Albert-Ludwigs-Universität Freiburg
Lecture: Introduction to Mobile Robotics
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Sheet 8

Topic: Landmark Mapping With Known Poses And Correspondences

Submission deadline: Fri 22.06.2007, 11:00 a.m. (before class)

Introduction

At time index t a robot performs a measurement $z_t = \langle d, \phi \rangle$ of a single unique landmark l, where d denotes the distance to the landmark and ϕ the bearing angle with respect to the robot's coordinate system. The robot's poses from which the measurements were performed are known and given by $\vec{x_t} = \langle x, y, \theta \rangle$ in world coordinates. The measurements are distorted by Gaussian noise, with $\sigma_d = 1$ and $\sigma_\phi = 0.1$.

The positions $\mu_t = \langle x_l, y_l \rangle$ of the landmarks expressed in world coordinates and the corresponding uncertainties Σ_t are integrated into the map and updated for each measurement using one Extended Kalman Filter (EKF) per landmark. Since the landmarks are assumed to be stationary, the prediction step of the EKF is omitted.

When a landmark is perceived for the first time, the corresponding measurement and the measurement noise matrix are transformed into world coordinates to get the initial estimates $\mu_0 = \bar{h}(z)$ and $\Sigma_0 = \bar{H} Q \bar{H}'$ where $\bar{h}$ is the inverse of h, and $\bar{H}$ the Jacobian of $\bar{h}$.

Exercise 1:

This exercise comprises calculations that you can do on paper or using a tool like Matlab or GNU Octave. Please provide formulas and numbers, no matter how you do it. Use $\Delta x = x_l - x$ and $\Delta y = y_l - y$.

(a) Determine the measurement function h, its linearization H (Jacobian), and the measurement noise matrix Q. This might help:

$$\frac{\partial \text{atan2}(y,x)}{\partial x} = \frac{-y}{x^2 + y^2} \ , \ \frac{\partial \text{atan2}(y,x)}{\partial y} = \frac{x}{x^2 + y^2}$$

(b) Derive the inverses $\bar{h}$ and $\bar{H}$ and compute μ_0 and Σ_0 using z_0 and $\vec{x}_0$.

(c) Calculate μ_t and Σ_t for $t = 1, \ldots, 3$. Draw all poses, measurements and landmark position estimates into a diagram (axes from 0 to 4), or plot them using octave/gnuplot.

(d) Visualize the uncertainties with ellipses, either as a rough sketch in the diagram or, if you use Octave, by calling drawCov(μ, Σ). The drawCov function is provided online in the zip file. Just as an explanation: the eigenvalues of a covariance matrix are proportional to the length of the semi-axes of the ellipse, and the eigenvectors denote the direction of the semi-axes. If the eigenvalues are scaled with $\chi^2_{2,0.05} = 5.991464$, the true value is in the ellipse with a probability of 95%.

Time	Pose			Measurement	
t	x	y	θ	d	ϕ
0	0	0	$0°$	2	$44°$
1	1	0.1	$10°$	4	$48°$
2	2	0	$-10°$	1	$102°$
3	3	-0.1	$-5°$	0.5	$114°$

Exercise 2:

No calculations for this exercise, only descriptions!

(a) Suppose there are multiple landmarks that are not unique but indistinguishable. What extra steps have to be performed? Sketch an approach to this problem.

(b) Suppose the true pose of the robot is not known, but estimated using an EKF. How can you include the pose uncertainties into the mapping process?