Albert-Ludwigs-Universität Freiburg
Lecture: Introduction to Mobile Robotics
Summer term 2008

Institut für Informatik

Prof. Dr. W. Burgard
Dipl.-Inf. B. Frank
Dipl.-Inf. D. Joho
MSc ACS H. Strasdat

Sheet 5

Topic: Kalman Filter

Submission deadline: Tue 10.6.2008, 11:00 a.m. (before class)

Exercise 1:

Consider a laser range finder observing a walking person in an environment. The laser range finder provides only informations about the position of the observed person. A Kalman Filter is used to track the person. It should estimate the person's position (x, y) and velocity $(\dot{x}, \dot{y})$ (no action can be executed by the system, it just observes and estimates). Consider that a new measurements can be incorporated every $\Delta t = 0.5s$.

(a) Specify the dimensions of the state vector.

(b) Specify the matrix A.

(c) Specify the matrix C.

Exercise 2:

Consider the situation in Exercise 1 and use your definitions for A and C. The initial state $\mathbf{x}_0$ of the system is given by: $(x = 0.8, y = 0, \dot{x} = 0.4, \dot{y} = 0)^T$. The estimate error covariance matrix $\overline{\Sigma}$ and the measurement error covariance matrix Q is given by:

$$\overline{\Sigma} = \begin{pmatrix} 0.5 & 0.2 & 0.3 & 0.3 \\ 0.2 & 0.5 & 0.3 & 0.3 \\ 0.3 & 0.3 & 0.5 & 0.2 \\ 0.3 & 0.3 & 0.2 & 0.5 \end{pmatrix}, \quad Q = \begin{pmatrix} 0.05 & 0.0 \\ 0.0 & 0.05 \end{pmatrix} \tag{1}$$

The Kalman Gain K is computed by: $K = \overline{\Sigma} C^T (C \overline{\Sigma} C^T + Q)^{-1}$. Since not everyone of you has access to MatLab, the result of this computation is:

$$K = \begin{pmatrix} 0.8952 & 0.0381 \\ 0.0381 & 0.8952 \\ 0.4000 & 0.4000 \\ 0.4000 & 0.4000 \end{pmatrix} \tag{2}$$

Consider that in this example the matrixes $\overline{\Sigma}$ and Q stay constant over time and are not updated. The first measurement, which is taken after time Δt is $(x = 1, y = 0.1)$. Compute the next state of the system $\mathbf{x}_1$.

Exercise 3:

Consider the odometry based motion model given by

$$g(\mathbf{x}, \mathbf{u}) = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} \delta_{trans} \cdot \cos(\theta + \delta_{rot1}) \\ \delta_{trans} \cdot \sin(\theta + \delta_{rot1}) \\ \delta_{rot1} + \delta_{rot2} \end{pmatrix}$$

with $\mathbf{x} = (x, y, \theta)$ and $\mathbf{u} = (\delta_{rot1}, \delta_{trans}, \delta_{rot2})$. Remark, g is not a linear function. Derive the corresponding Jacobians $G = \dfrac{\delta g(\mathbf{x}, \mathbf{u})}{\delta \mathbf{x}}$ and $V = \dfrac{\delta g(\mathbf{x}, \mathbf{u})}{\delta \mathbf{u}}$ needed by the prediction step of EKF-localization.