Introduction to Mobile Robotics

Information Gain-Based Exploration Using Rao-Blackwellized Particle Filters

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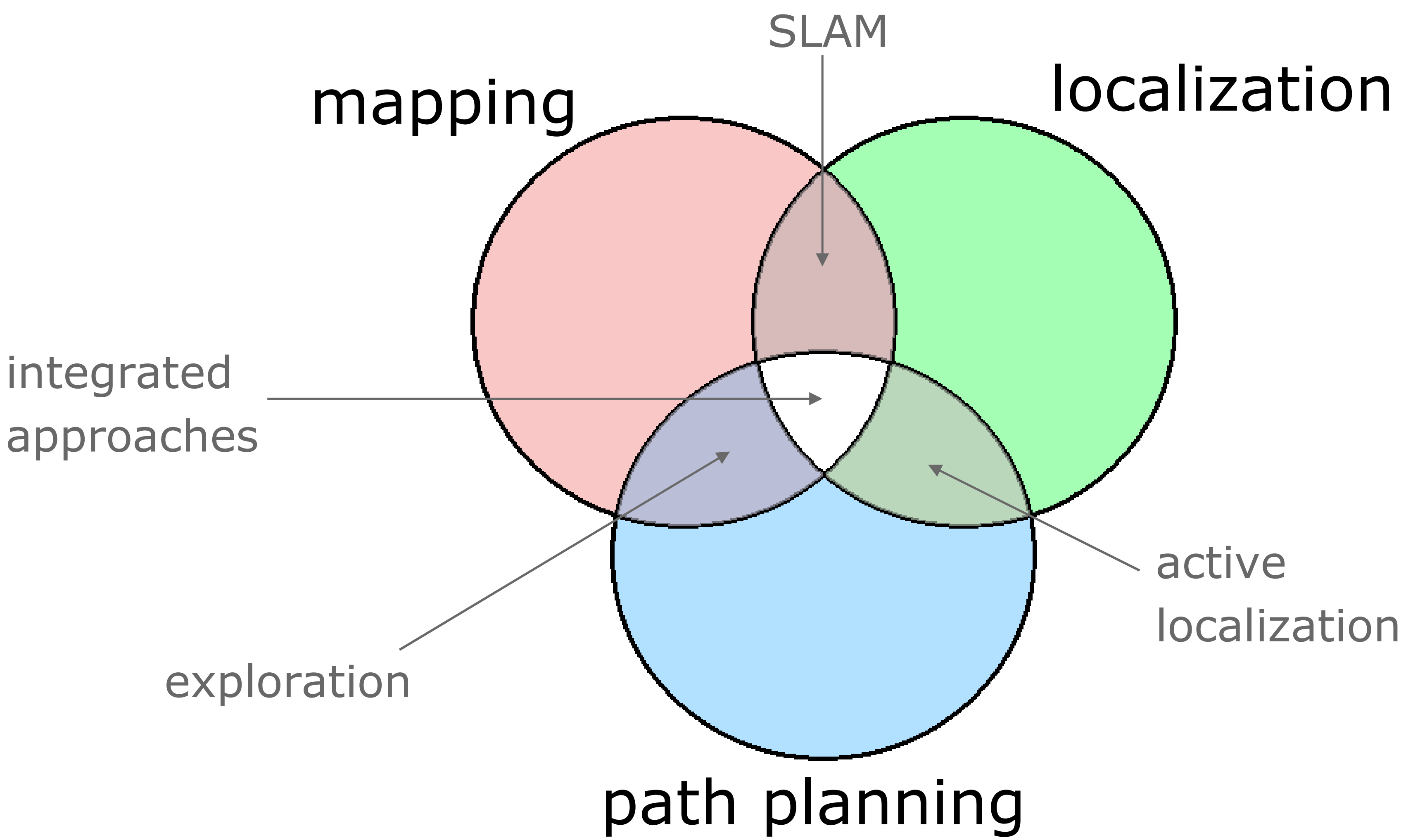
Tasks of Mobile Robots



Exploration and SLAM

- SLAM is typically passive, because it consumes incoming sensor data
- Exploration actively guides the robot to cover the environment with its sensors
- Exploration in combination with SLAM: Acting under pose and map uncertainty
- The uncertainty needs to be taken into account when selecting an action

Factorization Underlying Rao-Blackwellized Mapping

poses map observations & odometry

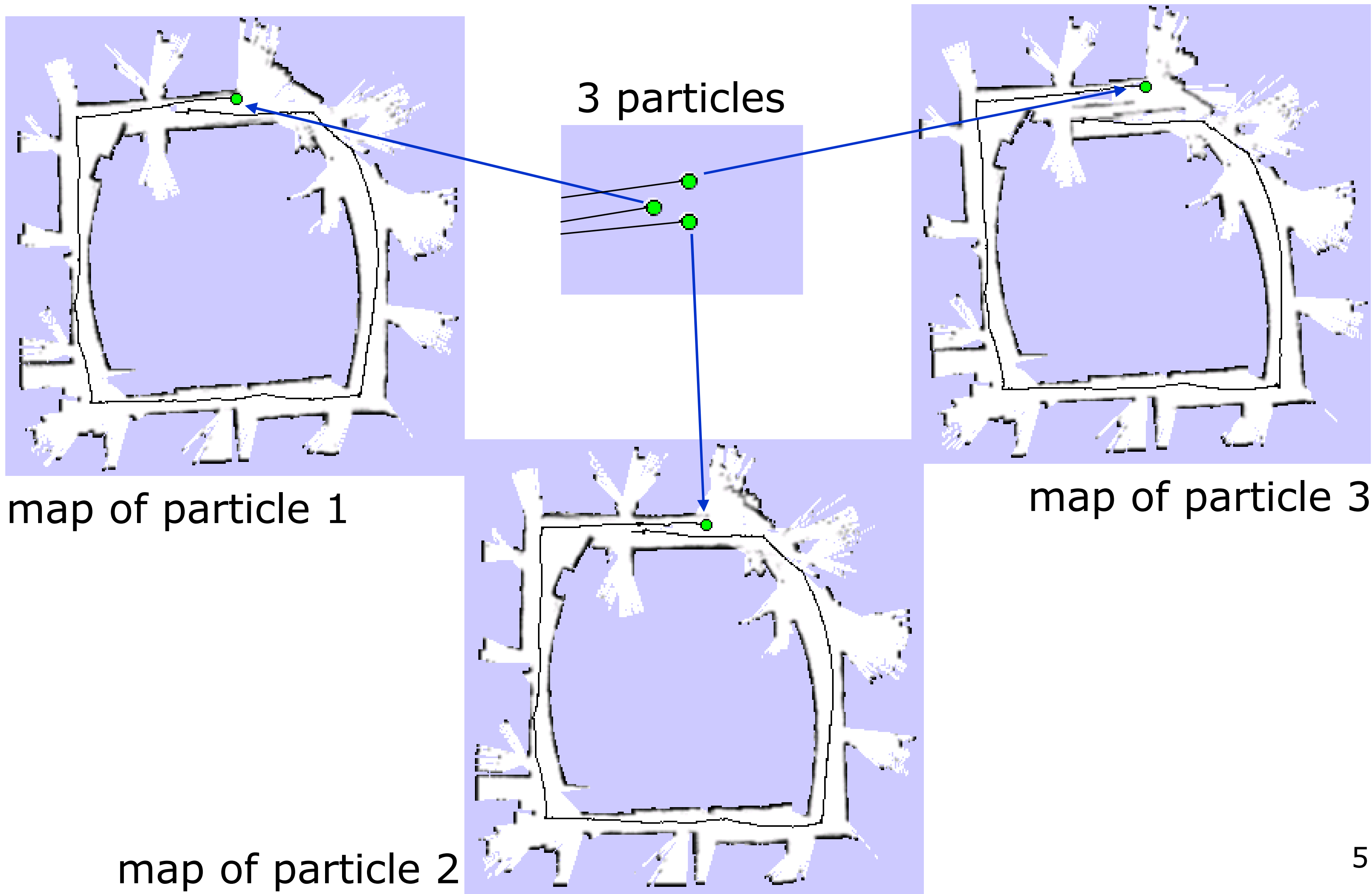
$$p(x, m \mid z, u)$$

$$= \; p(m \mid x, z, \cancel{u})\, p(x \mid z, u)$$

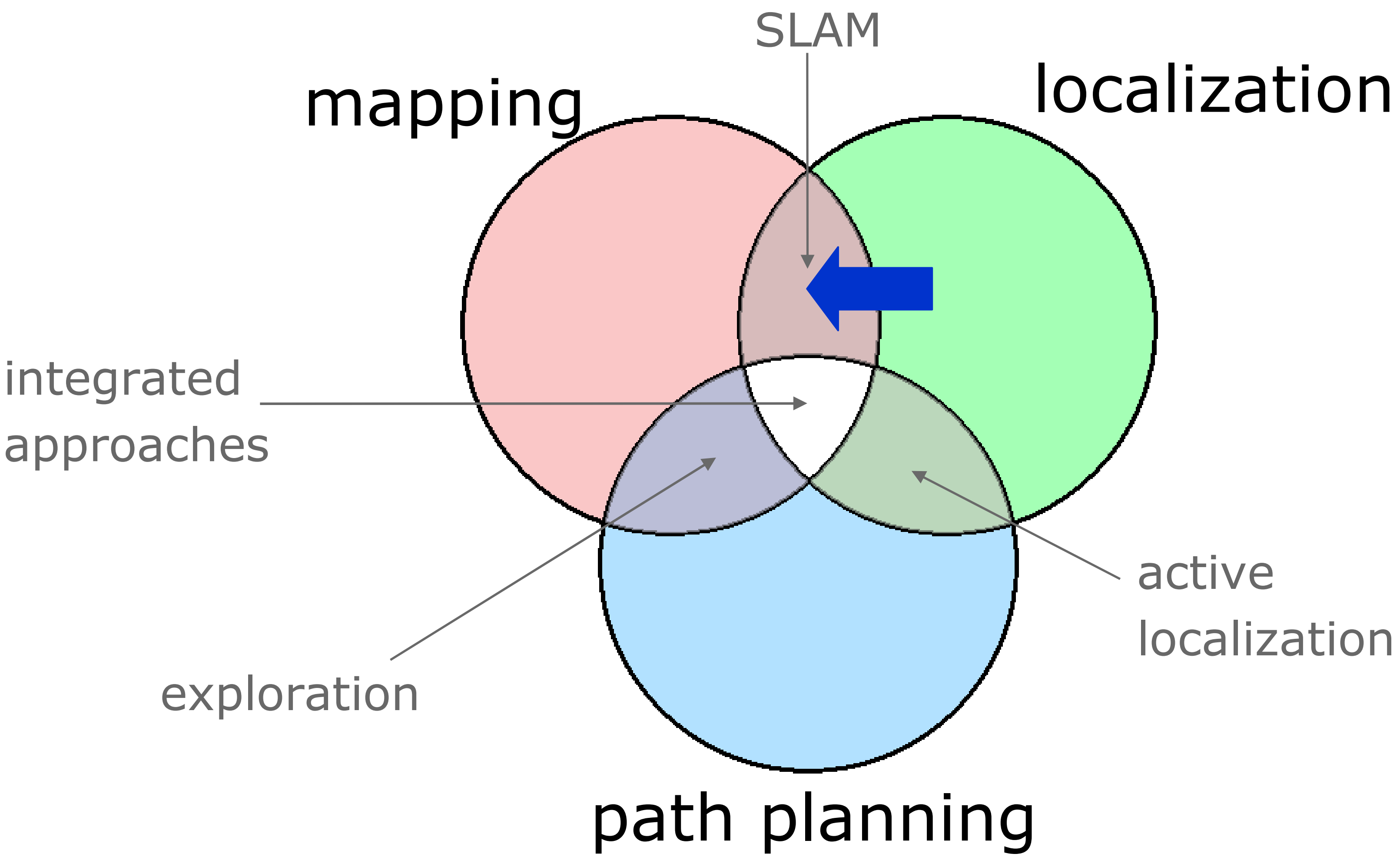
Mapping with known poses

Particle filter representing trajectory hypotheses

Example


3 particles
map of particle 1
map of particle 3
map of particle 2

Combining Rao-Blackwellized Mapping with Exploration



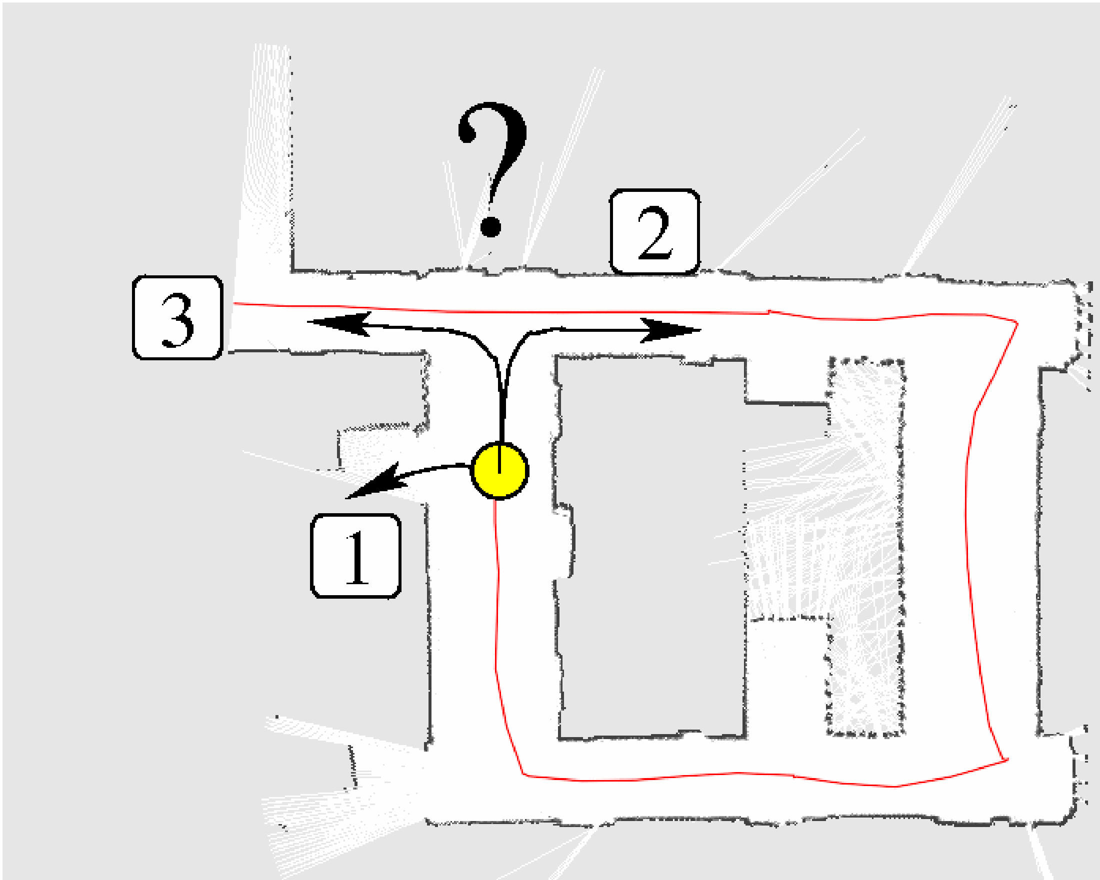
Exploration

- The approaches seen so far are purely passive

- By reasoning about control, the mapping process can be made much more effective

- Question: **Where to move next?**

Where to Move Next?



Decision-Theoretic Approach

- Learn the map using a Rao-Blackwellized particle filter
- Consider a set of potential actions
- Apply an exploration approach that minimizes the overall uncertainty

Utility = uncertainty reduction - cost

The Uncertainty of a Posterior

- Entropy is a general measure for the uncertainty of a posterior

$$\begin{aligned} H(p(x)) &= -\int_x p(x) \log p(x)\, dx \\ &= E_x[-\log(p(x))] \end{aligned}$$

- Information Gain = Uncertainty Reduction

$$I(t+1 \mid t) = H(p(x_t)) - H(p(x_{t+1}))$$

Entropy Computation

$$
\begin{aligned}
& H(p(x,y)) \\
&= E_{x,y}[-\log p(x,y)] \\
&= E_{x,y}[-\log(p(x)\, p(y \mid x))] \\
&= E_{x,y}[-\log p(x)] + E_{x,y}[-\log p(y \mid x)] \\
&= H(p(x)) + \int_{x,y} -p(x,y) \log p(y \mid x)\; dx\; dy \\
&= H(p(x)) + \int_{x,y} -p(y \mid x) p(x) \log p(y \mid x)\; dx\; dy \\
&= H(p(x)) + \int_{x} p(x) \int_{y} -p(y \mid x) \log p(y \mid x)\; dy\; dx \\
&= H(p(x)) + \int_{x} p(x) H(p(y \mid x))\; dx
\end{aligned}
$$

Computing the Map and Pose Uncertainty

data (laser and odometry)

$$H(p(x, m \mid d))$$

$$= H(p(x \mid d)) + \int_x p(x \mid d) H(p(m \mid x, d))\, dx$$

$$\approx H(p(x \mid d)) + \sum_{i=1}^{\#particles} \omega^{[i]} H(p(m^{[i]} \mid x^{[i]}, d))$$

trajectory uncertainty

particle weight

map uncertainty

Computing the Entropy of the Map Posterior

Occupancy Grid map *m*:

$$H(p(m)) = -\sum_{c \in m} p(c) \log p(c) + (1 - p(c)) \log(1 - p(c))$$

map uncertainty

grid cells

probability that the cell is occupied

Computing the Entropy of the Trajectory Posterior

1. High-dimensional Gaussian

$$H(\mathcal{G}(\mu, \Sigma)) \;=\; \log((2\pi e)^{(n/2)}|\Sigma|)$$

reduced rank for sparse particle sets

2. Grid-based approximation

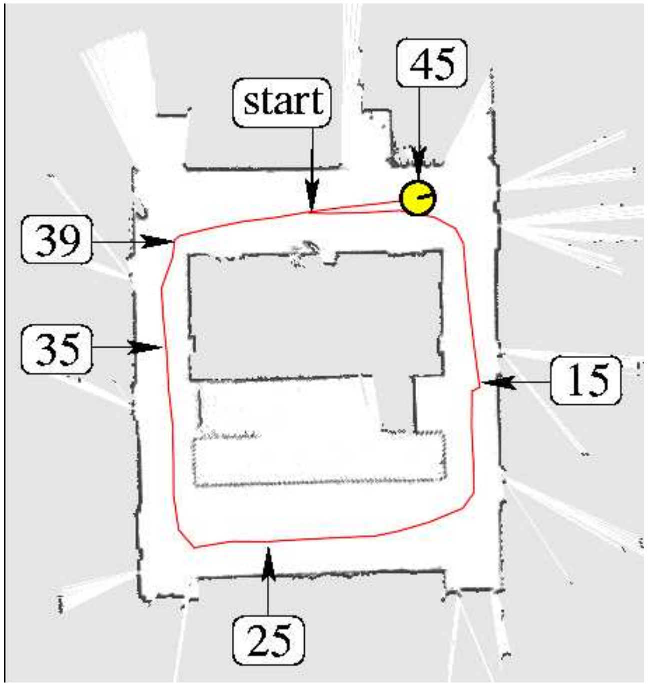
$$H(p(x \mid d)) \;\leadsto\; const.$$

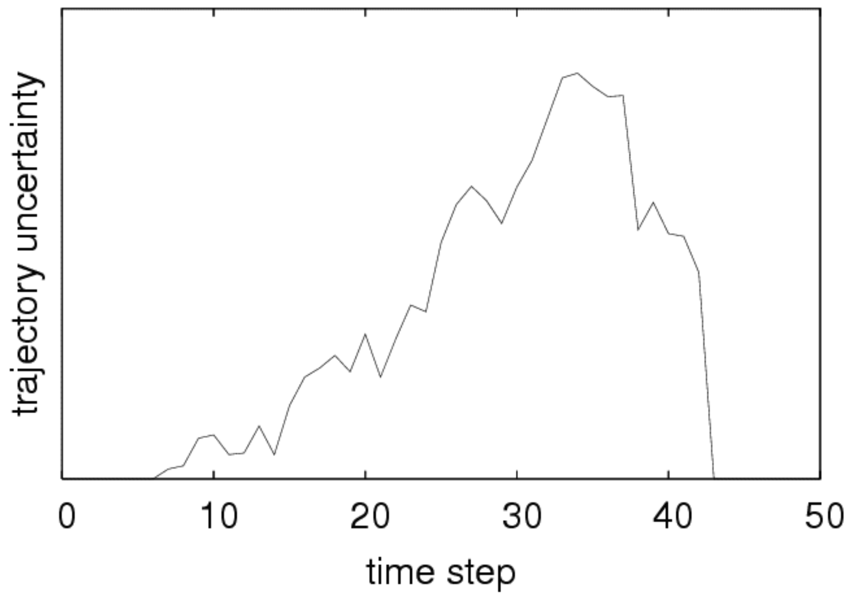
for sparse particle clouds

Approximation of the Trajectory Posterior Entropy

Average pose entropy over time:

$$H(p(x_{1:t} \mid d)) \approx \frac{1}{t} \sum_{t'=1}^{t} H(p(x_{t'} \mid d))$$





Information Gain

- The reduction of entropy in the model

observations
to be obtained

action

H before action
is carried out

$$I(\widehat{z}, a) =$$

$$H(p(m, x \mid d)) -$$

$$H(p(m, x, \widehat{x} \mid d, a, \widehat{z}))$$

new poses introduced
by action

H after action is
carried out

Computing the Expected Information Gain

- To compute the information gain one needs to know the observations obtained when carrying out an action

- This quantity is not known! Reason about potential measurements

$$E[I(a)] \;=\; \int_{\widehat{z}} p(\widehat{z} \mid a, d) \cdot I(\widehat{z}, a)\, d\widehat{z}$$

Reasoning about Measurements

- The filter represents a posterior about possible maps
- Use these maps to reason about possible observation
- Simulate laser measurements in the maps of the particles

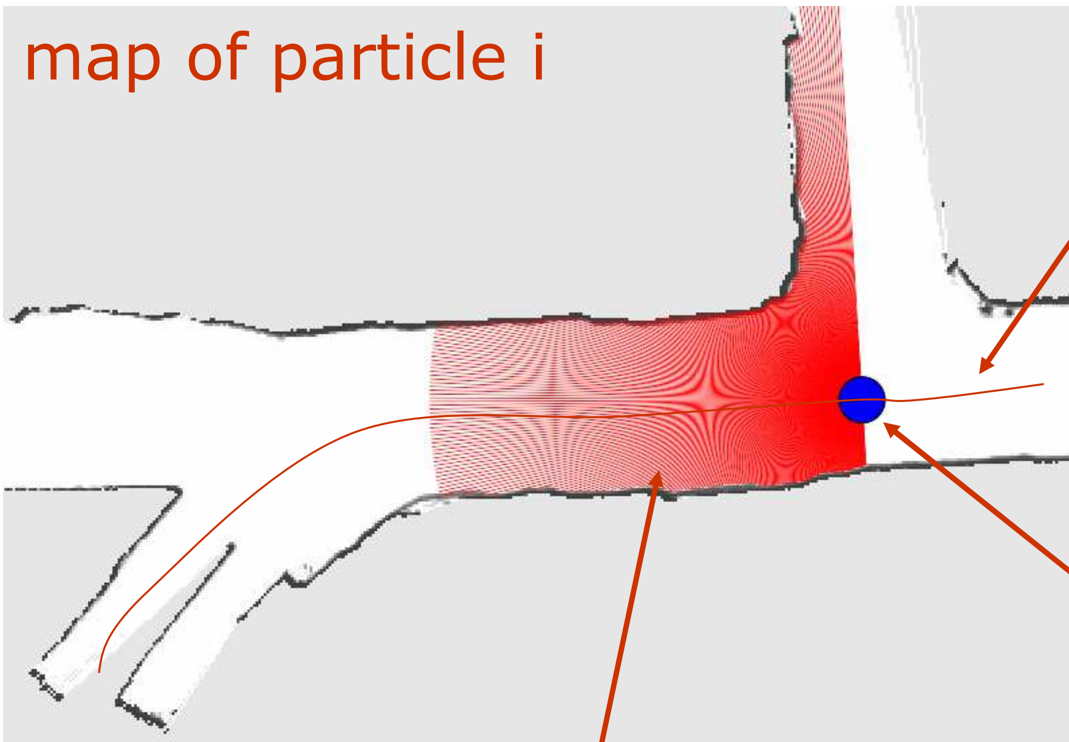
$$E[I(a)] = \int_{\widehat{z}} p(\widehat{z} \mid a, d) \cdot I(\widehat{z}, a)\, d\widehat{z}$$

measurement sequences simulated in the maps

likelihood (particle weight)

Reasoning about Measurements

- Ray-casting in the map of each particle to generate observation sequences



The Utility

- To take into account the cost of an action, we compute a utility

$$U(a) = I(a) - \alpha \cdot cost(a)$$

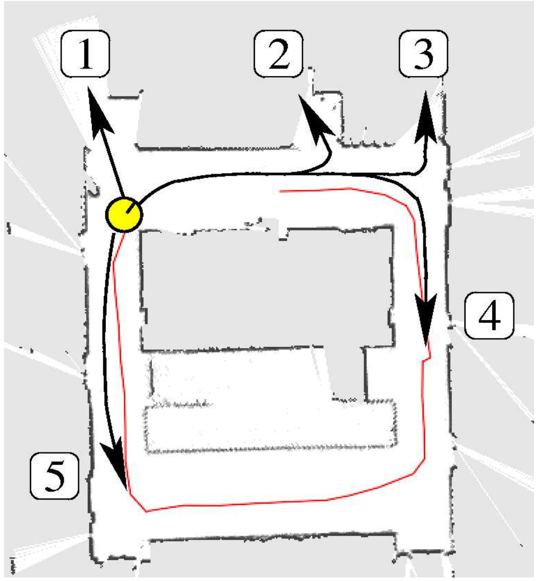
- Select the action with the highest expected utility

$$a^* = \underset{a}{\text{argmax}}\{E[U(a)]\}$$

Focusing on Specific Actions

To efficiently sample actions we consider

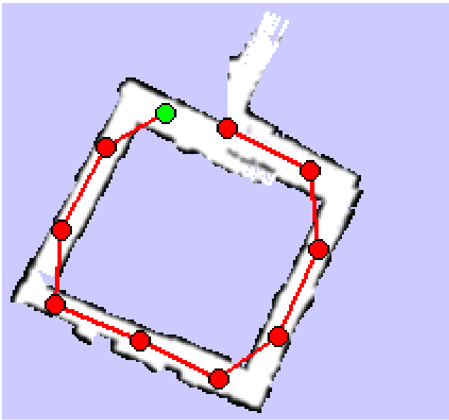
- **exploratory actions (1-3)**
- **loop closing actions (4)** and
- **place revisiting actions (5)**



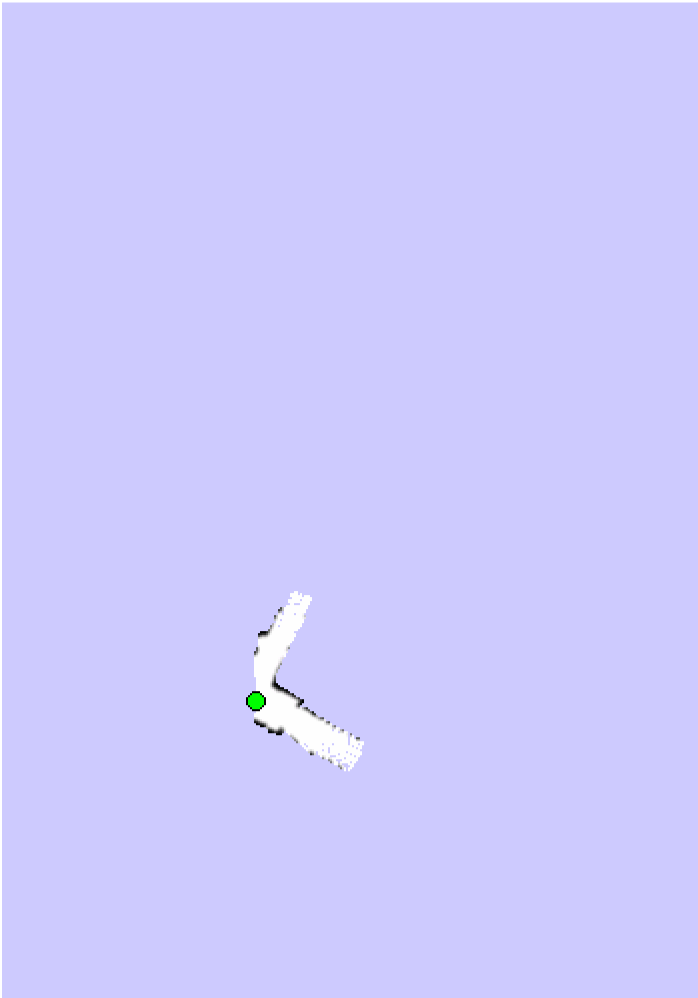
Dual Representation for Loop Detection

- **Trajectory graph** ("topological map") stores the **path traversed by the robot**
- **Occupancy grid** map represents the **space covered by the sensors**

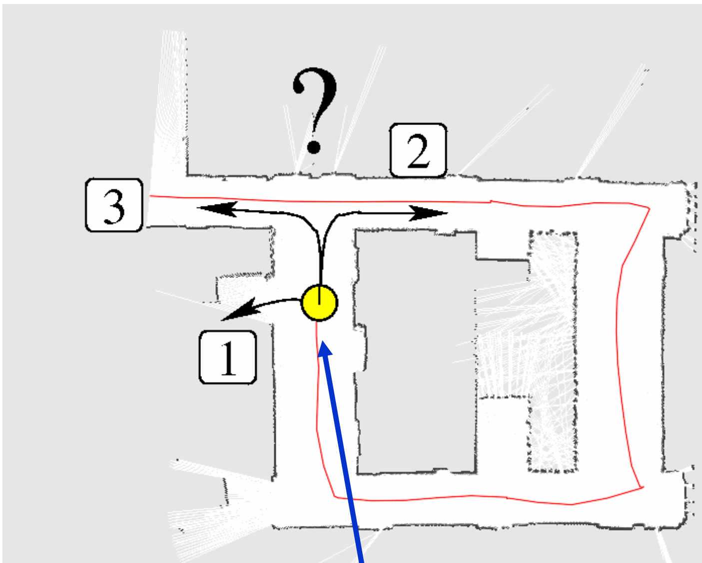
- **Loops** correspond to **long paths in the trajectory graph** and **short paths in the grid map**

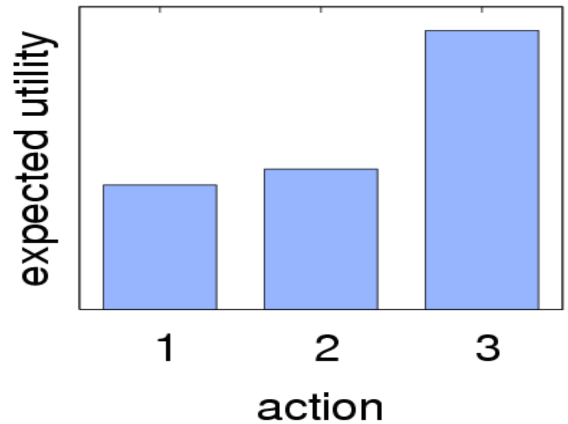


Example: Trajectory Graph

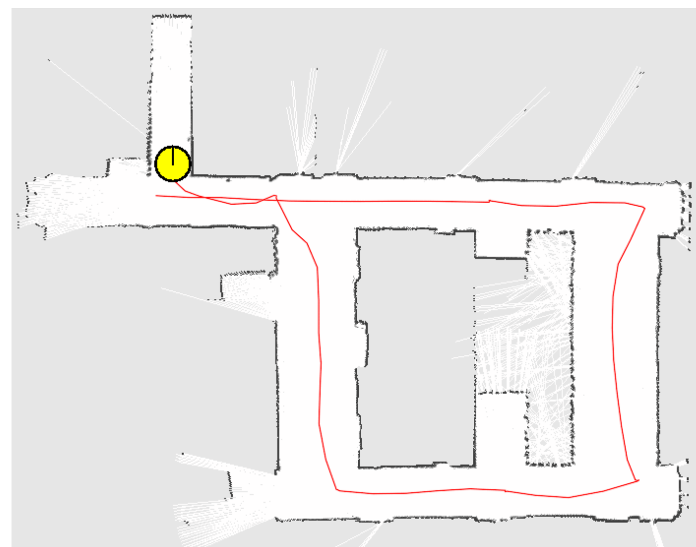


Application Example

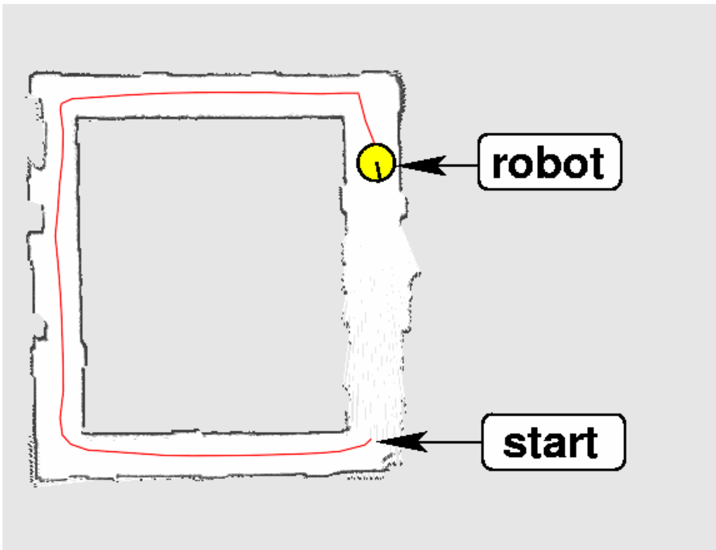


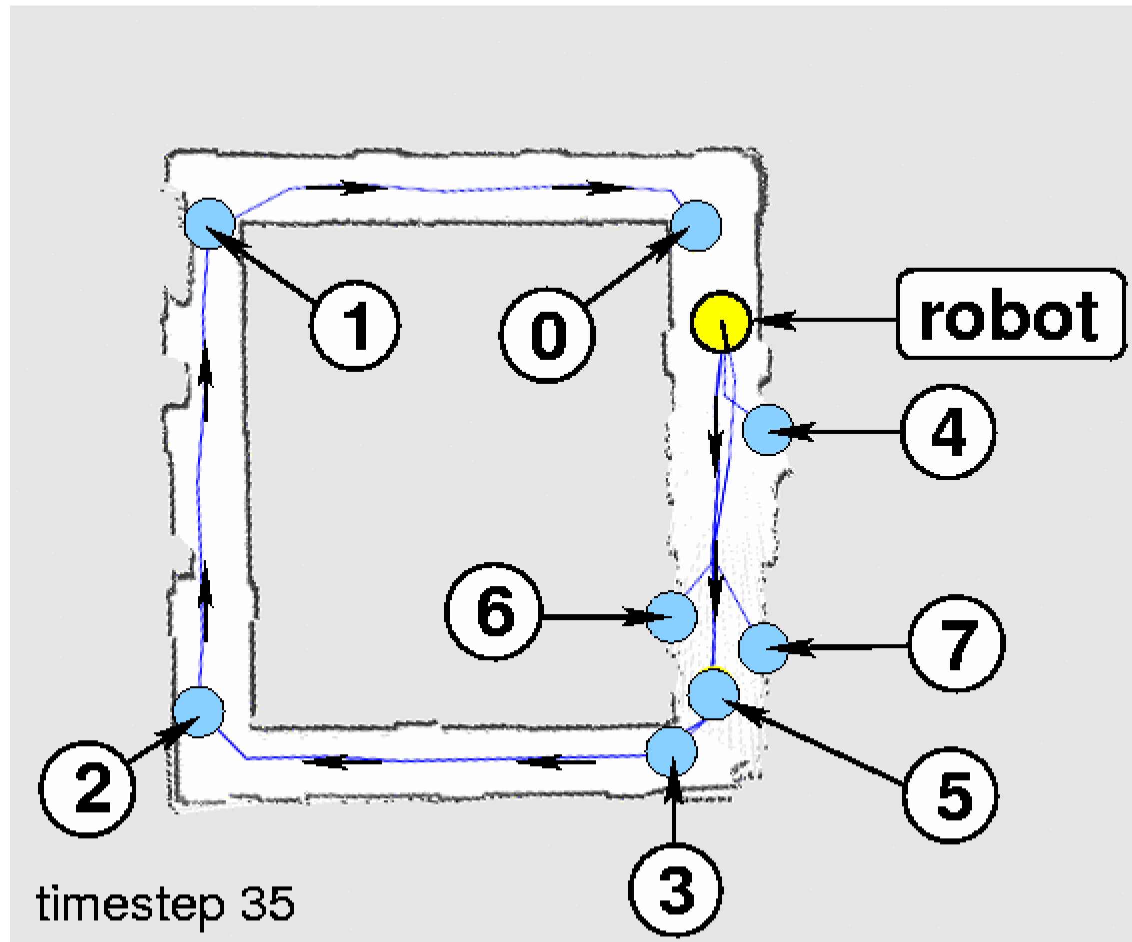


high pose uncertainty

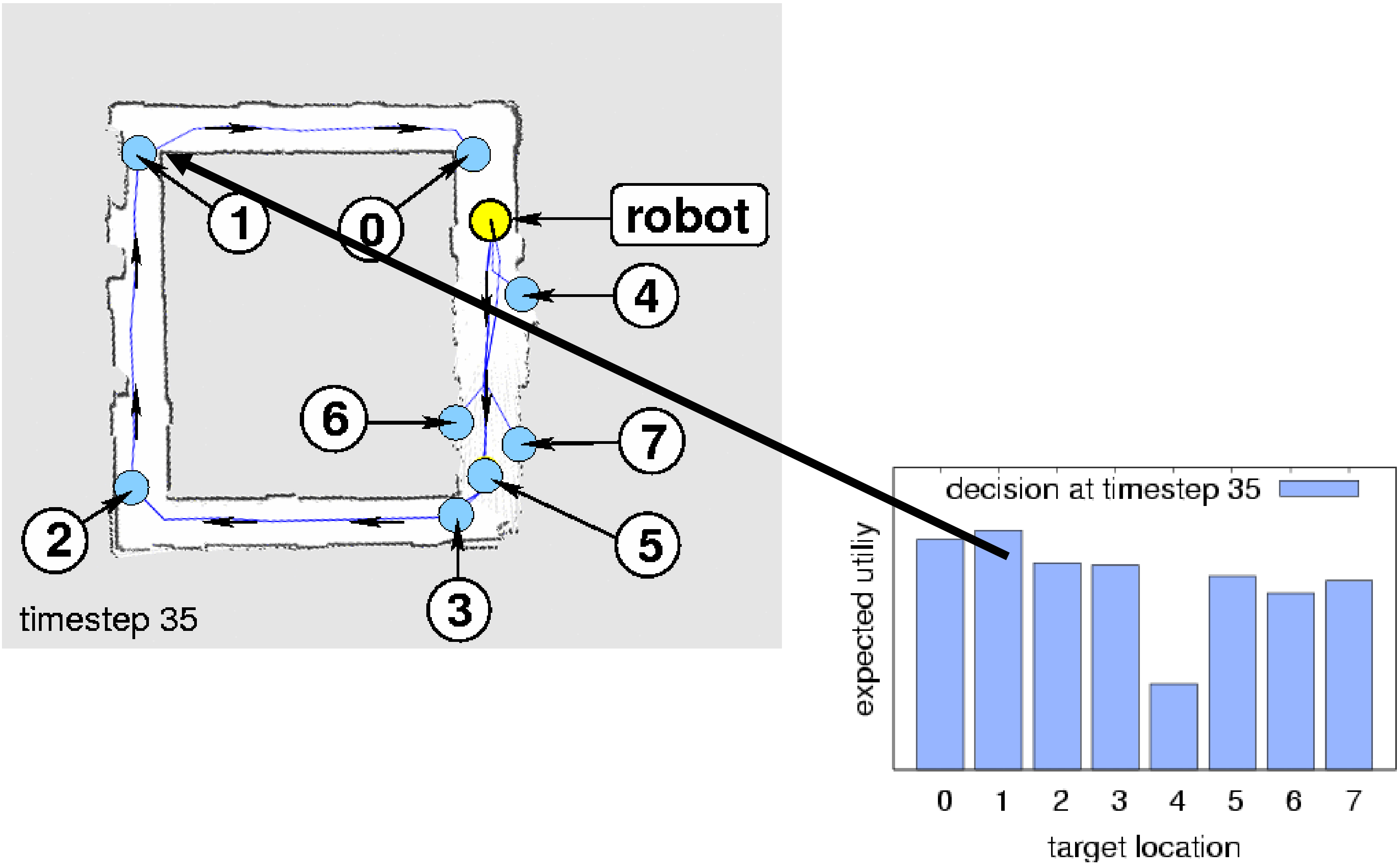


Example: Possible Targets

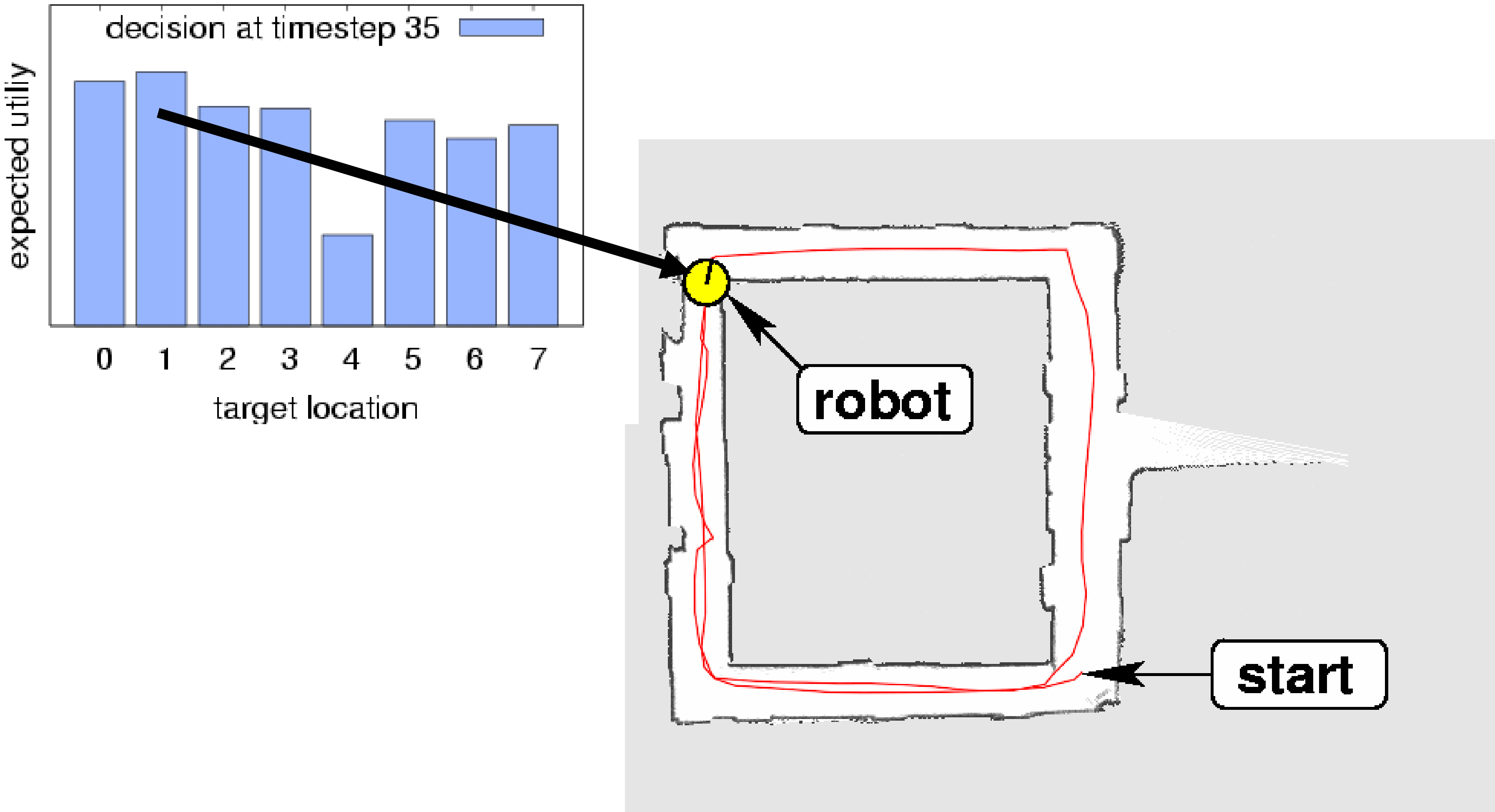




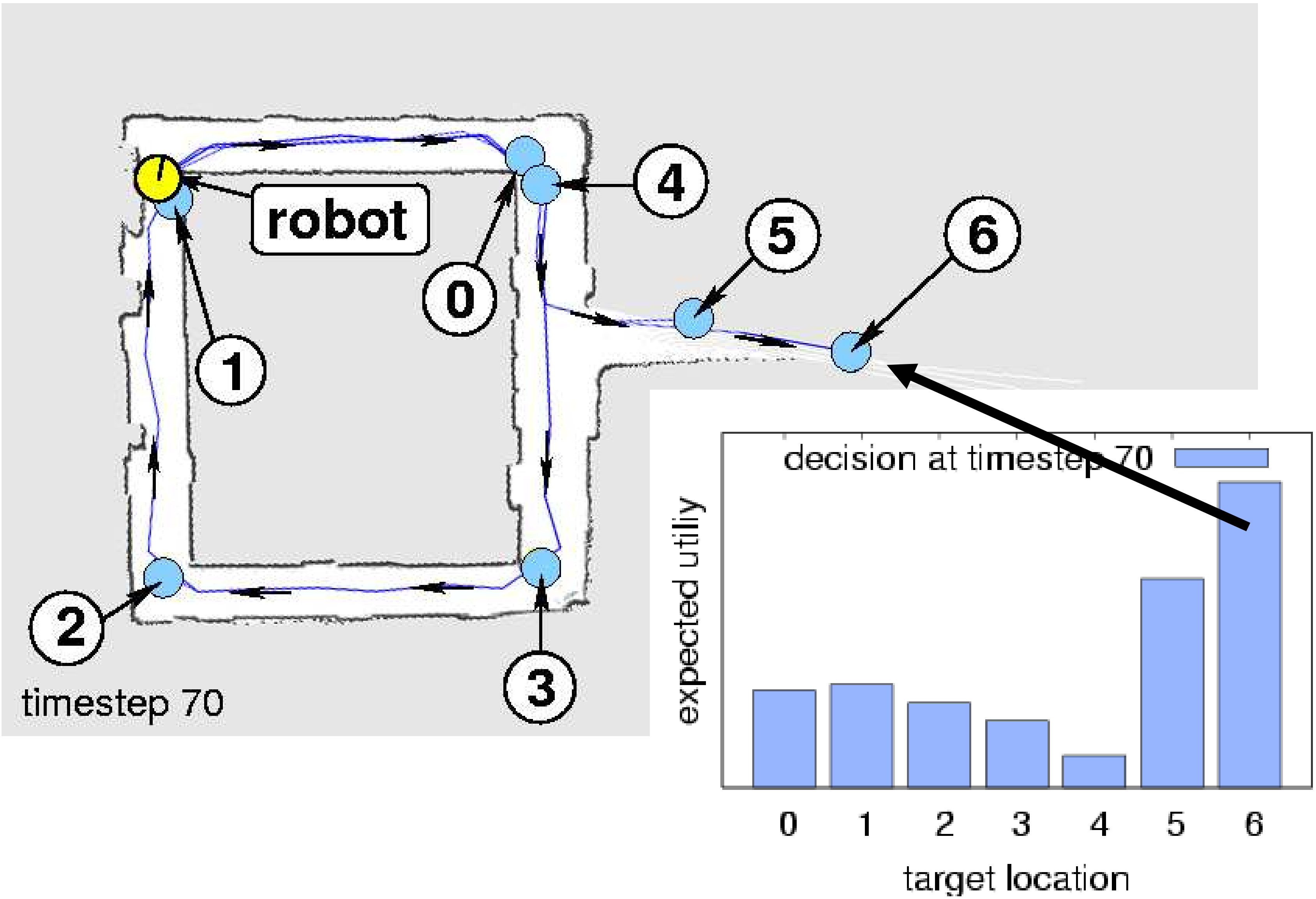
Example: Evaluate Targets



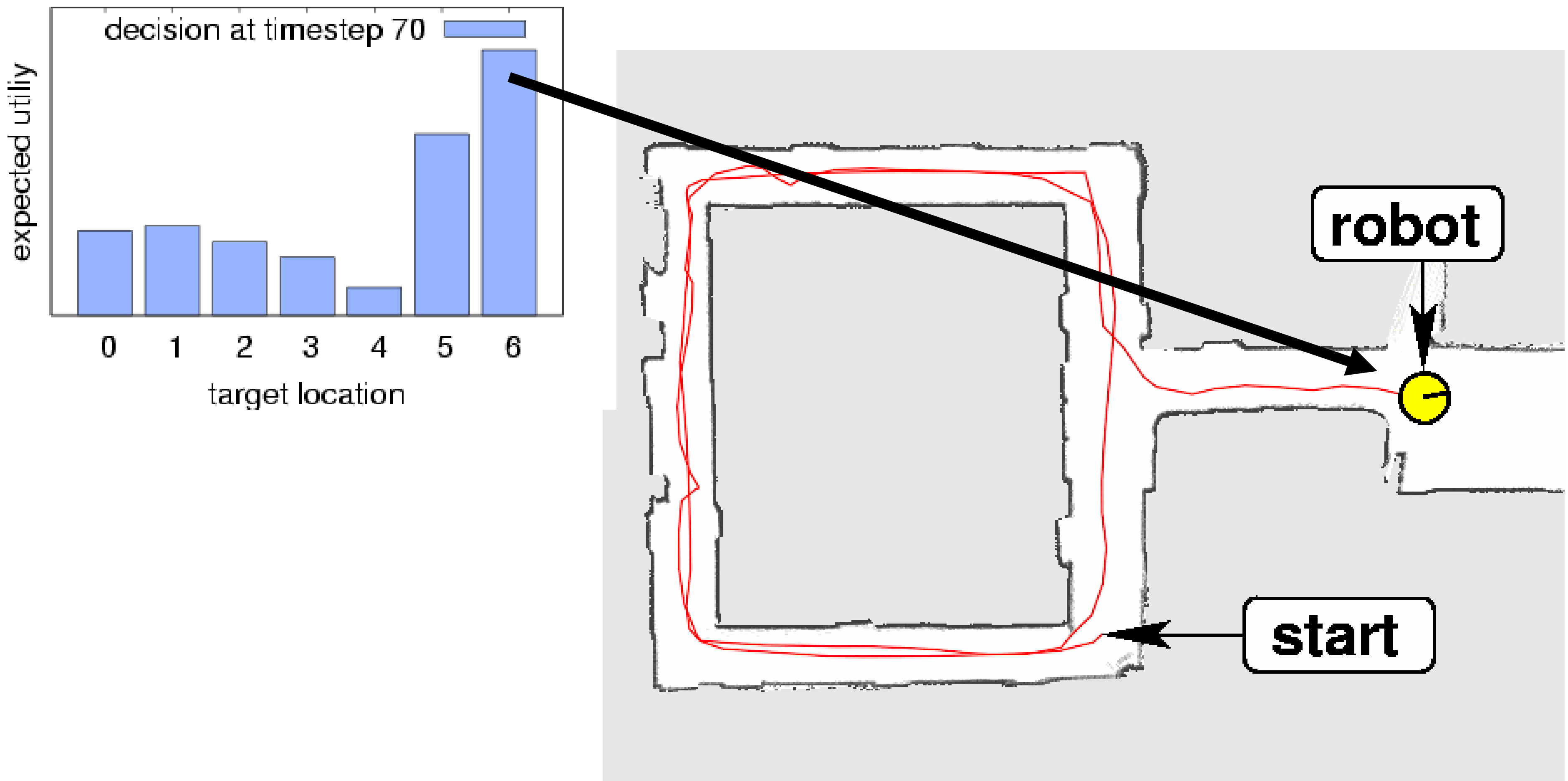
Example: Move Robot to Target



Example: Evaluate Targets

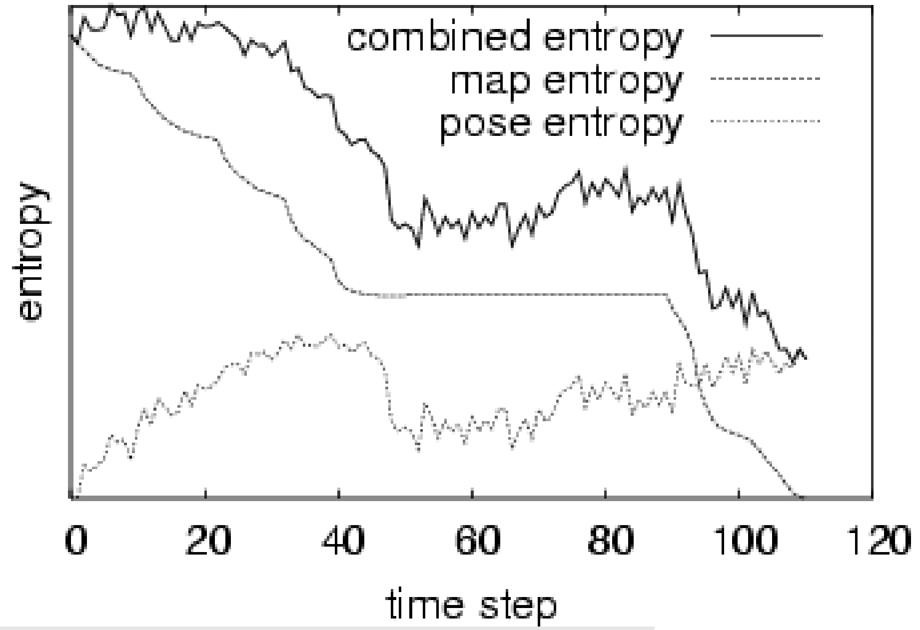


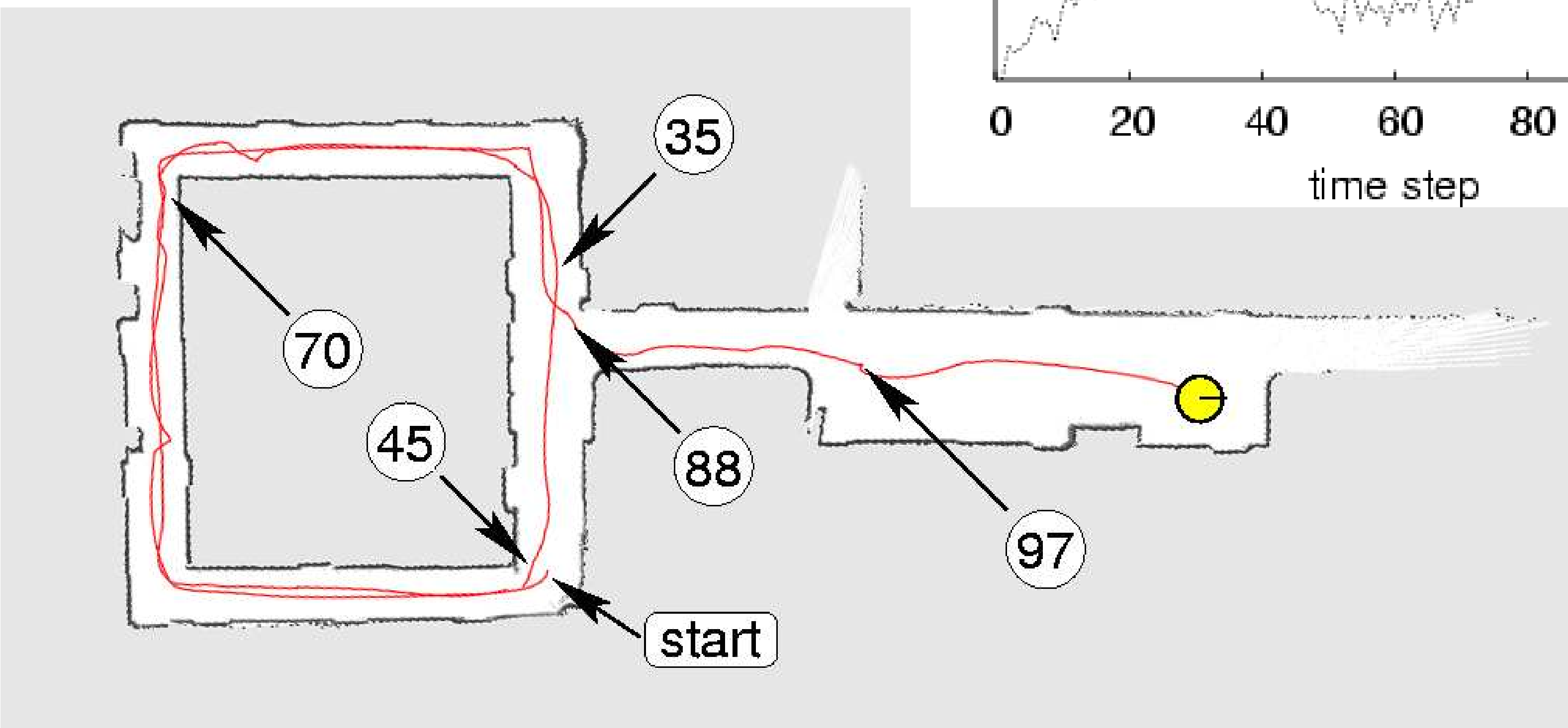
Example: Move Robot



... continue ...

Example: Entropy Evolution



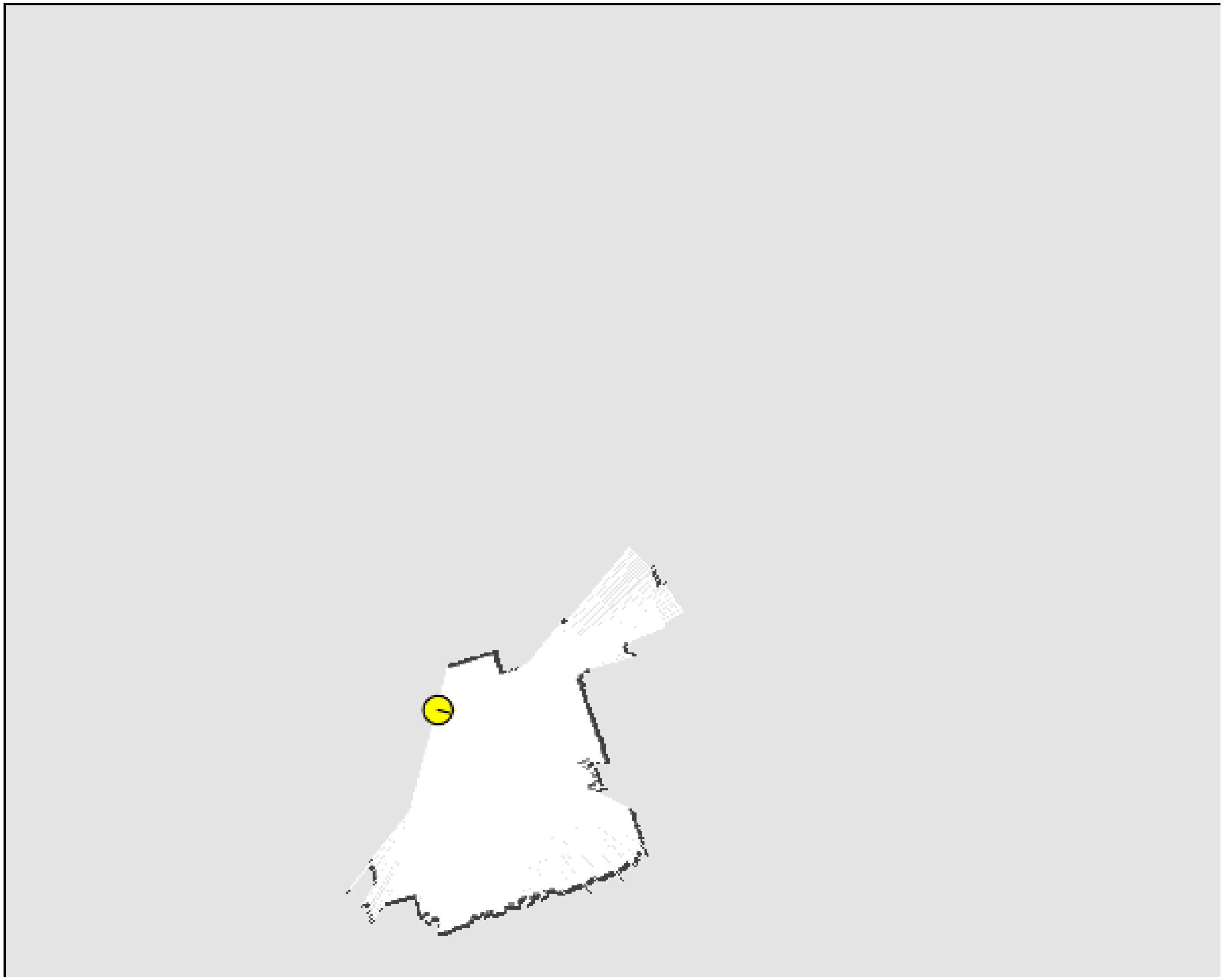


Comparison

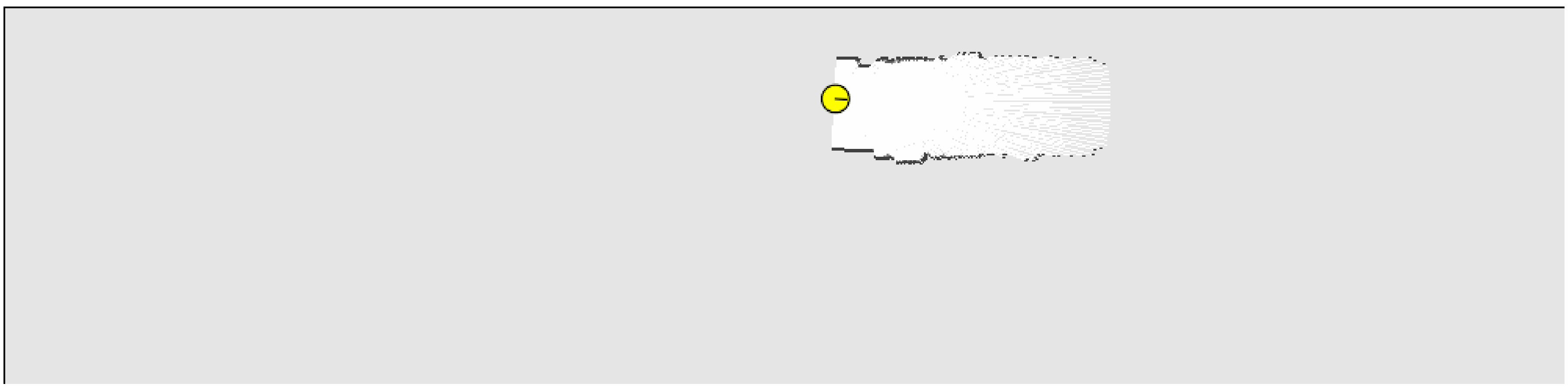
Map uncertainty only:

After loop closing action:

Real Exploration Example



Corridor Exploration



Summary

- A decision-theoretic approach to exploration in the context of RBPF-SLAM
- The approach utilizes the factorization of the Rao-Blackwellization to efficiently calculate the expected information gain
- Reasons about measurements obtained along the path of the robot
- Considers a reduced action set consisting of exploration, loop-closing, and place-revisiting actions
- Experimental results demonstrate the usefulness of the overall approach