Albert-Ludwigs-Universität Freiburg
Institut für Informatik
Lecture: Introduction to Mobile Robotics
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Prof. Dr. W. Burgard
Dipl.-Inf. B. Frank
Dipl.-Inf. D. Joho
MSc ACS H. Strasdat

Sheet 3

Topic: Data Explanation, Sensor Models, Discrete Bayes Filter

Submission deadline: Tue 27.5.2008, 11:00 a.m. (before class)

Exercise 1:

Consider the following situation: with a 50% probability we generate a data point from a normal distribution:

$$g(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

with mean $\mu = 0$ and $\sigma = 1$, while with the other 50% probability we generate a data point from a normal distribution with variance 1 and adjustable mean μ. We have observed some data points, and we wish to adjust μ to maximize their likelihood.

Suppose further that our data points are $a = 2$ and $b = -1.5$. We are told that either a or b comes from the Gaussian centered at 0 (but not both). In that case, there are only two data association possibilities: either a came from center 0 and b came from center μ, or vice versa.

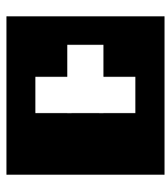
(a) Use gnuplot (or any other plotting tool) to plot the probability $P(a, b \mid \mu)$.

(b) Which is the mean μ so that a and b have the highest likelihood?

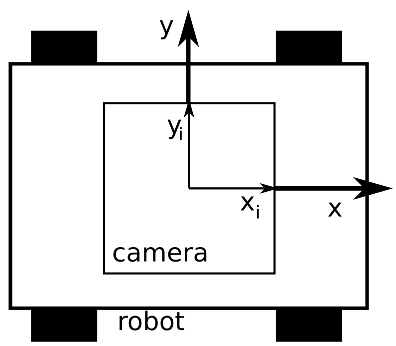
(c) What is the most likely data association given the mean μ from exercise (b). (Consider only the two data associations mentioned above.)

Exercise 2:

Many early robots navigating using features used artificial landmarks in the environment that were easy to recognize. A good place to mount such markers is a ceiling (why?). A classical example is a visual marker: Suppose we attach the following marker to the ceiling:



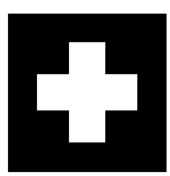
Let the world coordinates of the marker be (x_m, y_m, θ_m). We will denote the robot's pose by (x_r, y_r, θ_r). Now assume, we have a routine that can detect the marker in the image plane of a perspective camera. Let x_i and y_i denote the coordinates of the marker in the image plane, and θ_i its angular orientation. From the setup of our system (focal length of the camera, height of the ceiling, etc.) we know that each displacement d in x-y-space gets projected to a proportional displacement of $\alpha \cdot d$ in the image plane. The relation between the camera-centric coordinate system and the robot-centric coordinate system is as depicted:



Remember that the robot looks in the direction of the positive x-axis (of the global coordinate system) if its orientation is $\theta_r = 0$.

(a) Describe mathematically where to expect the marker (in global coordinates (x_m, y_m, θ_m)) when its image coordinates are (x_i, y_i, θ_i) and the robot is at position (x_r, y_r, θ_r).

(b) Now suppose we use several of the following symmetrical markers instead:



(i) How many markers do we need at least in order to determine the position unambiguously?

(ii) Sketch an unambiguous and an ambiguous placement of two such markers. How many hypotheses are there in the ambiguous case?

(iii) Sketch an unambiguous and an ambiguous placement of three such markers. How many hypotheses are there in the ambiguous case?

Exercise 3:

Programming task: A robot is living in a 1-dimensional world. It can move to the left and to the right; the world is 100m long ($x \in [0, 100]$). If the robot moves, it moves approximately 4m (with Gaussian error of $0.25m$). At the end of the world ($x = 100m$) there is a landmark. The robot can measure the distance to the landmark with a Gaussian error of $\sigma = 2m$. The landmark is only visible, if the robot is close to it ($x \geq 70m$).

Estimate the pose of the robot using a 1-dimensional grid. Thus, complete the method *update* in the *Grid* class. Use the following motion and sensor models:

- **Motion Model** $p(x' \mid x, u)$**:** Let assume that $u \in \{+4, -4\}$. Then, use the following motion model:

$$p(x' \mid x, u) = \begin{cases} 0.3750 & \text{if } x' - (x + u) = 0 \\ 0.2500 & \text{if } |x' - (x + u)| = 1 \\ 0.0625 & \text{if } |x' - (x + u)| = 2 \\ 0.0 & \text{else} \end{cases}$$

- **Sensor Model:** Use a Gaussian sensor model with standard deviation $\sigma = 2m$. (Thus, use only the "measurement noise" component of the "Beam-based Proximity Model" presented in the lecture.)

Remark: Make sure that the sum over all grid cells is always 1.