Introduction to Mobile Robotics

Mapping with Known Poses
Why Mapping?

- Learning maps is one of the fundamental problems in mobile robotics.
- Maps allow robots to efficiently carry out their tasks, allow localization ...
- Successful robot systems rely on maps for localization, path planning, activity planning etc.
The General Problem of Mapping

What does the environment look like?
The General Problem of Mapping

• Formally, mapping involves, given the sensor data,

\[ d = \{u_1, z_1, u_2, z_2, \ldots, u_n, z_n\} \]

to calculate the most likely map

\[ m^* = \arg \max_m P(m \mid d) \]
Mapping as a Chicken and Egg Problem

• So far we learned how to estimate the pose of the vehicle given the data and the map.
• Mapping, however, involves to simultaneously estimate the pose of the vehicle and the map.
• The general problem is therefore denoted as the simultaneous localization and mapping problem (SLAM).
• Throughout this section we will describe how to calculate a map given we know the pose of the vehicle.
Types of SLAM-Problems

• Grid maps or scans

• Landmark-based

[Lu & Milios, 97; Gutmann, 98; Thrun 98; Burgard, 99; Konolige & Gutmann, 00; Thrun, 00; Arras, 99; Haehnel, 01;...]

[Leonard et al., 98; Castelanos et al., 99; Dissanayake et al., 2001; Montemerlo et al., 2002;...]
Problems in Mapping

• **Sensor interpretation**
  - How do we extract relevant information from raw sensor data?
  - How do we represent and integrate this information over time?

• **Robot locations have to be estimated**
  - How can we identify that we are at a previously visited place?
  - This problem is the so-called data association problem.
Occupancy Grid Maps

- Introduced by Moravec and Elfes in 1985
- Represent environment by a grid.
- Estimate the probability that a location is occupied by an obstacle.

**Key assumptions**
- Occupancy of individual cells \((m_{xy})\) is independent

\[
Bel(m_t) = P(m_t \mid u_1, z_2 \ldots, u_{t-1}, z_t) = \prod_{x,y} Bel(m_{xy}^{[xy]})
\]

- Robot positions are known!
Updating Occupancy Grid Maps

• **Idea**: Update each individual cell using a binary Bayes filter.

\[
Bel(m_t^{[xy]}) = \eta \ p(z_t | m_t^{[xy]}) \int p(m_t^{[xy]} | m_{t-1}^{[xy]}, u_{t-1}) Bel(m_{t-1}^{[xy]}) \, dm_{t-1}^{[xy]}
\]

• **Additional assumption**: Map is static.
**Updating Occupancy Grid Maps**

- Update the map cells using the **inverse sensor model**

\[
Bel(m_t^{[xy]}) = 1 - \left( 1 + \frac{P(m_t^{[xy]} | z, u_{t-1})}{1 - P(m_t^{[xy]} | z, u_{t-1})} \right) \cdot \frac{1 - P(m_t^{[xy]})}{P(m_t^{[xy]})} \cdot \frac{Bel(m_{t-1}^{[xy]})}{1 - Bel(m_{t-1}^{[xy]})}^{-1}
\]

- Or use the **log-odds representation**

\[
\overline{B}(m_t^{[xy]}) = \log odds(m_t^{[xy]} | z, u_{t-1}) - \log odds(m_t^{[xy]}) + \overline{B}(m_{t-1}^{[xy]})
\]

\[
\overline{B}(m_t^{[xy]}) := \log odds(m_t^{[xy]})
\]

\[
\text{odds}(x) := \left( \frac{P(x)}{1 - P(x)} \right)
\]
Typical Sensor Model for Occupancy Grid Maps

Combination of a linear function and a Gaussian:
Key Parameters of the Model

\[ m_l = m_{d,\theta}(x_t) \]
Occupancy Value Depending on the Measured Distance

The graph shows the relationship between distance and occupancy probability. The distance is measured along the x-axis, while the occupancy probability is measured along the y-axis. The graph indicates that there are specific thresholds at $z-d_1$, $z$, $z+d_1$, $z+d_2$, and $z+d_3$, where the occupancy probability changes significantly.
Deviation from the Prior Belief
(the sphere of influence of the sensors)
Calculating the Occupancy Probability Based on Single Observations

\[
P(m_{d,\theta}(x(k)) \mid y(k), x(k)) = P(m_{d,\theta}(x(k)))
\]

\[
+ \begin{cases}
  -s(y(k), \theta) & d < y(k) - d_1 \\
  -s(y(k), \theta) + \frac{s(y(k), \theta)}{d_1} (d - y(k) + d_1) & d < y(k) + d_1 \\
  s(y(k), \theta) & d < y(k) + d_2 \\
  s(y(k), \theta) - \frac{s(y(k), \theta)}{d_3 - d_2} (d - y(k) - d_2) & d < y(k) + d_3 \\
  0 & \text{otherwise.}
\end{cases}
\]
Incremental Updating of Occupancy Grids (Example)
Resulting Map Obtained with Ultrasound Sensors
The maximum likelihood map is obtained by clipping the occupancy grid map at a threshold of 0.5.
Occupancy Grids: From scans to maps
Tech Museum, San Jose

CAD map

occupancy grid map
Alternative: Simple Counting

• For every cell count
  • \text{hits}(x,y): \text{number of cases where a beam ended at } <x,y>
  • \text{misses}(x,y): \text{number of cases where a beam passed through } <x,y>

\[
Bel(m^{[xy]}) = \frac{\text{hits}(x,y)}{\text{hits}(x,y) + \text{misses}(x,y)}
\]

• Value of interest: \(P(\text{reflects}(x,y))\)
The Measurement Model

1. pose at time $t$: $x_t$
2. beam $n$ of scan $t$: $z_{t,n}$
3. maximum range reading: $\zeta_{t,n} = 1$
4. beam reflected by an object: $\zeta_{t,n} = 0$

$$p(z_{t,n} \mid x_t, m) = \begin{cases} 
  \prod_{k=0}^{z_{t,n}-1} (1 - m_{f(x_t,n,k)}) & \text{if } \zeta_{t,n} = 1 \\
  m_{f(x_t,n,z_{t,n})} \prod_{k=0}^{z_{t,n}-1} (1 - m_{f(x_t,n,k)}) & \text{if } \zeta_{t,n} = 0 
\end{cases}$$
Computing the Most Likely Map

• Compute values for $m$ that maximize

$$m^* = \arg \max_m P(m \mid z_1, \ldots, z_t, x_1, \ldots, x_t)$$

• Assuming a uniform prior probability for $p(m)$, this is equivalent to maximizing (applic. of Bayes rule)

$$m^* = \arg \max_m P(z_1, \ldots, z_t \mid m, x_1, \ldots, x_t)$$

$$= \arg \max_m \prod_{t=1}^{T} P(z_t \mid m, x_t)$$

$$= \arg \max_m \sum_{t=1}^{T} \ln P(z_t \mid m, x_t)$$
Computing the Most Likely Map

\[ m^* = \arg \max_m \left[ \sum_{j=1}^{J} \sum_{t=1}^{T} \sum_{n=1}^{N} (I(f(x_t, n, z_{t,n}) = j) \cdot (1 - \zeta_{t,n}) \cdot \ln m_j \\
+ \sum_{k=0}^{z_{t,n}-1} I(f(x_t, n, k) = j) \cdot \ln (1 - m_j)) \right] \]

Suppose

\[ \alpha_j = \sum_{t=1}^{T} \sum_{n=1}^{N} I(f(x_t, n, z_{t,n}) = j) \cdot (1 - \zeta_{t,n}) \]

\[ \beta_j = \sum_{t=1}^{T} \sum_{n=1}^{N} \left[ \sum_{k=0}^{z_{t,n}-1} I(f(x_t, n, k) = j) \right] \]
Meaning of $\alpha_j$ and $\beta_j$

$$\alpha_j = \sum_{t=1}^{T} \sum_{n=1}^{N} I(f(x_t, n, z_{t,n}) = j) \cdot (1 - \zeta_{t,n})$$

corresponds to the number of times a beam that is not a maximum range beam ended in cell $j$ ($\text{hits}(j)$)

$$\beta_j = \sum_{t=1}^{T} \sum_{n=1}^{N} \left[ \sum_{k=0}^{z_{t,n}-1} I(f(x_t, n, k) = j) \right]$$

corresponds to the number of times a beam intercepted cell $j$ without ending in it ($\text{misses}(j)$).
Computing the Most Likely Map

We assume that all cells $m_j$ are independent:

$$m^* = \arg\max_m \left( \sum_{j=1}^{J} \alpha_j \ln m_j + \beta_j \ln(1 - m_j) \right)$$

If we set we obtain

$$\frac{\partial m}{\partial m_j} = \frac{\alpha_j}{m_j} - \frac{\beta_j}{1 - m_j} = 0$$

Computing the most likely map amounts to counting how often a cell has reflected a measurement and how often it was intercepted.
Difference between Occupancy Grid Maps and Counting

• The counting model determines how often a cell reflects a beam.
• The occupancy model represents whether or not a cell is occupied by an object.
• Although a cell might be occupied by an object, the reflection probability of this object might be very small.
Example Occupancy Map
Example Reflection Map

glass panes
Example

• Out of 1000 beams only 60% are reflected from a cell and 40% intercept it without ending in it.
• Accordingly, the reflection probability will be 0.6.
• Suppose $p(occ \mid z) = 0.55$ when a beam ends in a cell and $p(occ \mid z) = 0.45$ when a cell is intercepted by a beam that does not end in it.
• Accordingly, after $n$ measurements we will have

$$\left(\frac{0.55}{0.45}\right)^{n*0.6} \times \left(\frac{0.45}{0.55}\right)^{n*0.4} = \left(\frac{11}{9}\right)^{n*0.6} \times \left(\frac{11}{9}\right)^{-n*0.4} = \left(\frac{11}{9}\right)^{n*0.2}$$

• Whereas the reflection map yields a value of 0.6, the occupancy grid value converges to 1.
Summary

- Occupancy grid maps are a popular approach to represent the environment of a mobile robot given known poses.
- In this approach each cell is considered independently from all others.
- It stores the posterior probability that the corresponding area in the environment is occupied.
- Occupancy grid maps can be learned efficiently using a probabilistic approach.
- Reflection maps are an alternative representation.
- They store in each cell the probability that a beam is reflected by this cell.
- We provided a sensor model for computing the likelihood of measurements and showed that the counting procedure underlying reflection maps yield the optimal map.