Introduction to Mobile Robotics

Path Planning and Collision Avoidance

Cyrill Stachniss

Motion Planning

Latombe (1991):

"…eminently necessary since, by definition, a robot accomplishes tasks by moving in the real world."

Goals:

- Collision-free trajectories.
- Robot should reach the goal location as fast as possible.

… in Dynamic Environments

- How to react to unforeseen obstacles?
  - efficiency
  - reliability

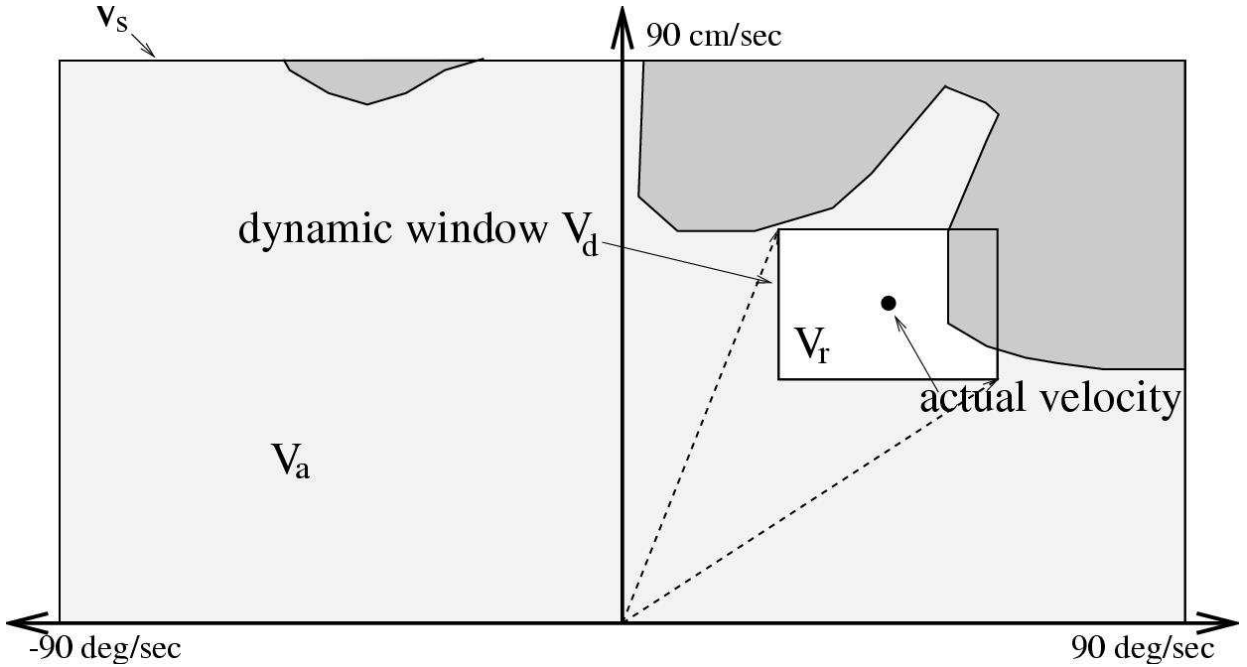
- Dynamic Window Approaches
  [Simmons, 96], [Fox et al., 97], [Brock & Khatib, 99]
- Grid map based Planning
  [Konolige, 00]
- Nearness Diagram Navigation
  [Minguez at al., 2001, 2002]
- Vector-Field-Histogram+
  [Ulrich & Borenstein, 98]
- A*, D*, D* Lite, ARA*, …

Dynamic Window Approach (1)

- Here: robot moves on circular arcs.
- Motion commands (v,ω).
- Which (v,ω) are admissible?
- **Collision Avoidance:** Check with geometric operations which trajectories are collision-free!

Dynamic Window Approach (2)

- Example for the Search-Space:



- Vs = all possible speeds of the robot.
- Va = obstacle free area.
- Vd = possible speeds based on possible accelerations.

$$Space = V_s \cap V_a \cap V_d$$

Dynamic Window Approach (3)

- How to choose <v,ω> now?
- Steering commands are chosen by a heuristic navigation function.
- This function tries to minimize the travel-time by:
  "**drive fast** in the **correct direction**".

Dynamic Window Approach (4)

- Heuristic navigation function.
- Planning restricted to <x,y>-space.
- No planning in the velocity space.

Navigation Function: [Brock & Khatib, 99]

$$NF = \alpha \cdot vel + \beta \cdot nf + \gamma \cdot \Delta nf + \delta \cdot goal$$

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Maximizes velocity.

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Maximizes velocity.

Considers cost to reach the goal.

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Considers cost to reach the goal.

Follows grid based path computed by A*.

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Navigation Function:

$$NF = \alpha \cdot vel + \beta \cdot nf + \gamma \cdot \Delta nf + \delta \cdot goal$$

Goal nearness.

Maximizes velocity.

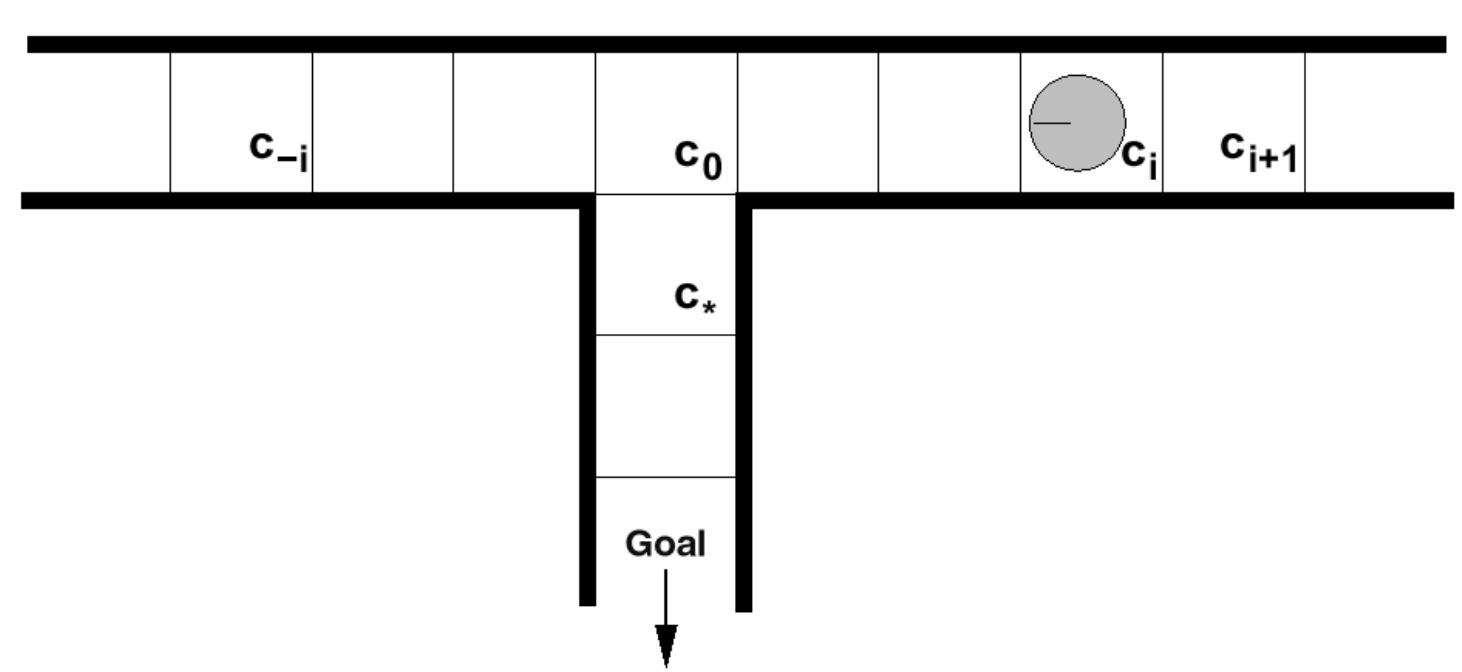
Considers cost to reach the goal.

Follows grid based path computed by A*.

Dynamic Window Approach (5)

- Reacts fast.
- Low CPU power required.
- Guides a robot on a collision free path.
- Successfully used in a lot of real world experiments.
- But not always good trajectories.
- Local Minima, sometimes no solution!

Problems of DWAs



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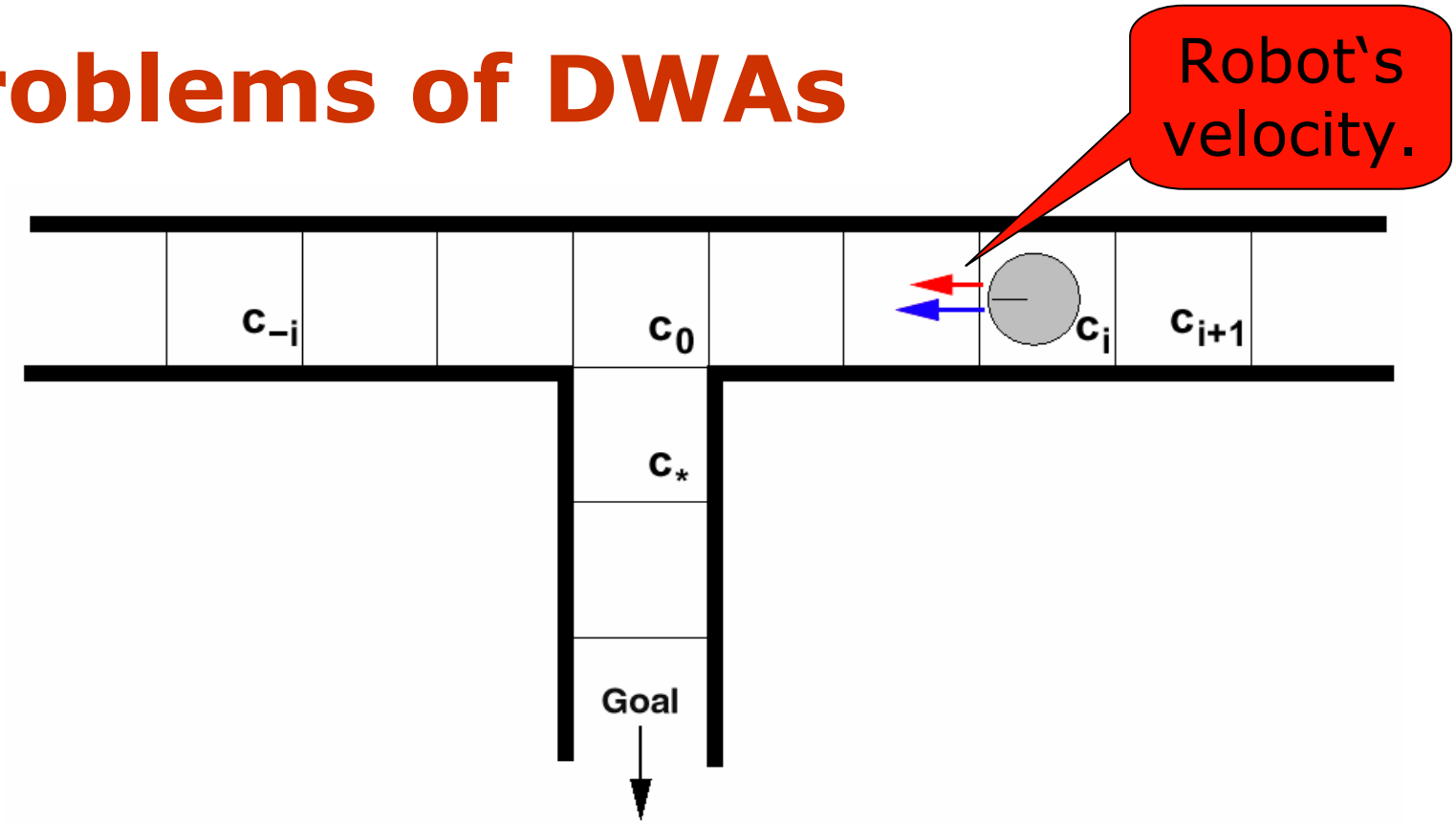
Problems of DWAs



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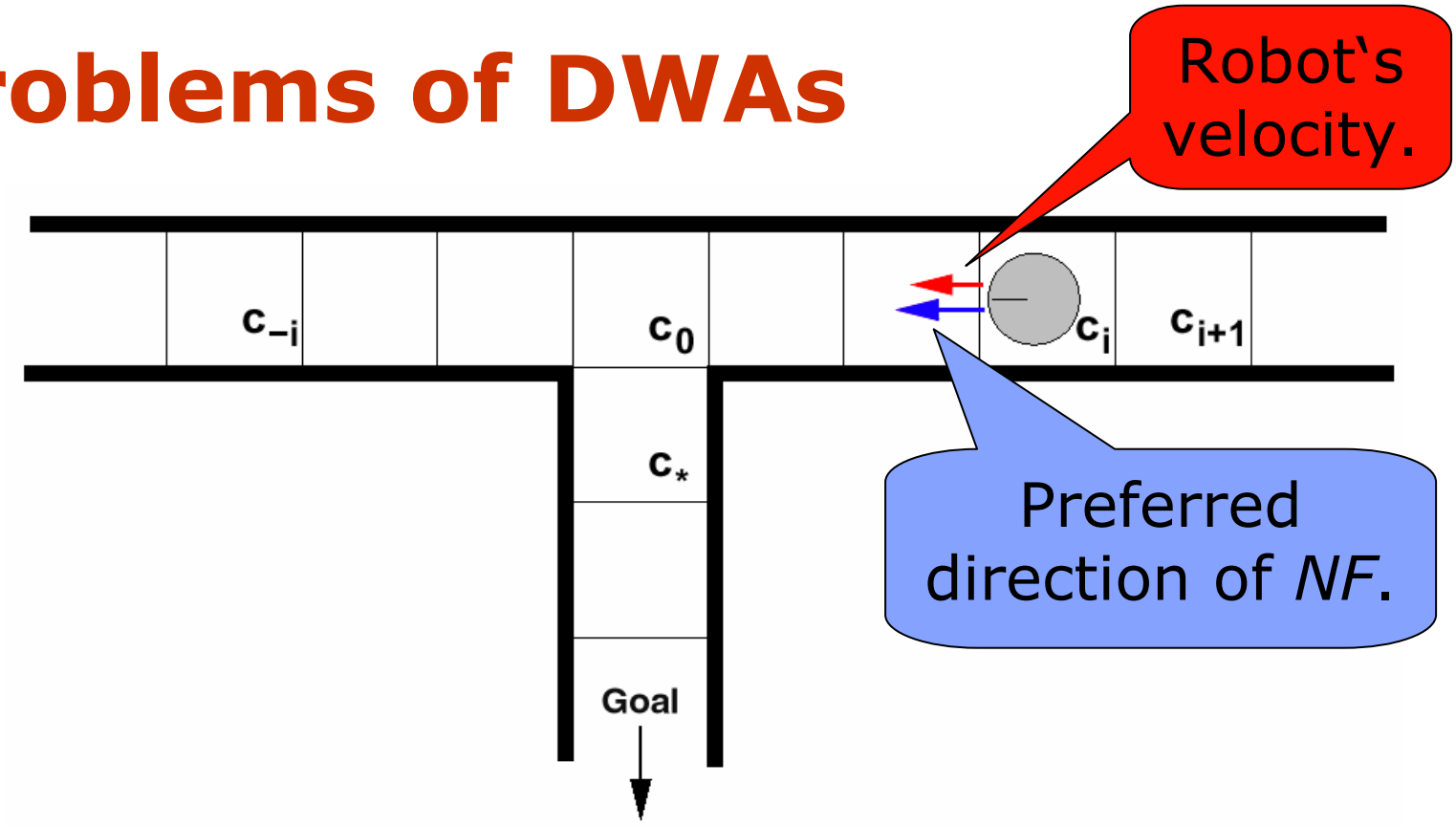

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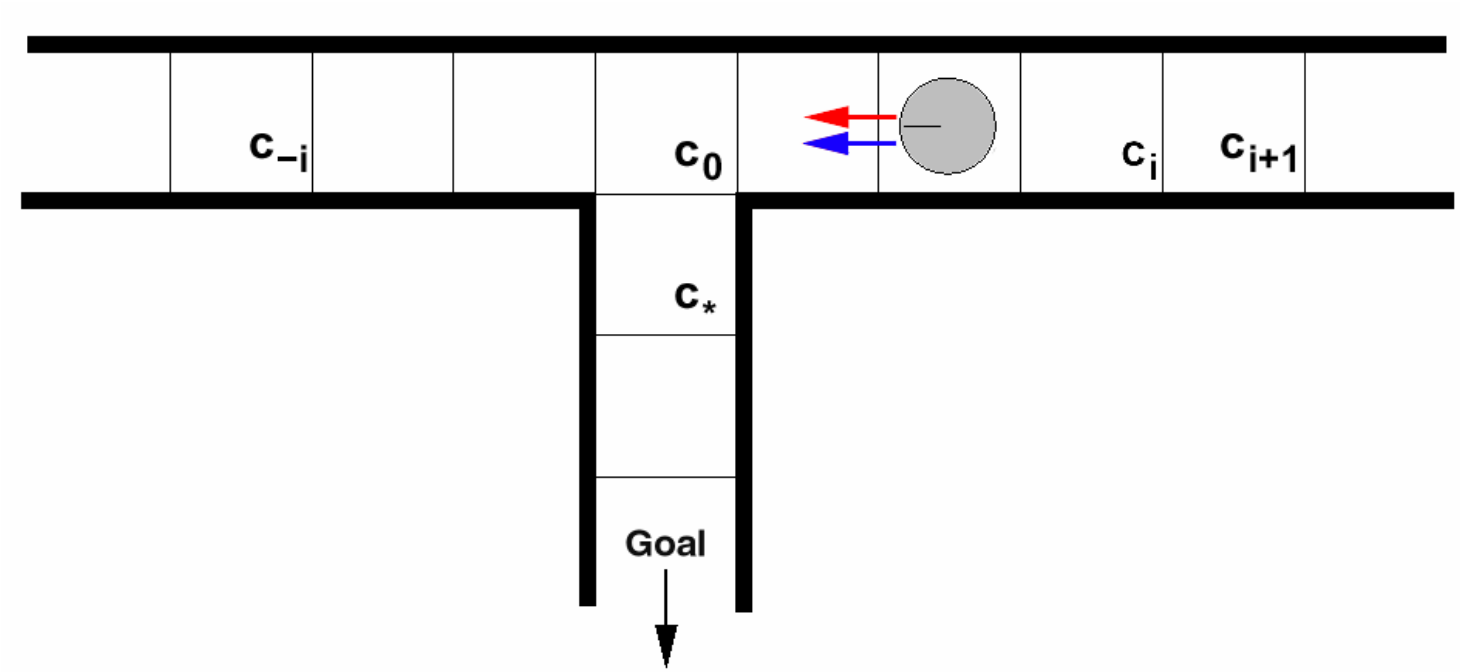

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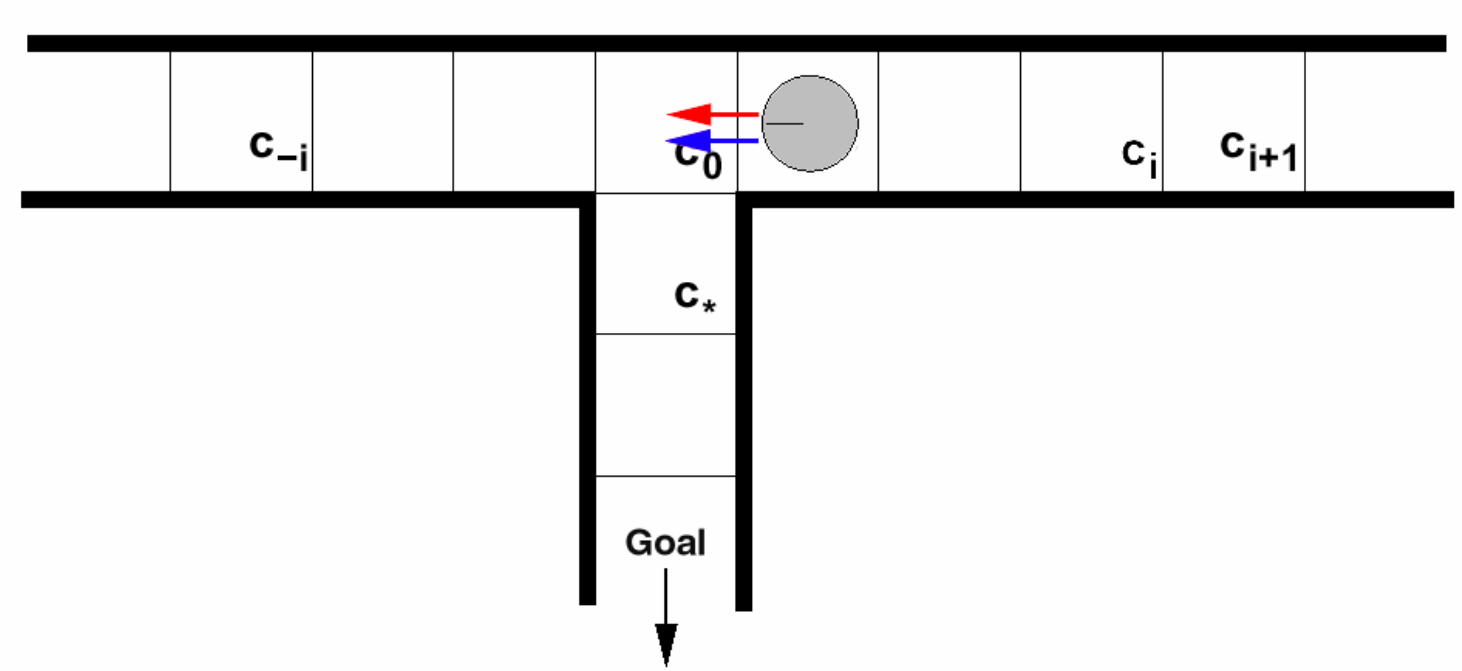

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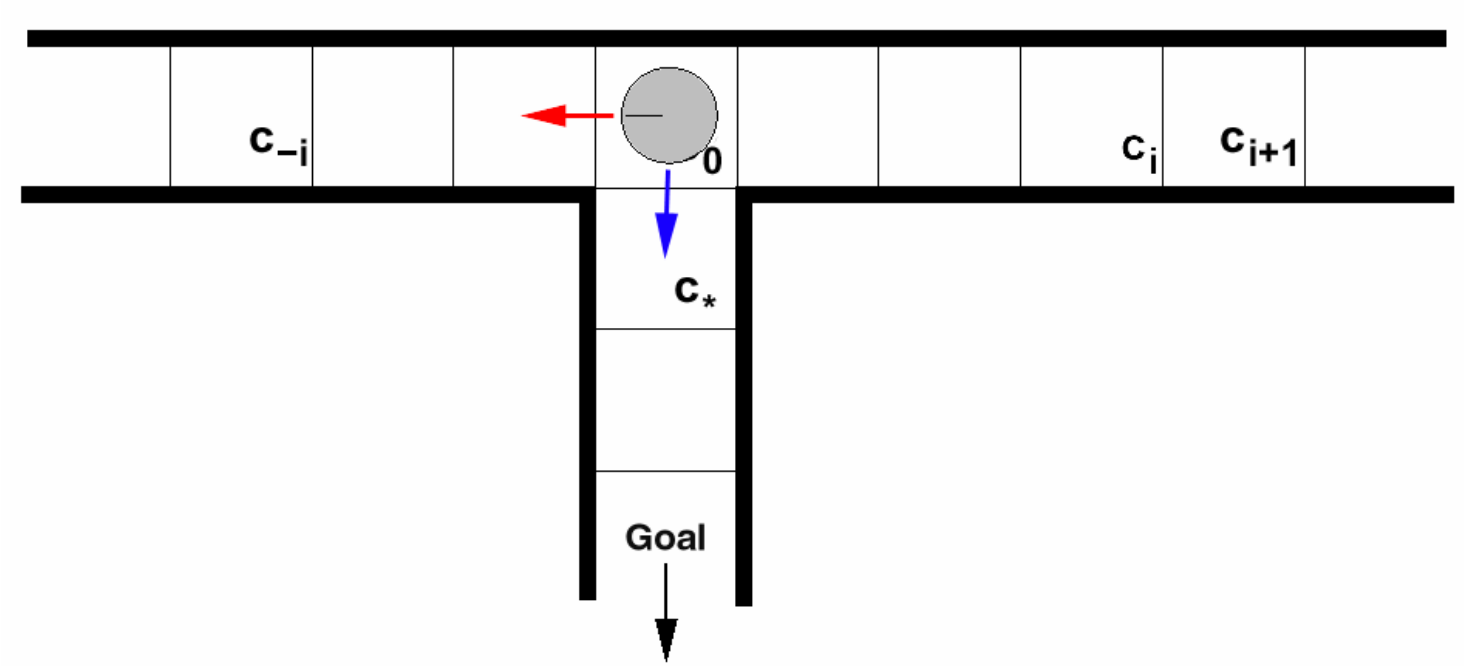
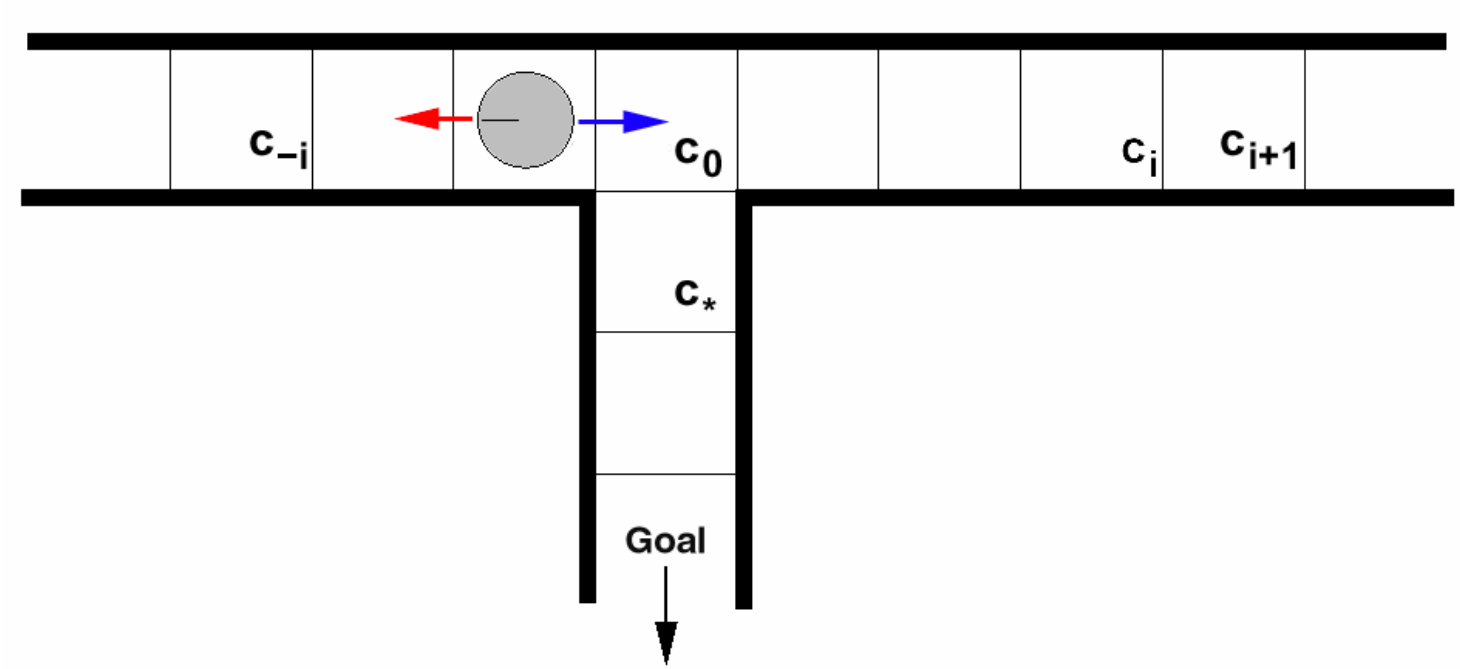


$$NF = \alpha \cdot vel + \beta \cdot nf + \gamma \cdot \Delta nf + \delta \cdot goal$$

- The robot drives too fast at c_0 to enter corridor facing south.

Problems of DWAs



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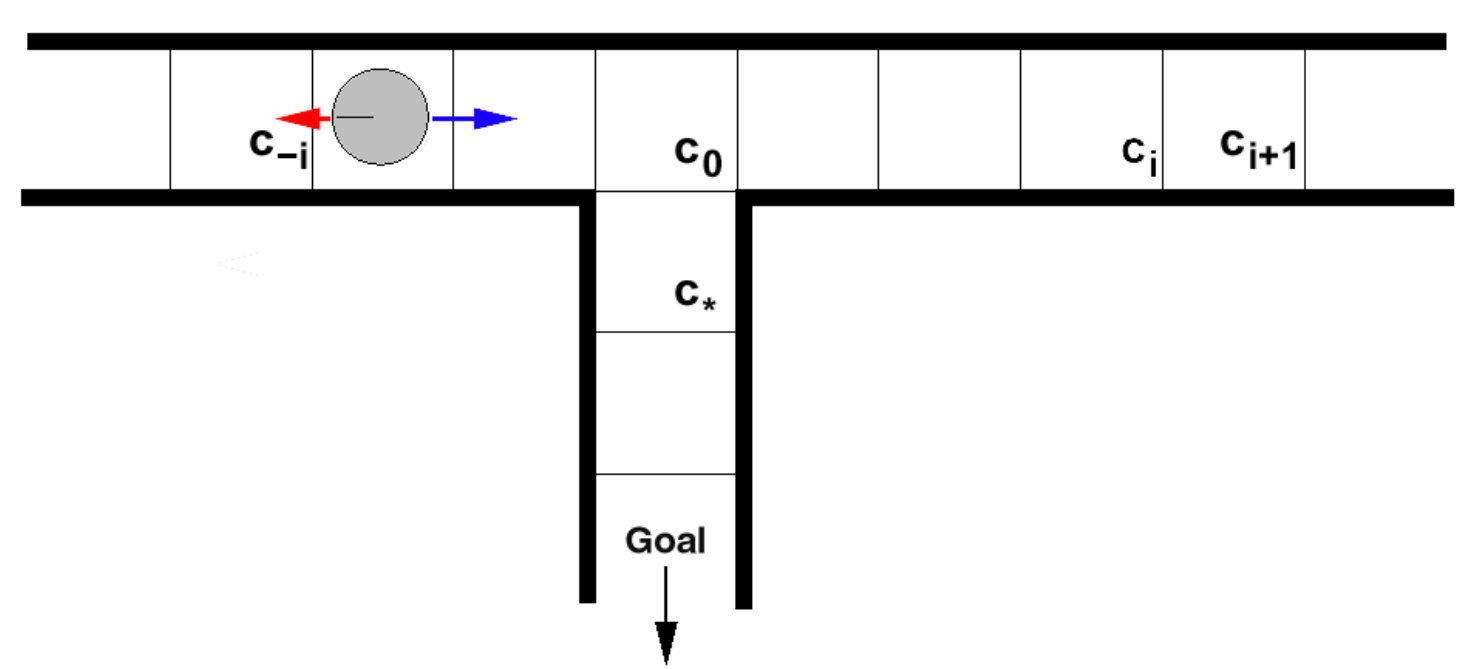

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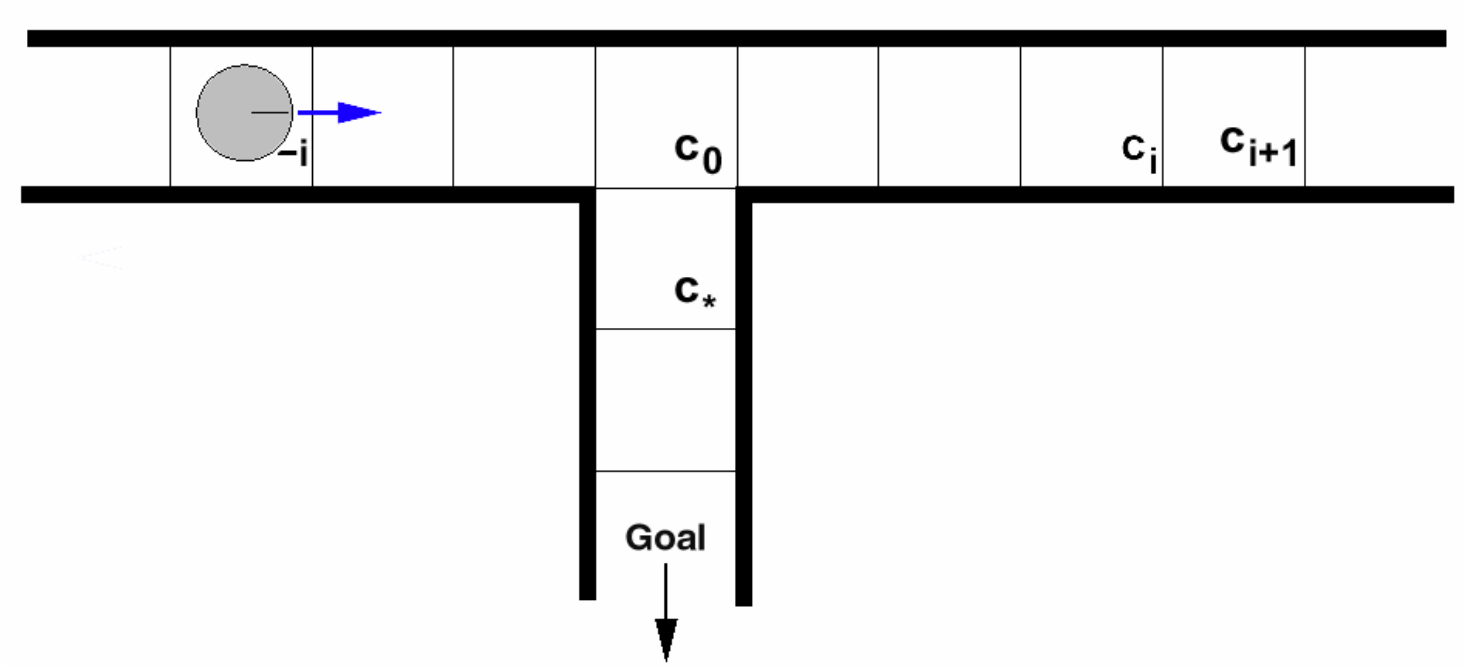
Problems of DWAs

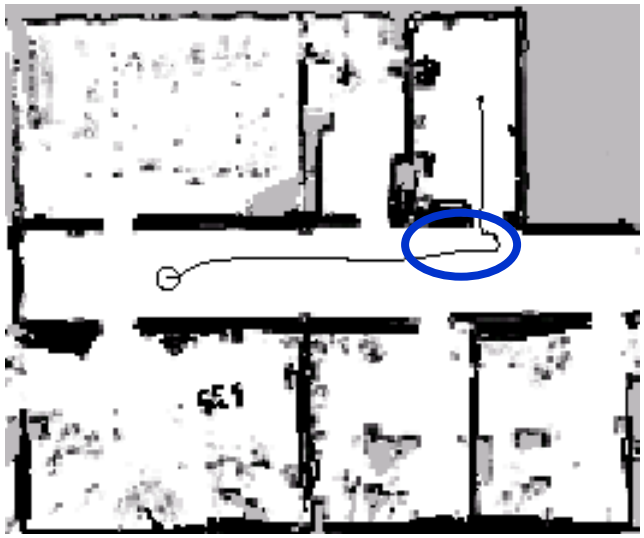


- Same situation as in the beginning.

  ➔ DWAs have problems to reach the goal.

Problems of DWAs

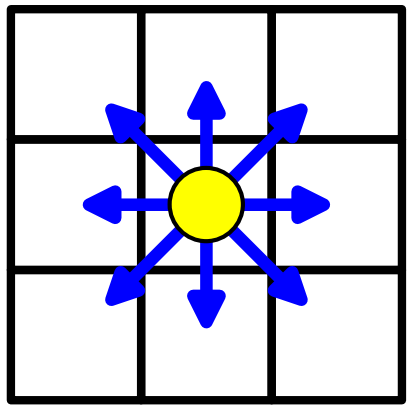
- Also problems in real world situations:



- Robot does not slow down early enough to enter the doorway.

Robot Path Planning with A*

- What about using A* to plan the path of a robot?
- Finds the shortest path
- Requires a graph structure
- Limited number of edges
- In robotics: planning on a 2d occupancy grid map



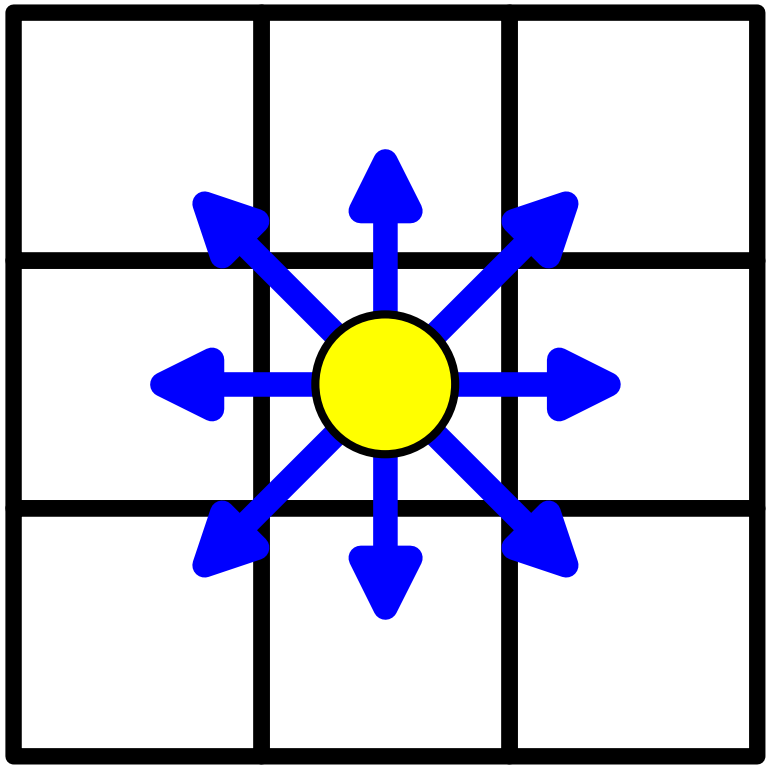
A*: Minimize the estimated path costs

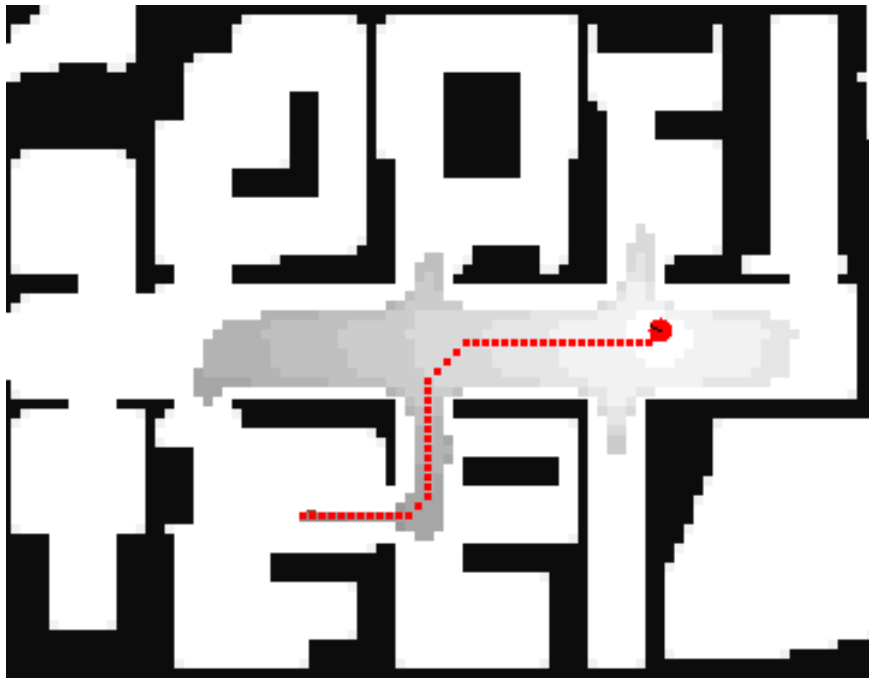
- $g(n)$ = actual cost from the initial state to n.
- $h(n)$ = estimated cost from n to the next goal.
- $f(n) = g(n) + h(n)$, the estimated cost of the cheapest solution through n.
- Let h*(n) be the actual cost of the optimal path from n to the next goal.
- h is admissible if the following holds for all n :

$$h(n) \leq h^*(n)$$

- We require that for A*, h is admissible (the straight-line distance is admissible in the Euclidean Space).

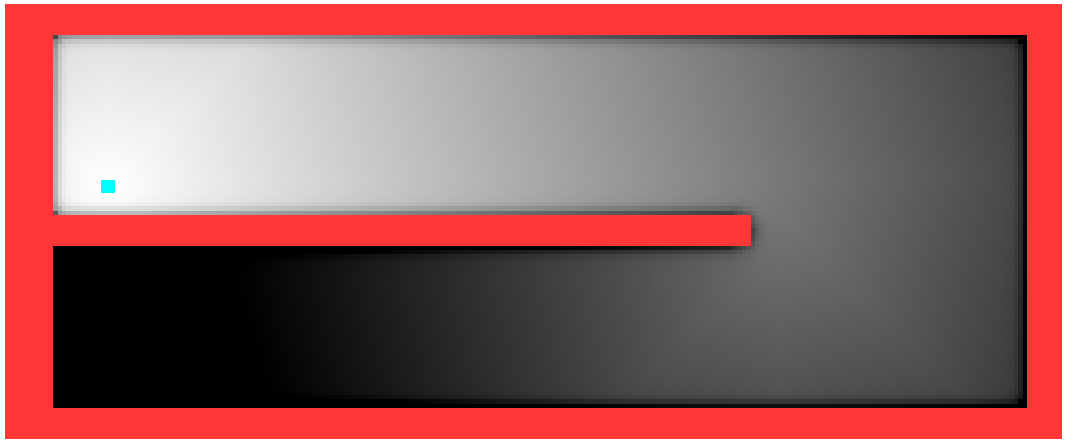
Example: Path Planning for Robots in a Grid-World





Deterministic Value Iteration

- To compute the shortest path from every state to one goal state, use (deterministic) value iteration.
- Very similar to Dijkstra's Algorithm.
- Such a cost distribution is the optimal heuristic for A*.



Typical Assumption in Robotics for A* Path Planning

- A robot is assumed to be localized.
- Often a robot has to compute a path based on an occupancy grid.
- Often the correct motion commands are executed (but no perfect world!).

Is this always true?

Problems

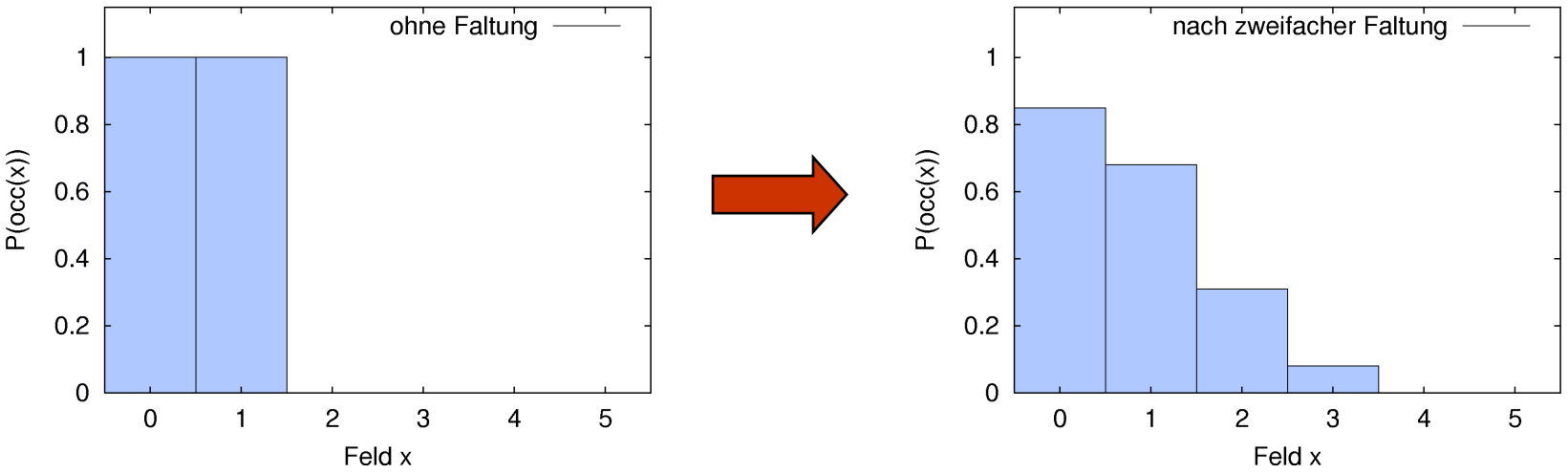
- What if the robot is slightly delocalized?
- Moving on the shortest path guides often the robot on a trajectory close to obstacles.

Convolve the Grid Map

- Convolution blurs out the map.
- Obstacles are assumed to be bigger than in reality.
- Perform an A* search in such a convolved map.
- Robots keeps distance to obstacles and moves on a short path!

Example: Map Convolution

- 1-d environment, cells c_0, …, c_5



- Cells before and after 2 convolution runs.

Convolution

- Consider an occupancy map. Than the convolution is defined as:

$$P(occ_{x_i,y}) = \frac{1}{4} \cdot P(occ_{x_{i-1},y}) + \frac{1}{2} \cdot P(occ_{x_i,y}) + \frac{1}{4} \cdot P(occ_{x_{i+1},y})$$

$$P(occ_{x_0,y}) = \frac{2}{3} \cdot P(occ_{x_0,y}) + \frac{1}{3} \cdot P(occ_{x_1,y})$$

$$P(occ_{x_{n-1},y}) = \frac{1}{3} \cdot P(occ_{x_{n-2},y}) + \frac{2}{3} \cdot P(occ_{x_{n-1},y})$$

- This is done for each row and each column of the map.
- “Gaussian blur”

A* in Convolved Maps

- The costs are a product of path length and occupancy probability of the cells.
- Cells with higher probability (e.g. caused by convolution) are shunned by the robot.
- Thus, it keeps distance to obstacles.
- This technique is fast and quite reliable.

Key Ideas of the Presented Approach (5d-Planning)

- Plans in the full <x,y,θ,v,ω> configuration space using A^*.
  - ➔ considers the robot's kinematic contraints.
- Generates a sequence of steering commands to reach the goal location.
- Maximizes tradeoff between driving time and distance to obstacles.

The Search Space (1)

- What is a state in this space?
  $\langle x,y,\theta,v,\omega \rangle$ = current position and speed of the robot

- How does a state transition look like?
  $\langle x_1,y_1,\theta_1,v_1,\omega_1 \rangle \rightarrow \langle x_2,y_2,\theta_2,v_2,\omega_2 \rangle$
  with motion command (v_2,ω_2) and
  $|v_1-v_2| < a_v$, $|\omega_1-\omega_2| < a_\omega$. Pose of the Robot is a result of the motion equations.

The Search Space (2)

Idea: Search in discretized <x,y,θ,v,ω>-space.

Problem: The search space is too huge to be explored within the time constraints (.25 secs for online control).

Solution: Restrict the full search space.

The Main Steps of Our Algorithm

1. Update (static) grid map based on sensory input.

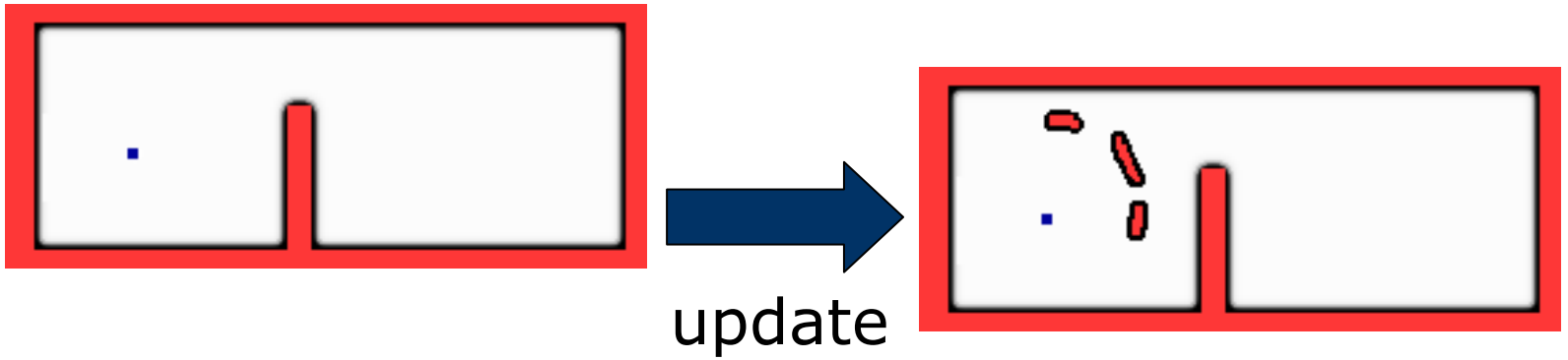
2. Use A^* to find a trajectory in the <x,y>-space using the updated grid map.

3. Determine a restricted 5d-configuration space based on step 2.

4. Find a trajectory by planning in the restricted <x,y,θ,v,ω>-space.

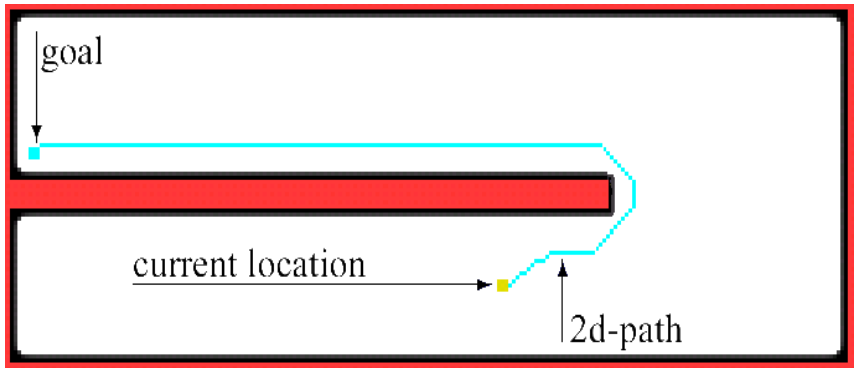
Updating the Grid Map

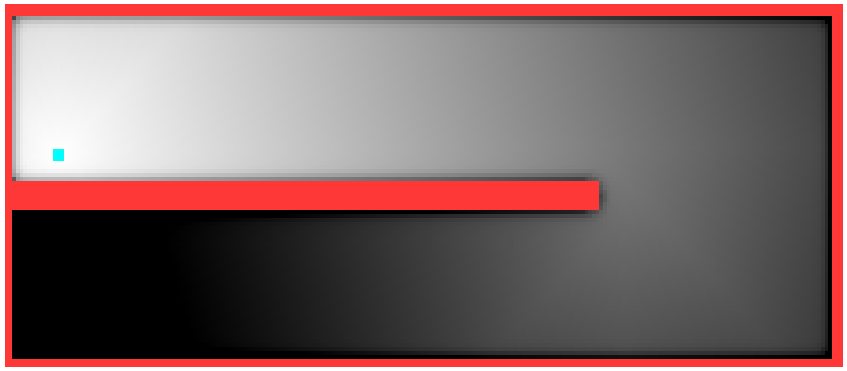
- The environment is represented as a 2d-occupency grid map.
- Use convolved map.
- All detected obstacles are added.
- We reset cells discovered free.



Find a Path in the 2d-Space

- Use A* to search for the optimal path in the 2d-grid map.
- Use heuristic based on a deterministic value iteration within the static map.





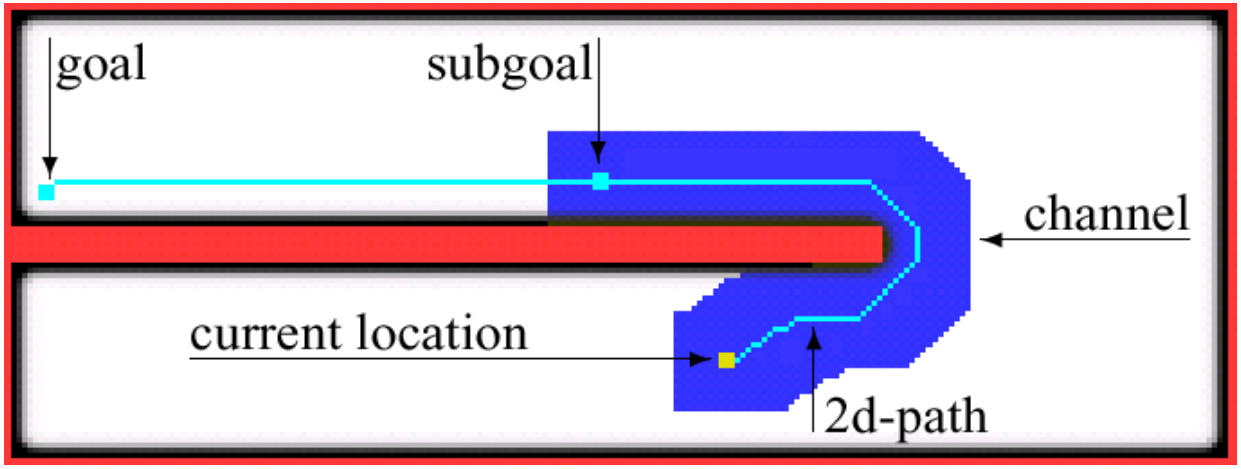
Restricting the Search Space

Assumption: The projection of the 5d-path onto the <x,y>-space lies close to the optimal 2d-path.

Therefore: Construct a restricted search space (channel) based on the 2d-path.

Space Restriction

- Resulting search space =
  $<x, y, \theta, v, \omega>$ with $(x,y) \in$ channel.
- Choose a subgoal lying on the 2d-path within the channel.

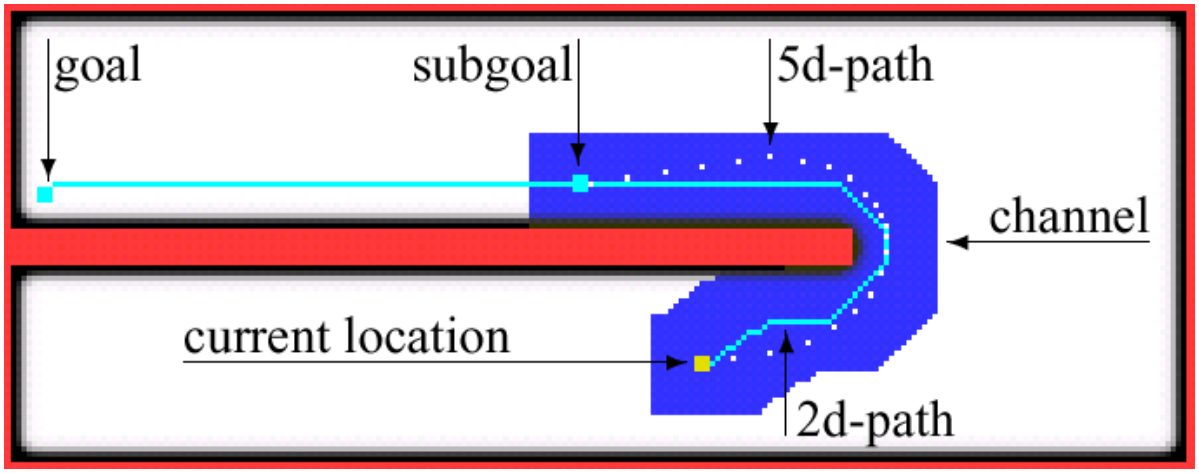


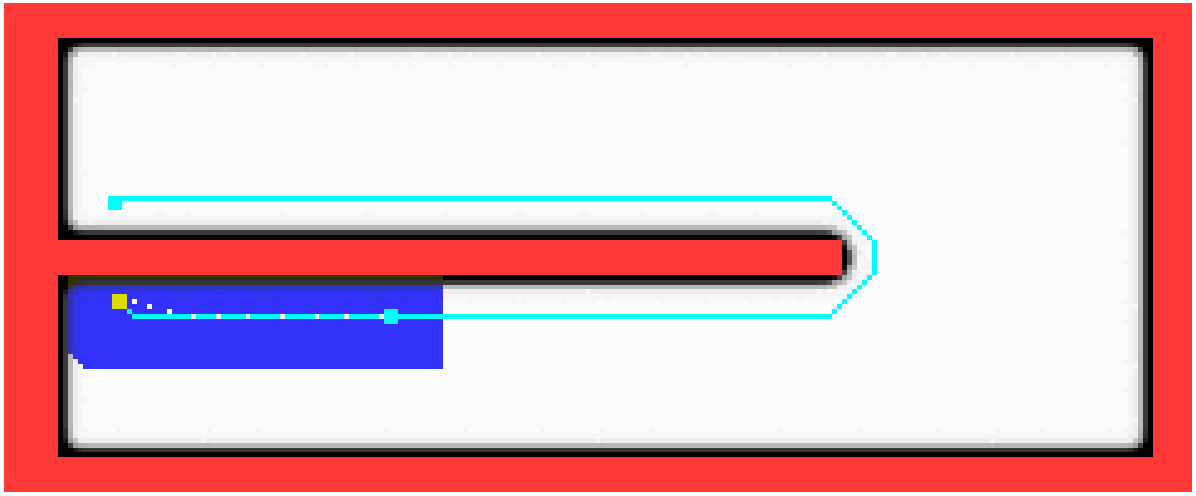
Find a Path in the 5d-Space

- Use A^* in the restricted 5d-space to find a sequence of steering commands to reach the subgoal.

- To estimate cell costs: perfom a deterministic 2d-value iteration within the channel.

Examples





Timeouts

- Online robot control:
  new steering command every .25 secs.

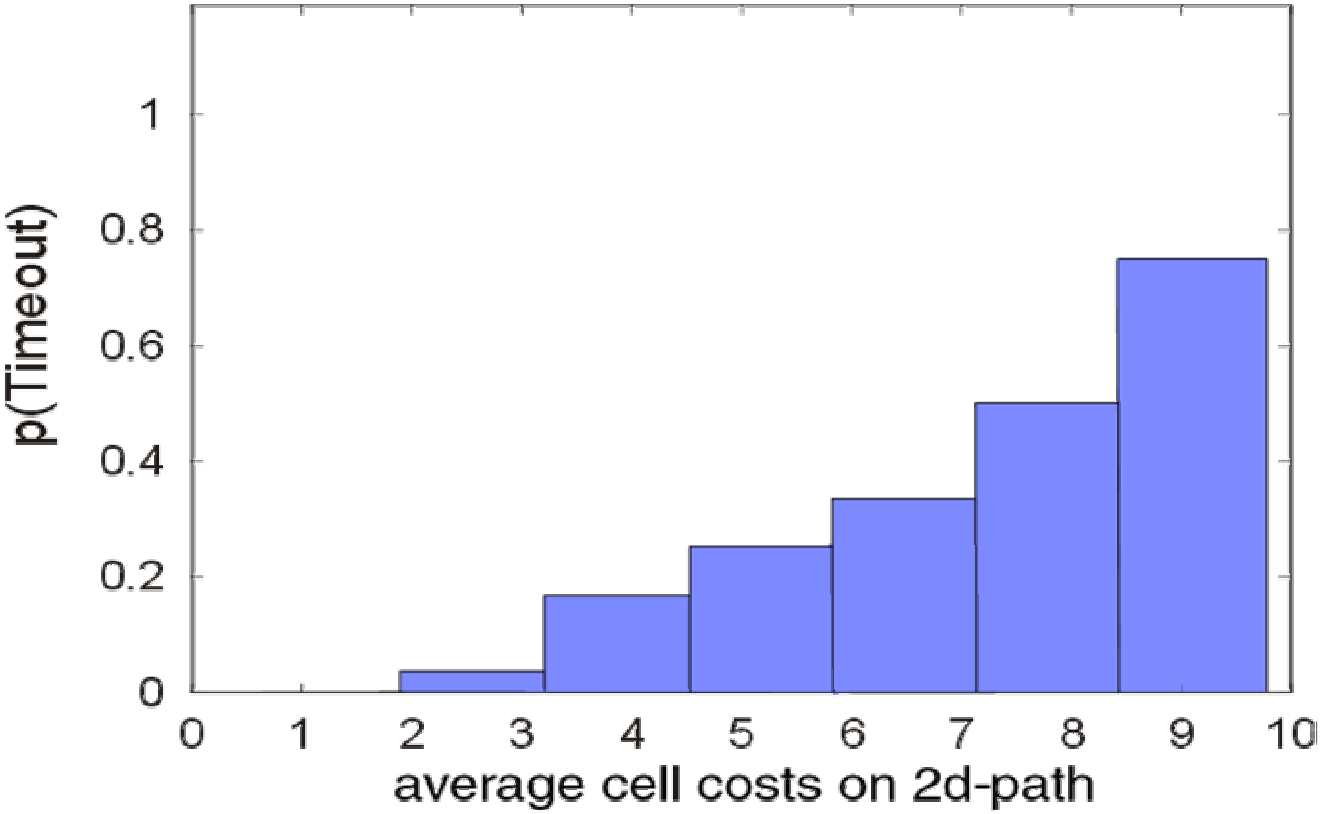
➔ Abort search after .25 secs.

How to find an admissible steering command?

Alternative Steering Command

- Previous trajectory still admissible? ➔ OK
- If not, drive on the 2d-path or use DWA to find new command.

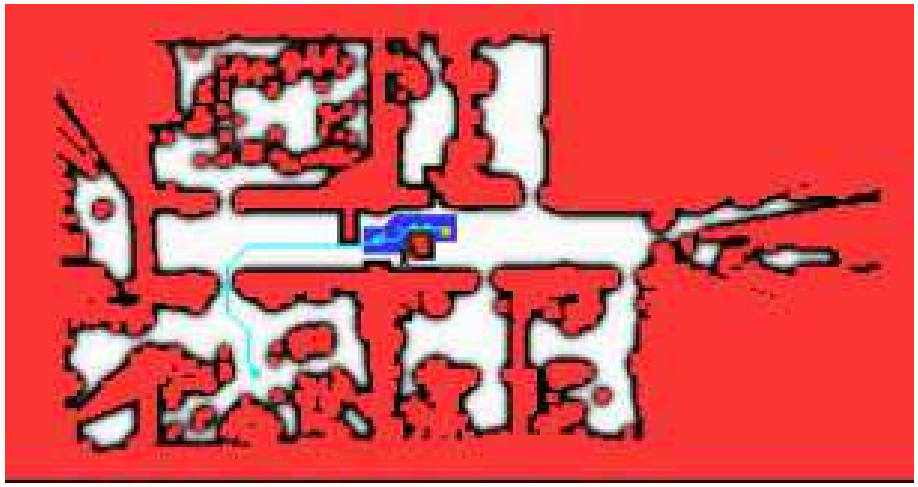
Timeout Avoidance



➔ Reduce the size of the channel if the 2d-path has high cost.

Examples

B21r robot Albert.



Planning state.

Comparison to the DWA (1)

- DWAs often have problems entering narrow passages.

DWA planned path.

Our Approach.

Comparison to the DWA (1)

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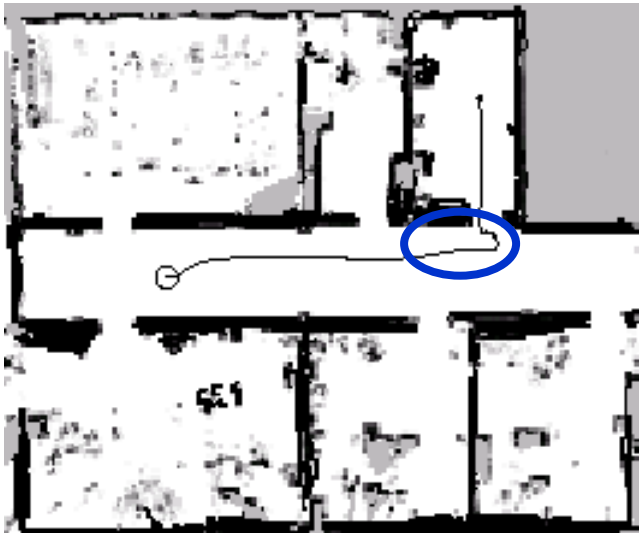
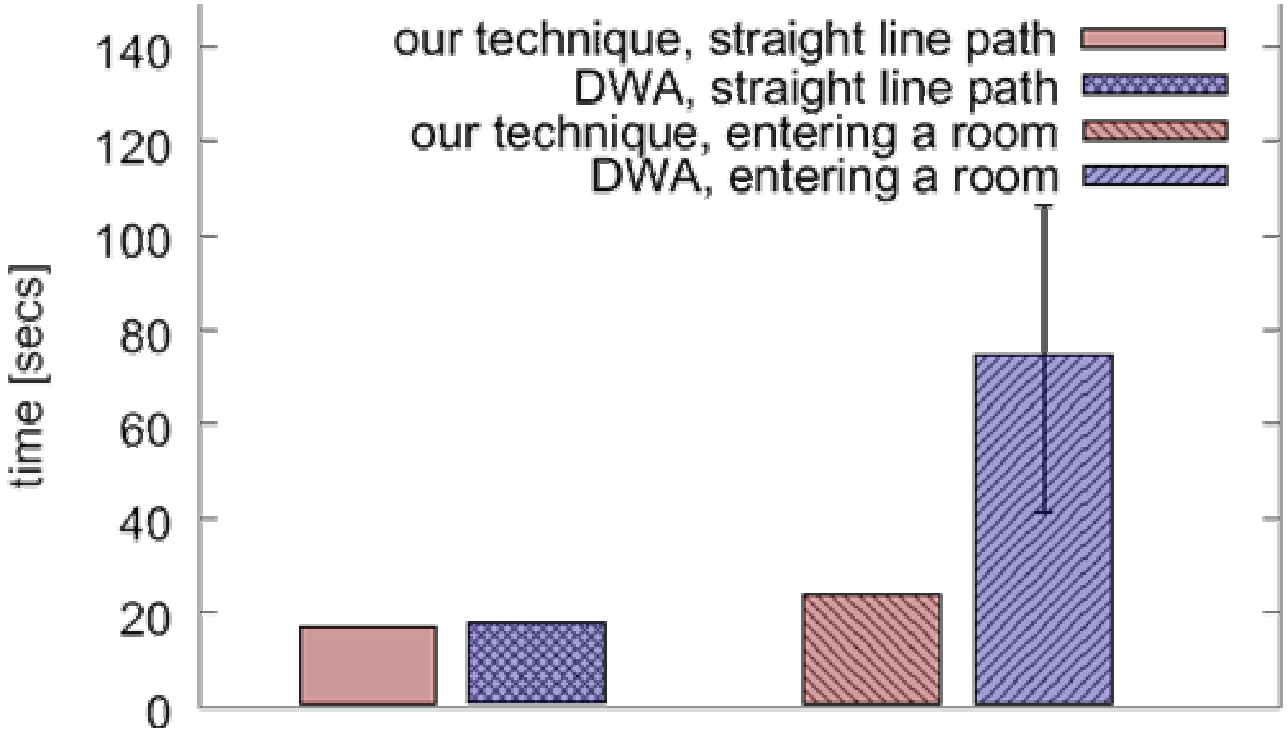


DWA planned path.

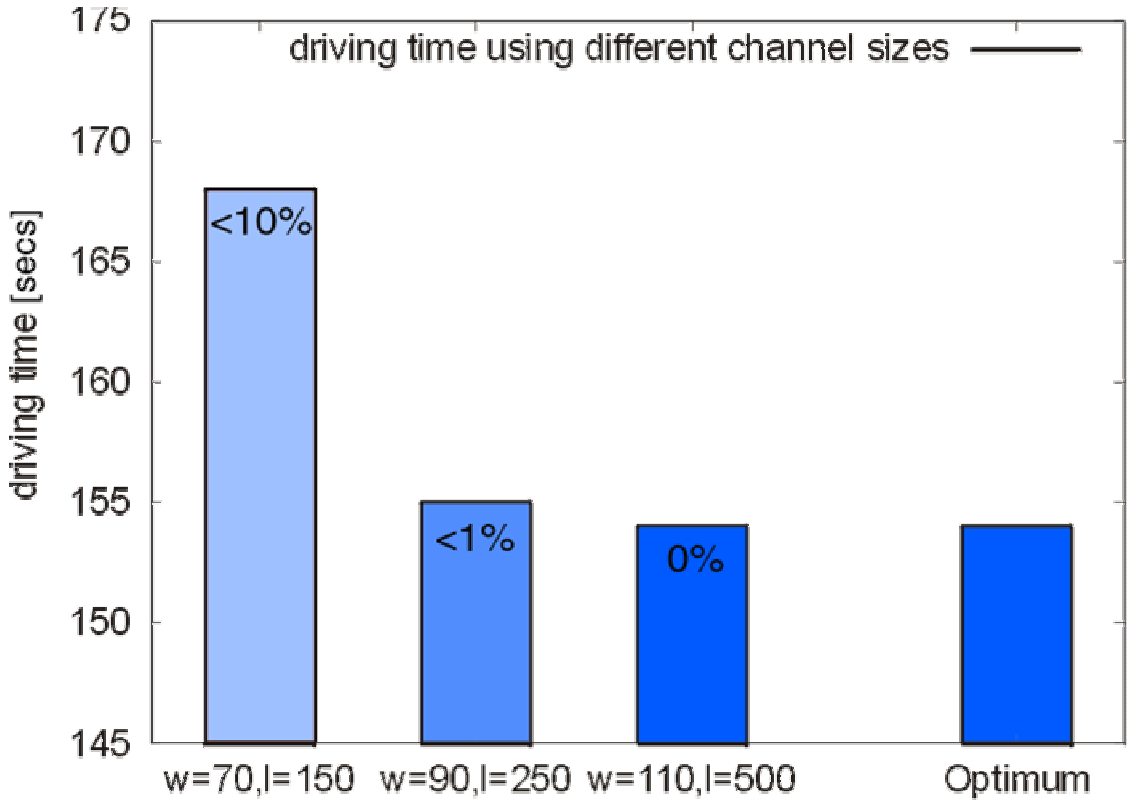
Our Approach.

Comparison to the DWA (2)



➔ The presented approach results in significantly faster motion when driving through narrow passages!

Comparison to the Optimum



- Channel: with length=5m, width=1.1m we are close to the optimal solution.

Summary

- New approach to reactive collision avoidance.
- Considers the robot's kinematic constraints and plans in the velocity space.
- Shows better results than the DWA in a variety of situations.
- The quality of the trajectory scales with the performance of the underlying hardware.
- The resulting paths are often close to the optimal ones.

What's Next?

- More complex vehicles (e.g., cars).
- Moving obstacles, motion prediction.
- Approximative Search
  (ARA*: Not-admissible heuristics
  for faster planning).
- …