Introduction to Mobile Robotics

Information Gain-Based Exploration Using Rao-Blackwellized Particle Filters

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Tasks of Mobile Robots

- SLAM
- Localization
- Mapping
- Path Planning

Exploration and SLAM

- SLAM is typically passive, because it consumes incoming sensor data
- Exploration actively guides the robot to cover the environment with its sensors
- Exploration in combination with SLAM: Acting under pose and map uncertainty
- The uncertainty needs to be taken into account when selecting an action

Factorization Underlying Rao-Blackwellized Mapping

\[ p(x, m \mid z, u) \]
\[ = p(m \mid x, z, u)p(x \mid z, u) \]

Mapping with known poses

Particle filter representing trajectory hypotheses
Combining Rao-Blackwellized Mapping with Exploration

Where to Move Next?

Example

Exploration

- The approaches seen so far are purely passive.
- By reasoning about control, the mapping process can be made much more effective.
- Question: Where to move next?
**Decision-Theoretic Approach**

- Learn the map using a Rao-Blackwellized particle filter
- Consider a set of potential actions
- Apply an exploration approach that minimizes the overall uncertainty

\[
\text{Utility} = \text{uncertainty reduction} - \text{cost}
\]

**The Uncertainty of a Posterior**

- Entropy is a general measure for the uncertainty of a posterior

\[
H(p(x)) = - \int_x p(x) \log p(x) \, dx
\]

\[
= \mathbb{E}_x[-\log(p(x))]
\]

- Information Gain = Uncertainty Reduction

\[
I(t+1 \mid t) = H(p(x_t)) - H(p(x_{t+1}))
\]

**Entropy Computation**

\[
H(p(x, y)) = \mathbb{E}_{x,y}[-\log p(x, y)]
\]

\[
= \mathbb{E}_{x,y}[-\log p(x)p(y \mid x)]
\]

\[
= \mathbb{E}_{x,y}[-\log p(x)] + \mathbb{E}_{x,y}[-\log p(y \mid x)]
\]

\[
= H(p(x)) + \int_{x,y} -p(x, y) \log p(y \mid x) \, dx \, dy
\]

\[
= H(p(x)) + \int_{x,y} -p(y \mid x)p(x) \log p(y \mid x) \, dx \, dy
\]

\[
= H(p(x)) + \int_x p(x) \int_y -p(y \mid x) \log p(y \mid x) \, dy \, dx
\]

\[
= H(p(x)) + \int_x p(x) H(p(y \mid x)) \, dx
\]

**Computing the Map and Pose Uncertainty**

\[
H(p(x, m \mid d)) = H(p(x \mid d)) + \int_x p(x \mid d) H(p(m \mid x, d)) \, dx
\]

\[
\approx H(p(x \mid d)) + \sum_{i=1}^{\text{#particles}} \omega[i] H(p(m[i] \mid x[i], d))
\]

\text{trajectory uncertainty} \quad \text{particle weight} \quad \text{map uncertainty}
Computing the Entropy of the Map Posterior

Occupancy Grid map \( m \):

\[
H(p(m)) = - \sum_{c \in m} p(c) \log p(c) + (1 - p(c)) \log(1 - p(c))
\]

- map uncertainty
- grid cells
- probability that the cell is occupied

Computing the Entropy of the Trajectory Posterior

1. High-dimensional Gaussian

\[
H(G(\mu, \Sigma)) = \log((2\pi e)^{W/2} |\Sigma|)
\]

reduced rank for sparse particle sets

2. Grid-based approximation

\[
H(p(x \mid d)) \sim \text{const.}
\]

for sparse particle clouds

Approximation of the Trajectory Posterior Entropy

Average pose entropy over time:

\[
H(p(x_{1:t} \mid d)) \approx \frac{1}{t} \sum_{t'=1}^{t} H(p(x_{t'} \mid d))
\]

Information Gain

- The reduction of entropy in the model

\[
I(\hat{z}, a) = H(p(m, x \mid d)) - H(p(m, x, \hat{x} \mid d, a, \hat{z}))
\]

observations to be obtained

action

H before action is carried out

new poses introduced by action

H after action is carried out
Computing the Expected Information Gain

- To compute the information gain one needs to know the observations obtained when carrying out an action.
- This quantity is not known! Reason about potential measurements:

\[
E[I(a)] = \int \tilde{z} p(\tilde{z} | a, d) \cdot I(\tilde{z}, a) \, d\tilde{z}
\]

Reasoning about Measurements

- The filter represents a posterior about possible maps.
- Use these maps to reason about possible observations.
- Simulate laser measurements in the maps of the particles:

\[
E[I(a)] = \int \tilde{z} p(\tilde{z} | a, d) \cdot I(\tilde{z}, a) \, d\tilde{z}
\]

The Utility

- To take into account the cost of an action, we compute a utility:

\[
U(a) = I(a) - \alpha \cdot \text{cost}(a)
\]

- Select the action with the highest expected utility:

\[
a^* = \arg\max_a \{ E[U(a)] \} 
\]
Focusing on Specific Actions

To efficiently sample actions we consider
- exploratory actions (1-3)
- loop closing actions (4) and
- place revisiting actions (5)

Dual Representation for Loop Detection

- **Trajectory graph** ("topological map") stores the path traversed by the robot
- **Occupancy grid** map represents the space covered by the sensors

- Loops correspond to long paths in the trajectory graph and short paths in the grid map

Example: Trajectory Graph

Application Example

High pose uncertainty
**Example: Move Robot**

![Graph showing expected utility of target locations over time steps.]  

**Example: Entropy Evolution**

![Graph showing entropy over time steps.]  

**Comparison**

Map uncertainty only:

![Map showing initial exploration path.]  

After loop closing action:

![Map showing updated exploration path.]  

**Real Exploration Example**

![Map showing real exploration example.]
Summary

- A decision-theoretic approach to exploration in the context of RBPF-SLAM
- The approach utilizes the factorization of the Rao-Blackwellization to efficiently calculate the expected information gain
- Reasons about measurements obtained along the path of the robot
- Considers a reduced action set consisting of exploration, loop-closing, and place-revisiting actions
- Experimental results demonstrate the usefulness of the overall approach