Introduction

This exercise deals with the prediction step of the Extended Kalman Filter (EKF). A pose estimate in time step $t$ is represented by a Gaussian with mean $\mu_t$ and a covariance matrix $\Sigma_t$. These are the corresponding update equations:

$$
\mu_t := g(\mu_{t-1}, u_t)
$$

$$
\Sigma_t := G\Sigma GT + VMV^T
$$

Exercise 1:

As with the odometry model introduced in the lecture and Ex. 1 of the last sheet, a control vector $u$ consists of an initial rotation angle $\delta_{rot_1}$, a straight forward motion distance $\delta_{trans}$, and a final rotation angle $\delta_{rot_2}$. The mean poses (or states) $\mu_t$ are defined as usual:

$$
u_t = \begin{pmatrix}
\delta_{rot_1} \\
\delta_{trans} \\
\delta_{rot_2}
\end{pmatrix}
$$

$$
\mu_t = \begin{pmatrix}
x_t \\
y_t \\
\theta_t
\end{pmatrix}
$$

Given a pair of absolute odometry readings $s_1 = \langle x_1, y_1, \theta_1 \rangle$ and $s_2 = \langle x_2, y_2, \theta_2 \rangle$ as in the CARMEN log files, compute the relative pose like $s2.subtract(s1)$ would do. Determine the equivalent control vector $u = \langle \delta_{rot_1}, \delta_{trans}, \delta_{rot_2} \rangle$. Note that this is the inverse transformation of Sheet 4, Ex. 1 (a).

Exercise 2:

The function $g(\mu_{t-1}, u_t)$ defines how the new pose mean estimate $\mu_t$ depends on the last pose $\mu_{t-1}$ and the control vector $u_t$. Determine $g$ for the motion and control model of Ex. 1.

Exercise 3:

To perform the linearization of the previously defined function $g$, derive the Jacobians

$$
G = \frac{\partial g(\mu_{t-1}, u_t)}{\partial \mu_{t-1}}
$$

$$
V = \frac{\partial g(\mu_{t-1}, u_t)}{\partial u_t}
$$
If you are not sure about the form of Jacobians in general, please look it up (e.g. Wikipedia).

**Exercise 4:**

$M$ is the motion noise matrix in control space. With $VMV^T$ it is linearized and mapped into state space:

$$M = \begin{pmatrix} \sigma_{\delta_{rot_1}}^2 & 0 & 0 \\ 0 & \sigma_{\delta_{trans}}^2 & 0 \\ 0 & 0 & \sigma_{\delta_{rot_2}}^2 \end{pmatrix}.$$  

Assume $\sigma_{\delta_{rot_1}} = 5^\circ$, $\sigma_{\delta_{trans}} = 0.5m$, and $\sigma_{\delta_{rot_2}} = 10^\circ$. Given the initial estimates $\mu_0$ and $\Sigma_0$, and the control vectors $u_1$ and $u_2$, compute the resulting estimates $\mu_1$, $\Sigma_1$ and $\mu_2$, $\Sigma_2$ using the Kalman filter equations.

$$\mu_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad \Sigma_0 = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{pmatrix}, \quad u_1 = \begin{pmatrix} 10^\circ \\ 3m \\ 10^\circ \end{pmatrix}, \quad u_2 = \begin{pmatrix} -20^\circ \\ 10m \\ -10^\circ \end{pmatrix}.$$