Albert-Ludwigs-Universität Freiburg
Lecture: Introduction to Mobile Robotics
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Institut für Informatik

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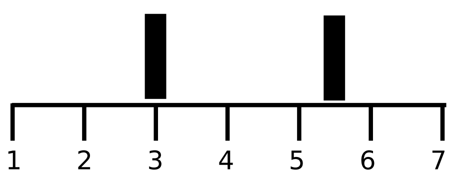
Sheet 2

Topic: Bayes Filter, Motion Models

Submission deadline: Tue 20.5.2008, 11:00 a.m. (before class)

Exercise 1:

Suppose your robot is equipped with a simple but noisy sensor that can detect a pole and returns the relative position of the pole. The possible measurements are: the pole is at the location to your left ($z = -1$), to your right ($z = 1$) or at your current position ($z = 0$). The maximum reading range is 1 meter, so sometimes you will not detect a pole at all ($z = n$). Your robot lives in a one-dimensional world and can be at one of seven possible locations ($x = 1, \ldots, x = 7$), each 1 meter apart. There a two poles at positions $x = 3$ and $x = 5.5$.



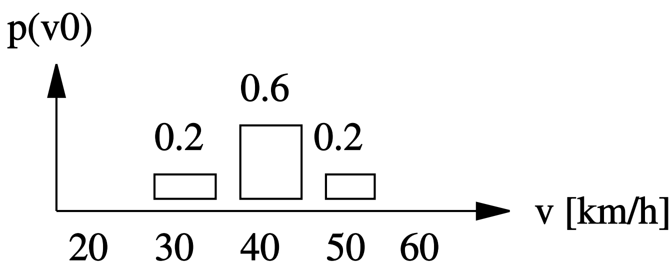
You are given the following sensor model:

$p(z \mid x)$	$x = 1$	$x = 2$	$x = 3$	$x = 4$	$x = 5$	$x = 6$	$x = 7$
$z = -1$	0.0	0.00	0.25	0.50	0.0	0.5	0.0
$z = 0$	0.0	0.25	0.50	0.25	0.5	0.5	0.0
$z = 1$	0.0	0.50	0.25	0.00	0.5	0.0	0.0
$z = n$	1.0	0.25	0.00	0.25	0.0	0.0	1.0

Compute the corresponding matrix $p(x \mid z)$ with the probabilities for being at location x given that you have observed a measurement z (Hint: Your are given no prior information about your location, that is, before making a measurement each location is equally likely.)

Exercise 2:

Consider the following every-day situation: You help your grandma to buy some groceries. Unfortunately, her car is rather old and the speed indicator is not working any more. Since you cannot afford another speeding ticket, you have to reason about your speed using just the public speed indicators on the side of the street (see the picture below). You guess, that your current speed v_0 is distributed as follows:



Of course, the acceleration is not perfect for such an old car. For each possible action, $a = -10$ (slowing down 10km/h), $a = +10$ (accelerating by 10 km/h), $a = 0$ (keeping the speed), the transition probabilities for the speed v of your car are given in the following table.

	$v_{i+1} = v_i - 10$	$v_{i+1} = v_i$	$v_{i+1} = v_i + 10$
$a_i = -10$	0.6	0.4	0
$a_i = 0$	0	1	0
$a_i = +10$	0	0.2	0.8

The public speed indicators that provide you with speed measurements m_i have the following measurement accuracy:

	$m_i < v_i - 10$	$m_i = v_i - 10$	$m_i = v_i$	$m_i = v_i + 10$	$m_i > v_i + 10$
probability	0	0.1	0.7	0.2	0

On the ride to the supermarket, you perform the following actions and obtain the following measurements. Each measurement m_i is obtained after the according action a_i has had its effect on the speed.

time i:	1	2	3
action a_i:	$+10$	0	-10
measurement m_i:	60	50	40

Please use the Bayesian Filtering technique to calculate your belief about the car speed after each time step i. Is it likely that you have exceeded the speed limit of 50 km/h at some point?

Exercise 3:

Let a robot be equipped with wheel encoders and on-board software that transforms the physical measuring data into time-discrete odometry measurements $\langle \hat{\delta}_{rot_1}, \hat{\delta}_{trans}, \hat{\delta}_{rot_2} \rangle$.

(a) Derive equations for the calculation of the end-pose $\langle x', y', \theta' \rangle$ after a performed motion $\langle \delta_{rot_1}, \delta_{trans}, \delta_{rot_2} \rangle$ from a start-pose $\langle x, y, \theta \rangle$.

(b) Let the robot start at pose $\langle x, y, \theta \rangle = \langle 0m, 0m, 0^\circ \rangle$ and obtain the following subsequent odometry measurements:

$$\begin{aligned} \hat{\delta}^1_{rot_1} &= 10^\circ \\ \hat{\delta}^1_{trans} &= 3m \\ \hat{\delta}^1_{rot_2} &= 10^\circ \end{aligned}$$

$$\begin{aligned} \hat{\delta}^2_{rot_1} &= -20^\circ \\ \hat{\delta}^2_{trans} &= 10m \\ \hat{\delta}^2_{rot_2} &= -10^\circ \end{aligned}$$

Please assume perfect measurements and calculate the exact pose of the robot.

(c) How would your pose estimate for the first movement look like under the following simple error model? Please draw the movements and pose estimates into one diagram.

$$\begin{aligned} \hat{\delta}_{rot_1} &= \delta_{rot_1} \pm \varepsilon_{rot_1}, & \varepsilon_{rot_1} &= 5^\circ \\ \hat{\delta}_{trans} &= \delta_{trans} \pm \varepsilon_{trans}, & \varepsilon_{trans} &= 0.5m \\ \hat{\delta}_{rot_2} &= \delta_{rot_2} \pm \varepsilon_{rot_2}, & \varepsilon_{rot_2} &= 10^\circ \end{aligned}$$