

Sheet 2

Topics: Differential Drive, Conditional Probabilities

Submission deadline: Fri 04.05.2007, 11:00 a.m. (before class)

General Notice: Updated Requirements

To be admitted to the final exam, every student has to

- attend at least 50% of the seminars in person,
- submit solutions for at least 50% of the exercises, and
- present at least one of the solutions in class.

The exercises should be done in groups of one or two students. One bonus point for the final exam is granted for every reasonable solution of a complete exercise sheet.

Exercise 1:

A robot equipped with a differential drive starts at the position $x = 0$, $y = 0$ and with heading $\alpha = \frac{\pi}{2}$ ($\frac{\pi}{2}$ is the direction of the y -axis). It has to move to the position $x = 300\text{cm}$, $y = 0$, $\alpha = \frac{\pi}{2}$. The movement of the vehicle is described by steering commands (v_l = speed of left wheel, v_r = speed of right wheel, t = driving time).

- What is the minimal number of steering commands (v_l, v_r, t) needed to guide the vehicle to the desired target location?
- What is the length of the shortest trajectory under this constraint?
- Which sequence of steering commands guides the robot on the shortest trajectory to the desired location if an arbitrary number of steering commands can be used?
- What is the length of this trajectory?

Note: the length of a trajectory refers to the scalar traveled distance along the trajectory.

Exercise 2:

The robot introduced in Exercise 1 starts at the position $x = 0$, $y = 0$ and with heading $\alpha = 0$ (0 is the direction of the x -axis). The distance between both wheels is $w = 50\text{cm}$. It executes the following sequence of steering commands:

$$\begin{aligned}c_1 &= (v_l = 30\text{cm/s}, v_r = 20\text{cm/s}, t = 3\text{s}), \\c_2 &= (v_l = 30\text{cm/s}, v_r = 30\text{cm/s}, t = 2\text{s}), \\c_3 &= (v_l = 10\text{cm/s}, v_r = -10\text{cm/s}, t = 1\text{s}).\end{aligned}$$

What is the position (x, y, α) of the robot after executing these commands?

Exercise 3:

(a) Prove the conditionalized version of the general product rule:

$$P(A, B | E) = P(A | B, E) \cdot P(B | E)$$

(b) Prove the conditionalized version of Bayes' rule:

$$P(A | B, C) = \frac{P(B | A, C) \cdot P(A | C)}{P(B | C)}$$

Exercise 4:

Suppose you are a witness to a nighttime hit-and-run accident involving a taxi in Athens. All taxi cars in Athens are blue or green. You swear, under oath, that the taxi was blue. Extensive testing shows that, under the dim lighting conditions, discrimination between blue and green is 75% reliable. Is it possible to calculate the most likely color for the taxi? (Hint: distinguish carefully between the proposition that the taxi is blue and the proposition that the taxi appears blue.) What is your resulting estimate, given that 9 out of 10 Athenian taxis are green?