Introduction to Mobile Robotics

Bayes Filter Implementations

Particle filters
Sample-based Localization (sonar)
Particle Filters

- Represent belief by random samples
- Estimation of non-Gaussian, nonlinear processes

- Monte Carlo filter, Survival of the fittest, Condensation, Bootstrap filter, Particle filter

- Filtering: [Rubin, 88], [Gordon et al., 93], [Kitagawa 96]
- Computer vision: [Isard and Blake 96, 98]
- Dynamic Bayesian Networks: [Kanazawa et al., 95]
Importance Sampling

Weight samples: \( w = \frac{f}{g} \)
Importance Sampling with Resampling: Landmark Detection Example
Distributions
Distributions

Wanted: samples distributed according to $p(x \mid z_1, z_2, z_3)$
This is Easy!

We can draw samples from \( p(x|z_i) \) by adding noise to the detection parameters.
Importance Sampling with Resampling

Target distribution \( f : p(x \mid z_1, z_2, \ldots, z_n) = \frac{\prod_{k} p(z_k \mid x) \cdot p(x)}{p(z_1, z_2, \ldots, z_n)} \)

Sampling distribution \( g : p(x \mid z_l) = \frac{p(z_l \mid x) \cdot p(x)}{p(z_l)} \)

Importance weights \( w : \frac{f}{g} = \frac{p(x \mid z_1, z_2, \ldots, z_n)}{p(x \mid z_l)} = \frac{p(z_l) \prod_{k \neq l} p(z_k \mid x)}{p(z_1, z_2, \ldots, z_n)} \)
Importance Sampling with Resampling

samples drawn from $p(x|z_1)$

likelihood functions $p(z_2|x)$ and $p(z_3|x)$

After re-sampling
Particle Filters
Sensor Information: Importance Sampling

\[
\begin{align*}
Bel(x) & \leftarrow \alpha p(z \mid x) Bel^{-}(x) \\
w & \leftarrow \frac{\alpha p(z \mid x) Bel^{-}(x)}{Bel^{-}(x)} = \alpha p(z \mid x)
\end{align*}
\]
Robot Motion

\[ Bel^{-}(x) \leftarrow \int p(x | u, x') Bel(x') \, dx' \]
Sensor Information: Importance Sampling

\[ Bel(x) \leftarrow \alpha \ p(z \mid x) \ Bel^-(x) \]

\[ w \leftarrow \frac{\alpha \ p(z \mid x) \ Bel^-(x)}{Bel^-(x)} = \alpha \ p(z \mid x) \]
Robot Motion

\[
Bel^{-}(x) \leftarrow \int p(x \mid u, x') Bel(x') \, dx'
\]
Particle Filter Algorithm

1. Algorithm \texttt{particle\_filter}( S_{t-1}, u_{t-1}, z_t ):
2. \hspace{1em} S_t = \emptyset, \hspace{1em} \eta = 0
3. \hspace{1em} \textbf{For} \hspace{1em} i = 1 \ldots n \hspace{2em} \textit{Generate new samples}
4. \hspace{1em} \text{Sample index } j(i) \text{ from the discrete distribution given by } w_{t-1}
5. \hspace{1em} \text{Sample } x_t^i \text{ from } p(x_t \mid x_{t-1}, u_{t-1}) \text{ using } x_{t-1}^{j(i)} \text{ and } u_{t-1}
6. \hspace{1em} w_t^i = p(z_t \mid x_t^i) \hspace{2em} \textit{Compute importance weight}
7. \hspace{1em} \eta = \eta + w_t^i \hspace{2em} \textit{Update normalization factor}
8. \hspace{1em} S_t = S_t \cup \{ < x_t^i, w_t^i > \} \hspace{2em} \textit{Insert}
9. \hspace{1em} \textbf{For} \hspace{1em} i = 1 \ldots n
10. \hspace{1em} w_t^i = w_t^i / \eta \hspace{2em} \textit{Normalize weights}
**Particle Filter Algorithm**

\[ Bel(x_t) = \eta \ p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_{t-1}) \ Bel(x_{t-1}) \ dx_{t-1} \]

- **Importance factor for** \( x^i_{t-1} \):\[ w_t^i = \frac{\text{target distribution}}{\text{proposal distribution}} = \frac{\eta \ p(z_t \mid x_t) \ p(x_t \mid x_{t-1}, u_{t-1}) \ Bel(x_{t-1})}{p(x_t \mid x_{t-1}, u_{t-1}) \ Bel(x_{t-1})} \]

- **draw** \( x^i_{t-1} \) **from** \( Bel(x_{t-1}) \)
- **draw** \( x^i_t \) **from** \( p(x_t \mid x^i_{t-1}, u_{t-1}) \)

\[ \propto p(z_t \mid x_t) \]
Resampling

• **Given**: Set $S$ of weighted samples.

• **Wanted**: Random sample, where the probability of drawing $x_i$ is given by $w_i$.

• Typically done $n$ times with replacement to generate new sample set $S'$.
Resampling

- Roulette wheel
- Binary search, $n \log n$
- Stochastic universal sampling
- Systematic resampling
- Linear time complexity
- Easy to implement, low variance
Resampling Algorithm

1. Algorithm `systematic_resampling(S,n)`:

2. $S' = \emptyset, c_1 = w^1$

3. **For** $i = 2 \ldots n$ \hspace{1cm} Generate cdf

4. $c_i = c_{i-1} + w^i$

5. $u_1 \sim U[0, n^{-1}], i = 1$ \hspace{1cm} Initialize threshold

6. **For** $j = 1 \ldots n$ \hspace{1cm} Draw samples …

7. **While** $(u_j > c_i)$ \hspace{1cm} Skip until next threshold reached

8. $i = i + 1$

9. $S' = S' \cup \{< x^i, n^{-1} > \}$ \hspace{1cm} Insert

10. $u_{j+1} = u_j + n^{-1}$ \hspace{1cm} Increment threshold

11. **Return** $S'$

Also called **stochastic universal sampling**
Motion Model Reminder

Start

10 meters
Proximity Sensor Model Reminder

Laser sensor

Sonar sensor
Sample-based Localization (sonar)
Initial Distribution
After Incorporating Ten Ultrasound Scans
After Incorporating 65 Ultrasound Scans
Estimated Path
Using Ceiling Maps for Localization

[Dellaert et al. 99]
Vision-based Localization

\[ P(z|x) \]

\[ h(x) \]
Under a Light

Measurement $z$: $P(z|x)$:
Next to a Light

Measurement $z$:  

$P(z|x)$:
Elsewhere

Measurement $z$: $P(z|x)$:
Global Localization Using Vision
Robots in Action: Albert
Application: Rhino and Albert Synchronized in Munich and Bonn

[Robotics And Automation Magazine, to appear]
Localization for AIBO robots
Limitations

• The approach described so far is able to
  • track the pose of a mobile robot and to
  • globally localize the robot.

• How can we deal with localization errors (i.e., the kidnapped robot problem)?
Approaches

- Randomly insert samples (the robot can be teleported at any point in time).
- Insert random samples proportional to the average likelihood of the particles (the robot has been teleported with higher probability when the likelihood of its observations drops).
Random Samples
Vision-Based Localization

936 Images, 4MB, .6 secs/image

Trajectory of the robot:
Odometry Information
Image Sequence
Resulting Trajectories

Position tracking:
Resulting Trajectories

Global localization:
Global Localization
Kidnapping the Robot
Recovery from Failure
Summary

• Particle filters are an implementation of recursive Bayesian filtering.
• They represent the posterior by a set of weighted samples.
• In the context of localization, the particles are propagated according to the motion model.
• They are then weighted according to the likelihood of the observations.
• In a re-sampling step, new particles are drawn with a probability proportional to the likelihood of the observation.