^*$ is never overestimated, we obtain the **A* search**, which is complete and optimal.
- **IDA** is a combination of the iterative-deepening and A* searches.
- **Local search methods** only ever work on one state, attempting to improve it step-wise.
- **Genetic algorithms** imitate evolution by combining good solutions.