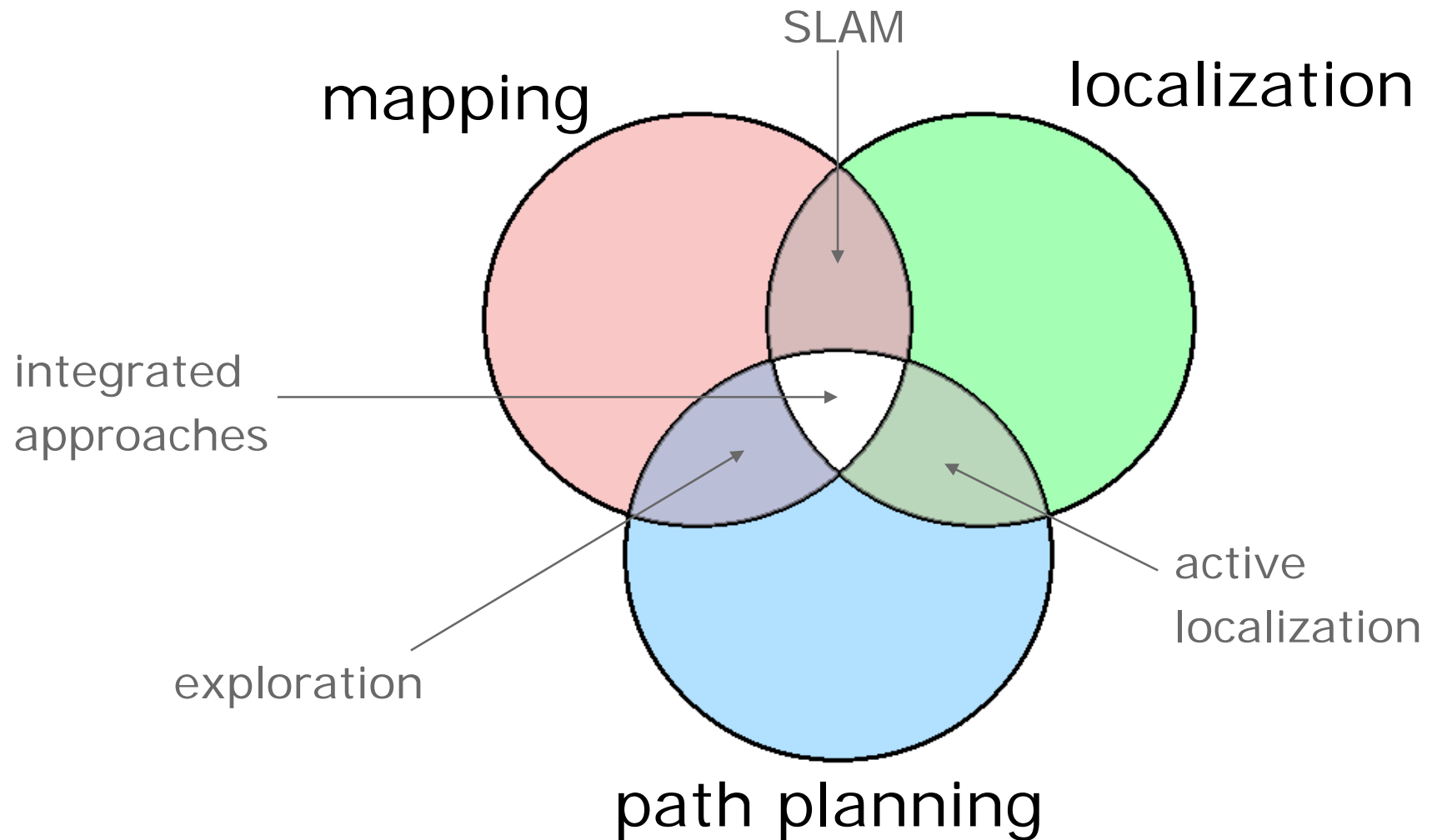


Introduction to Mobile Robotics

Information Gain-Based Exploration

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Maren Bennewitz
Christian Plagemann

Tasks of Mobile Robots



Exploration and SLAM

- SLAM is typically **passive**, because it consumes incoming sensor data
- Exploration **actively guides the robot** to cover the environment with its sensors
- Exploration in combination with SLAM: **Acting under pose and map uncertainty**
- Uncertainty should/needs to be taken into account when selecting an action

Mapping with Rao-Blackwellized Particle Filter (Brief Summary)

- Each particle represents a possible trajectory of the robot
- Each particle
 - maintains its own map and
 - updates it upon “mapping with known poses”
- Each particle survives with a probability proportional to the likelihood of the observations relative to its own map

Factorization Underlying Rao-Blackwellized Mapping

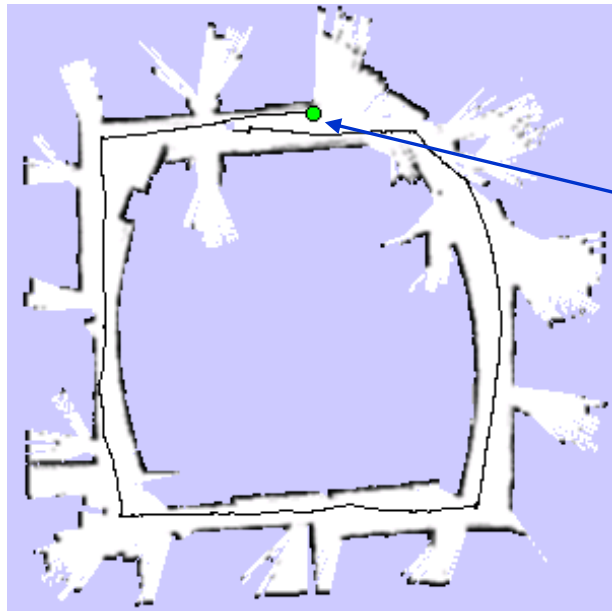
poses map observations & odometry

$$p(x, m \mid z, u) = p(m \mid x, z, u) p(x \mid z, u)$$

Mapping with known poses

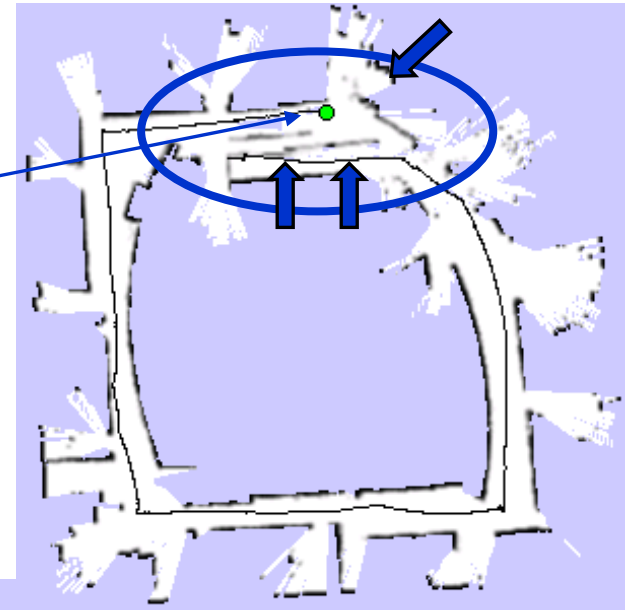
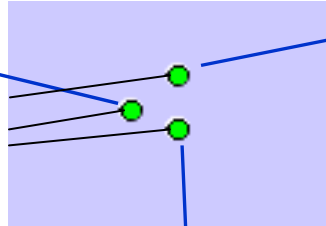
Particle filter representing trajectory hypotheses

Example: Particle Filter for Mapping

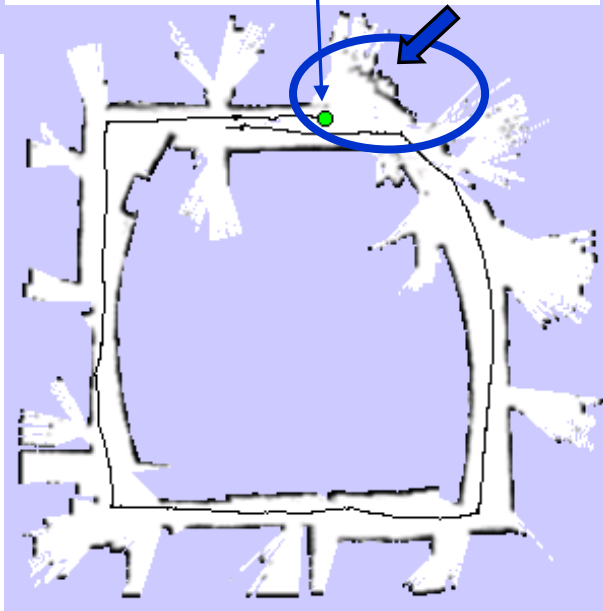


map of particle 1

3 particles



map of particle 2



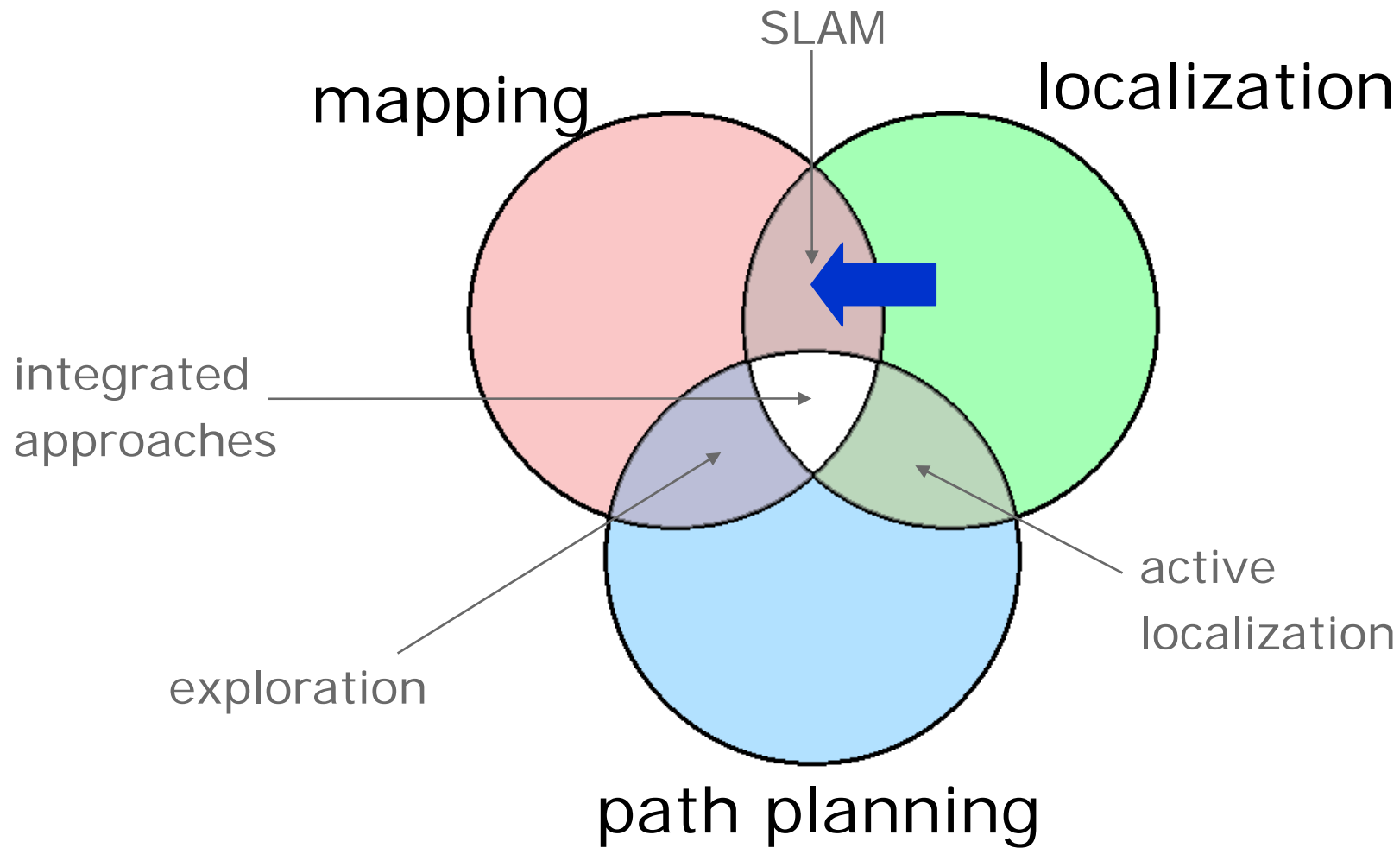
map of particle 3

Outdoor Campus Map



- **30 particles**
- 250x250m²
- 1.75 km (odometry)
- 20cm resolution during scan matching
- 30cm resolution in final map

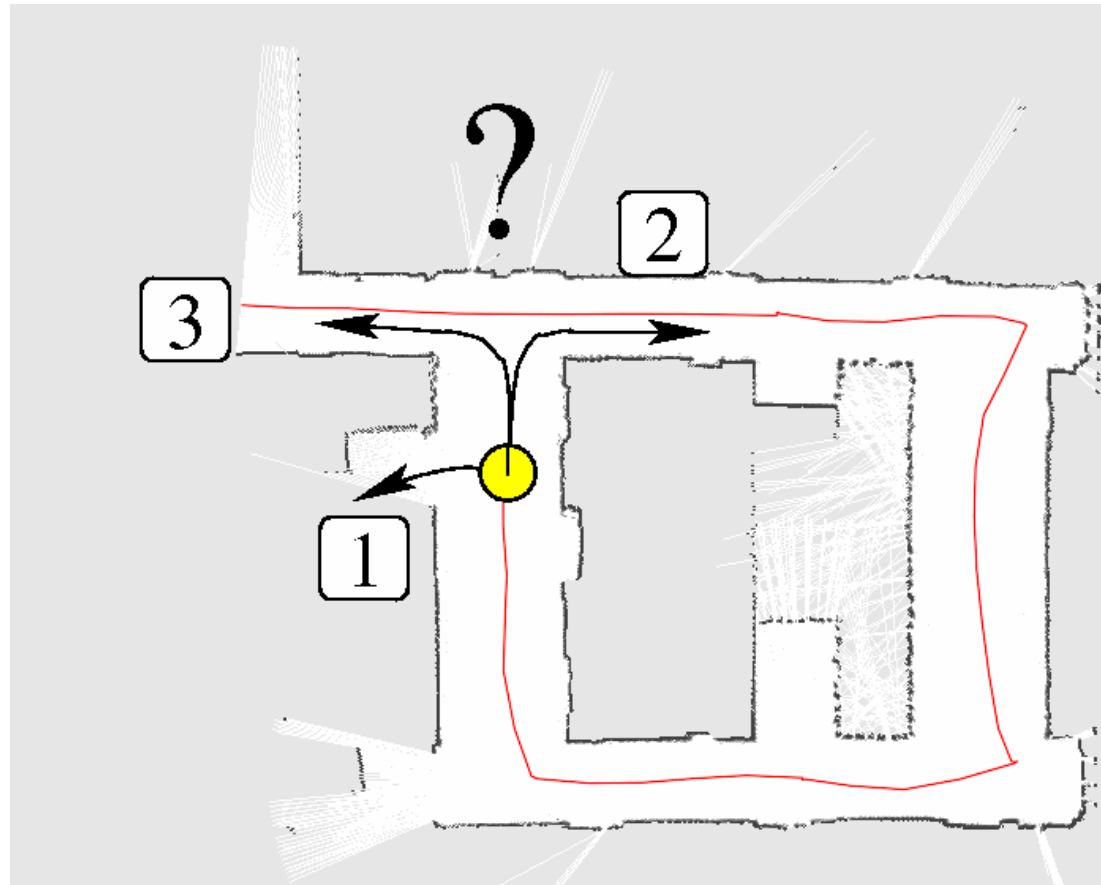
Combining Exploration and SLAM



Exploration

- The approaches seen so far are purely passive
- By reasoning about control, the mapping process can be made much more effective
- Question: **Where to move next?**

Where to Move Next?



Decision-Theoretic Approach

- Learn the map using a Rao-Blackwellized particle filter
- Consider a set of potential actions
- Apply an exploration approach that minimizes the overall uncertainty

Utility = uncertainty reduction - cost

The Uncertainty of a Posterior

- Entropy is a general measure for the uncertainty of a posterior

$$\begin{aligned} H(p(x)) &= - \int_x p(x) \log p(x) dx \\ &= E_x[-\log(p(x))] \end{aligned}$$

- Information Gain = Uncertainty Reduction

$$I(t + 1 | t) = H(p(x_t)) - H(p(x_{t+1}))$$

Entropy Computation

$$\begin{aligned} H(p(x, y)) &= E_{x,y}[-\log p(x, y)] \\ &= E_{x,y}[-\log(p(x) p(y | x))] \\ &= E_{x,y}[-\log p(x)] + E_{x,y}[-\log p(y | x)] \\ &= H(p(x)) + \int_{x,y} -p(x, y) \log p(y | x) dx dy \\ &= H(p(x)) + \int_{x,y} -p(y | x)p(x) \log p(y | x) dx dy \\ &= H(p(x)) + \int_x p(x) \int_y -p(y | x) \log p(y | x) dy dx \\ &= H(p(x)) + \int_x p(x) H(p(y | x)) dx \end{aligned}$$

Computing the Map and Pose Uncertainty

The diagram illustrates the decomposition of the joint entropy $H(p(x, m | d))$ into three components. A blue arrow points from the text "data (laser and odometry)" to the d in the first term of the equation. Another blue arrow points from the text "trajectory uncertainty" to the $H(p(x | d))$ term. A third blue arrow points from the text "particle weight" to the $\omega^{[i]}$ term. A fourth blue arrow points from the text "map uncertainty" to the $H(p(m^{[i]} | x^{[i]}, d))$ term.

$$\begin{aligned} & H(p(x, m | d)) \quad \text{data (laser and odometry)} \\ &= H(p(x | d)) + \int_x p(x | d) H(p(m | x, d)) dx \\ &\approx H(p(x | d)) + \sum_{i=1}^{\#particles} \omega^{[i]} H(p(m^{[i]} | x^{[i]}, d)) \\ &\quad \text{trajectory uncertainty} \quad \text{particle weight} \quad \text{map uncertainty} \end{aligned}$$

Computing the Entropy of the Map Posterior

Occupancy Grid map m :

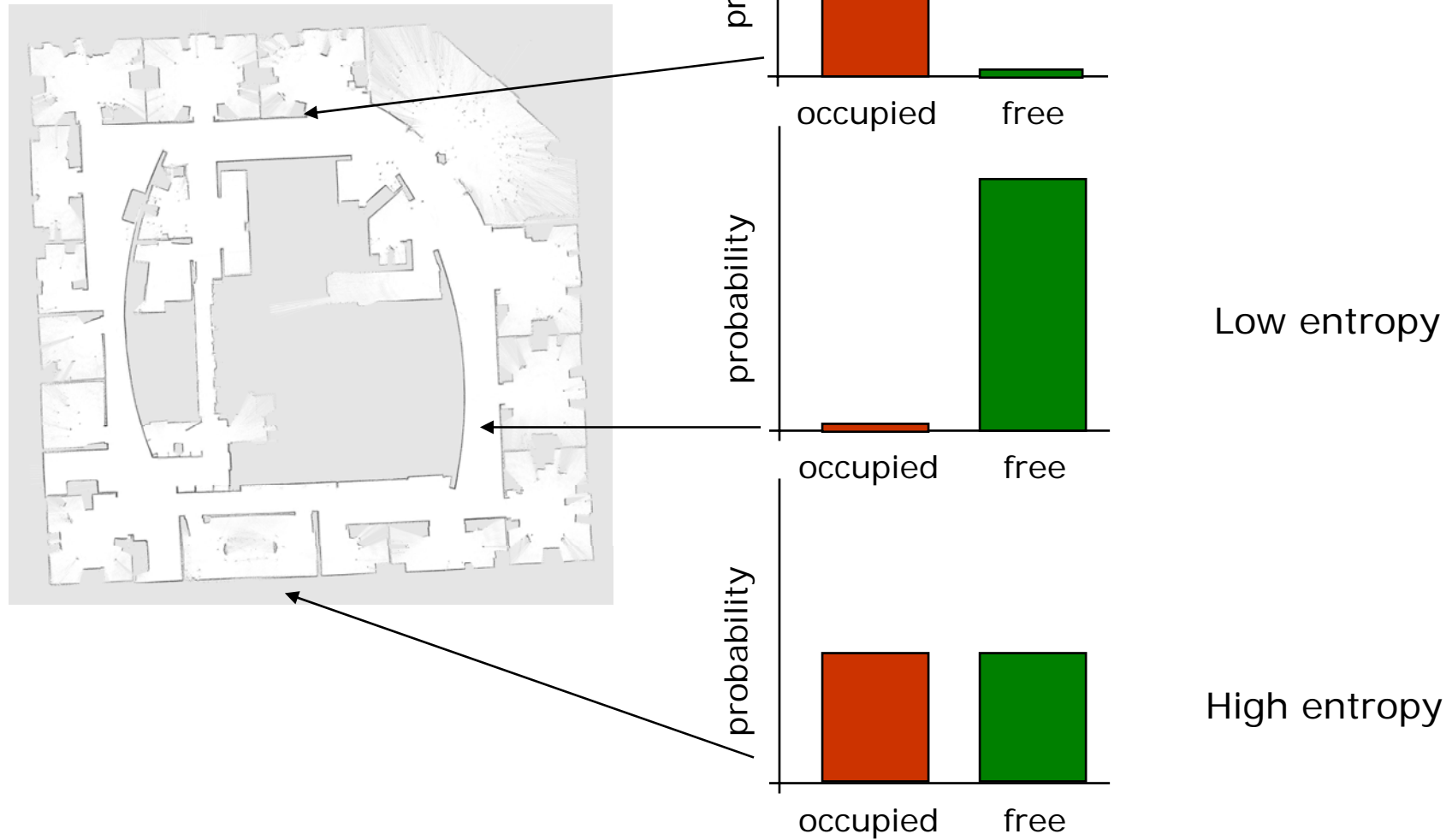
$$H(p(m)) = - \sum_{c \in m} p(c) \log p(c) + (1 - p(c)) \log(1 - p(c))$$

map uncertainty

grid cells

probability that the cell is occupied

Map Entropy



The overall entropy is the sum of the individual entropy values

Computing the Entropy of the Trajectory Posterior

1. High-dimensional Gaussian

$$H(\mathcal{G}(\mu, \Sigma)) = \log((2\pi e)^{(n/2)} |\Sigma|)$$

reduced rank for sparse particle sets

2. Grid-based approximation

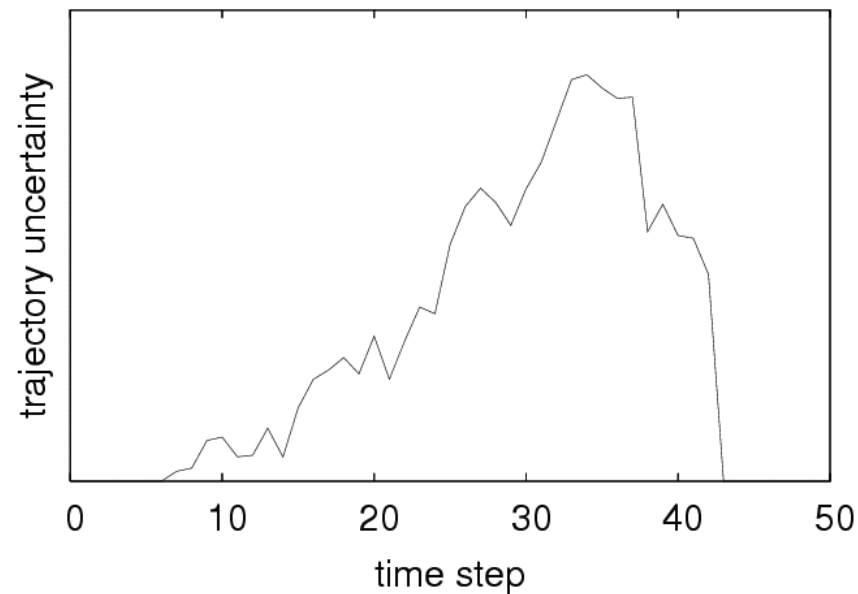
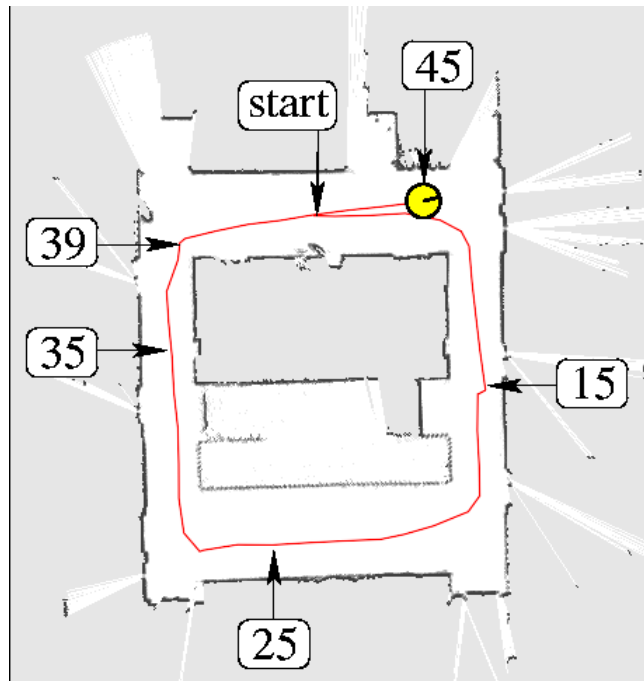
$$H(p(x | d)) \rightsquigarrow \text{const.}$$

for sparse particle clouds

Approximation of the Trajectory Posterior Entropy

Average pose entropy over time:

$$H(p(x_{1:t} | d)) \approx \frac{1}{t} \sum_{t'=1}^t H(p(x_{t'} | d))$$



Information Gain

- The reduction of entropy in the model

observations
to be obtained

action

$$I(\hat{z}, a) = H(p(m, x | d)) - H(p(m, x, \hat{x} | d, a, \hat{z}))$$

H before action
is carried out

H after action is
carried out

new poses introduced
by action

Computing the Expected Information Gain

- To compute the information gain one needs to know the observations obtained when carrying out an action
- This quantity is not known! Reason about potential measurements

$$E[I(a)] = \int_{\hat{z}} p(\hat{z} | a, d) \cdot I(\hat{z}, a) d\hat{z}$$

Reasoning about Measurements

- The filter represents a posterior about possible maps
- Use these maps to reason about possible observation
- Simulate laser measurements in the maps of the particles

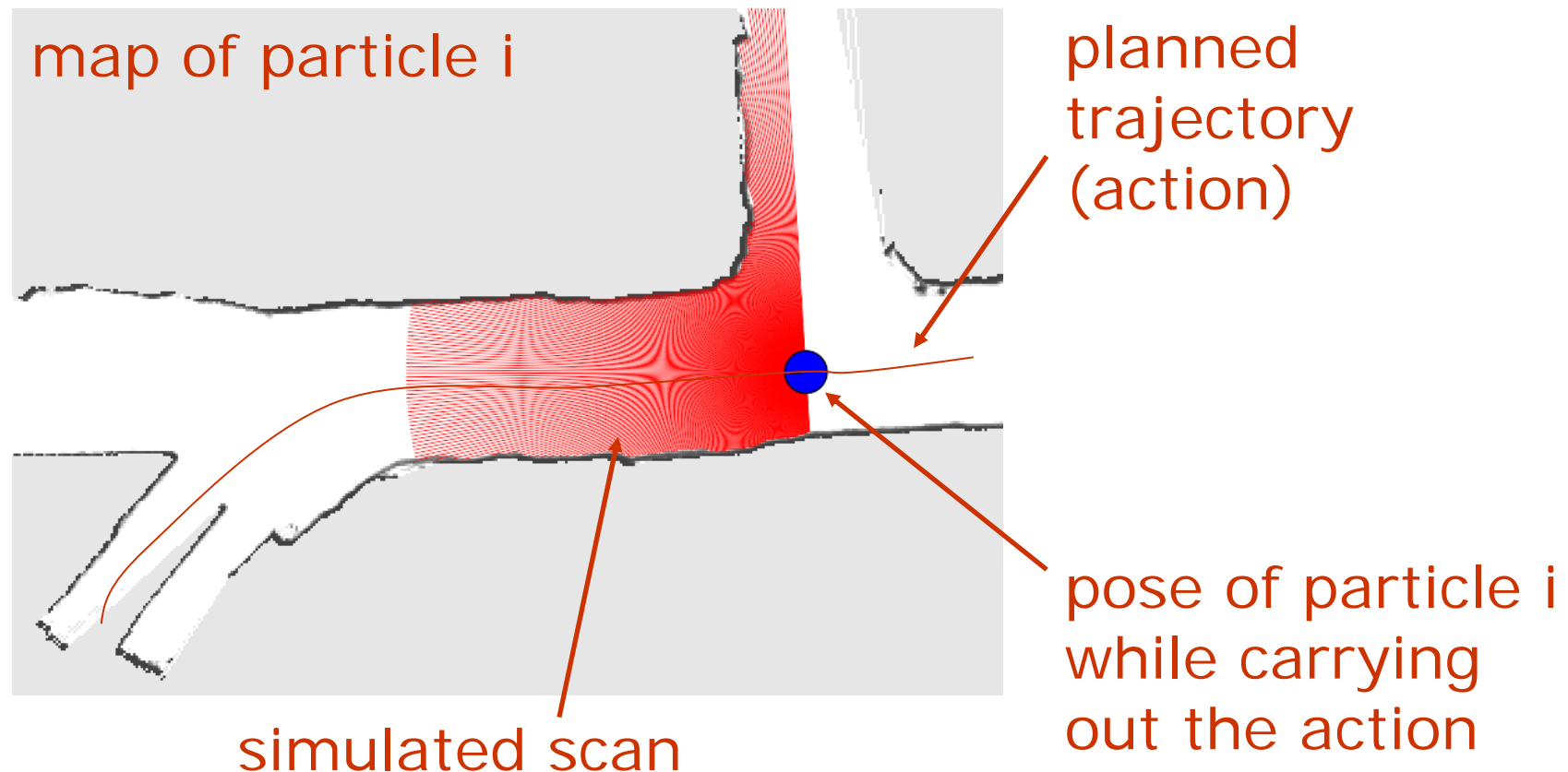
$$E[I(a)] = \int_{\hat{z}} p(\hat{z} | a, d) \cdot I(\hat{z}, a) d\hat{z}$$

measurement sequences
simulated in the maps

likelihood
(particle weight)

Reasoning about Measurements

- Ray-casting in the map of each particle to generate observation sequences



The Utility

- To take into account the cost of an action, we compute a utility

$$U(a) = I(a) - \alpha \cdot \text{cost}(a)$$

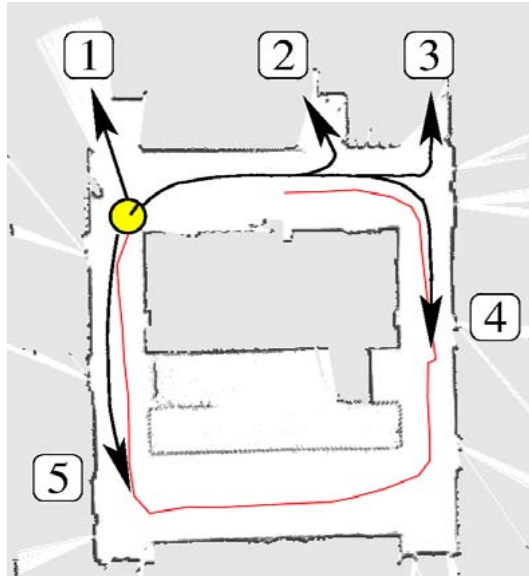
- Select the action with the highest expected utility

$$a^* = \underset{a}{\operatorname{argmax}} \{E[U(a)]\}$$

Focusing on Specific Actions

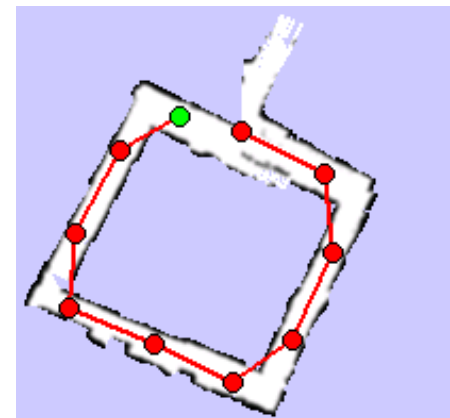
To efficiently sample actions we consider

- **exploratory actions (1-3)**
- **loop closing actions (4)** and
- **place revisiting actions (5)**

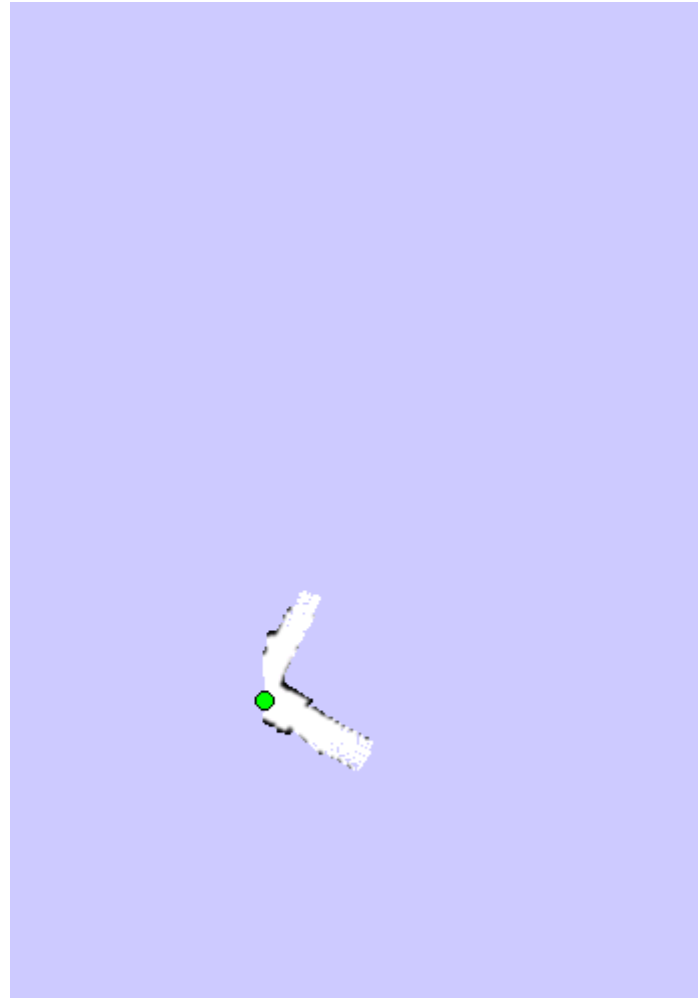


Dual Representation for Loop Detection

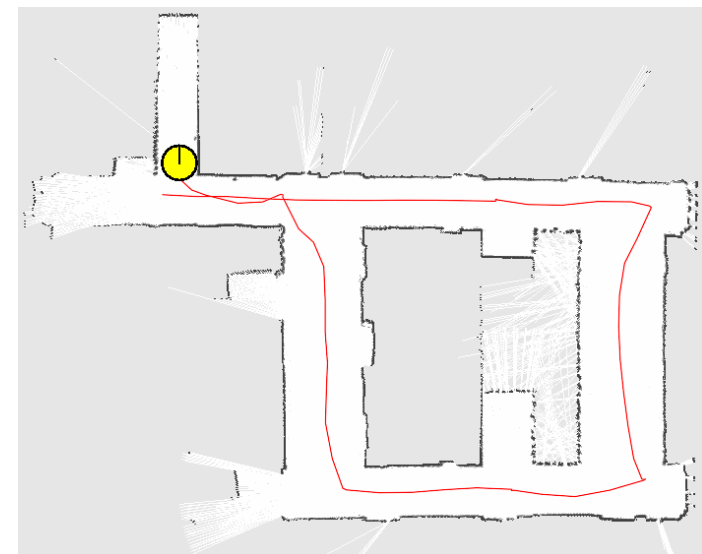
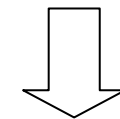
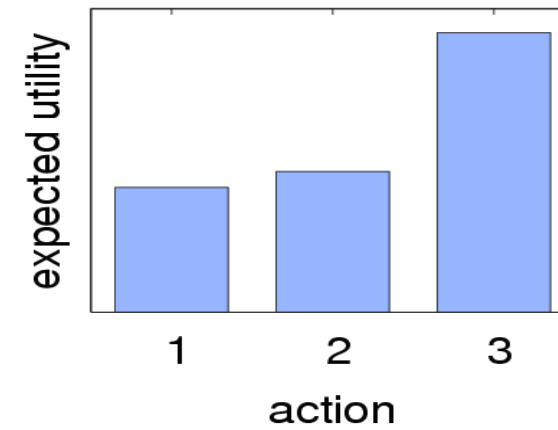
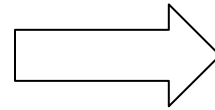
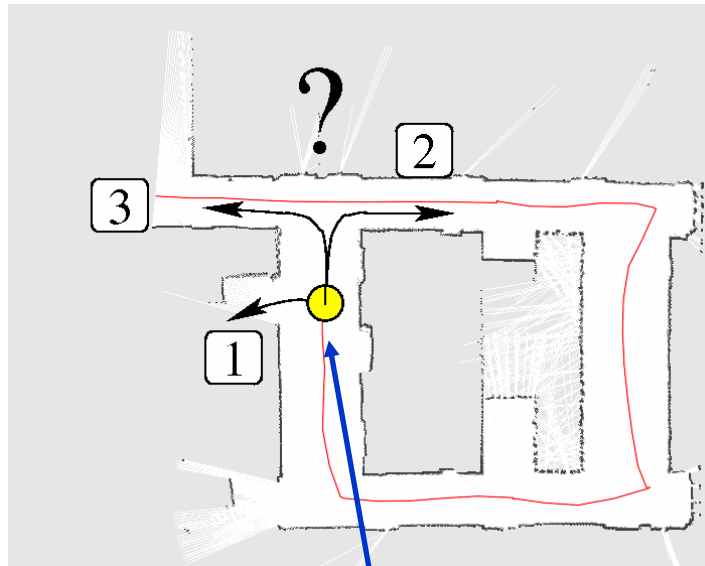
- **Trajectory graph** (“topological map”) stores the **path traversed by the robot**
- **Occupancy grid** map represents the **space covered by the sensors**
- **Loops** correspond to **long paths in the trajectory graph** and **short paths in the grid map**



Example: Trajectory Graph

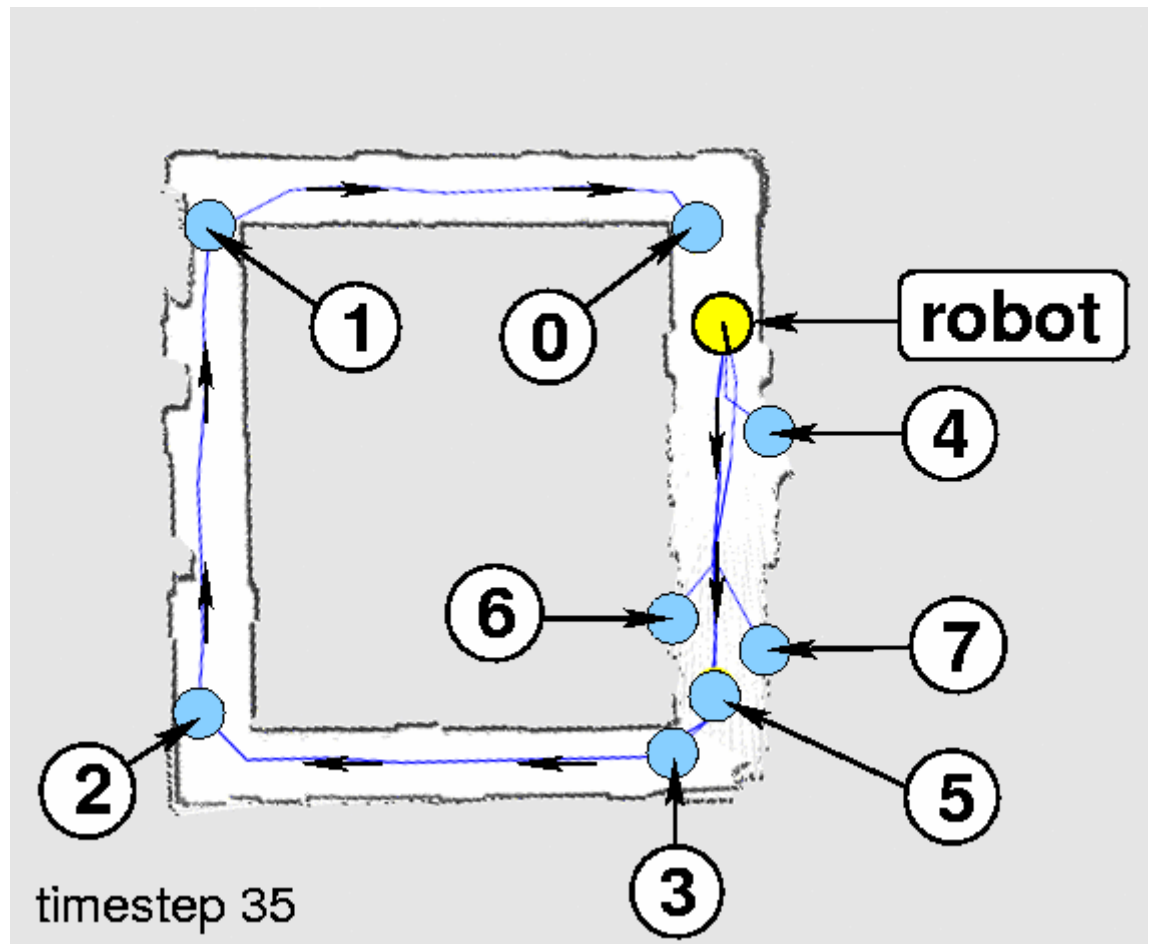
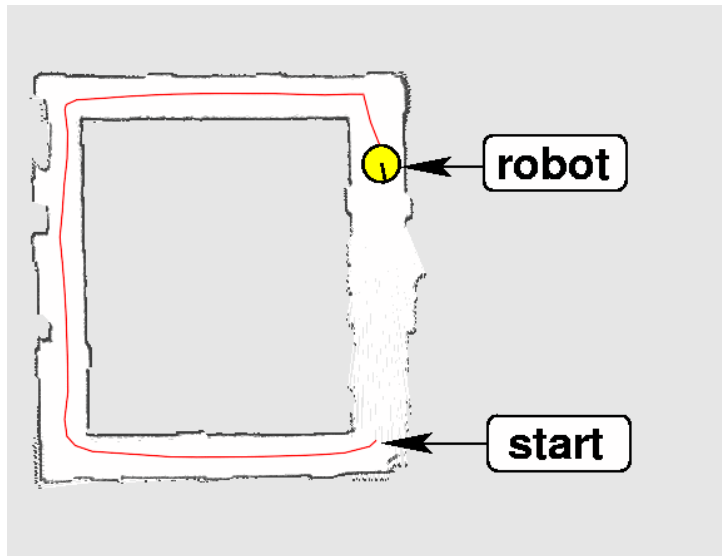


Application Example

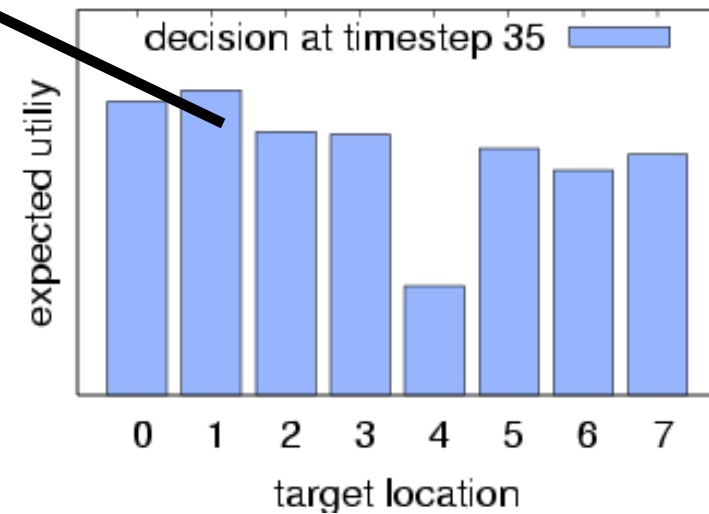
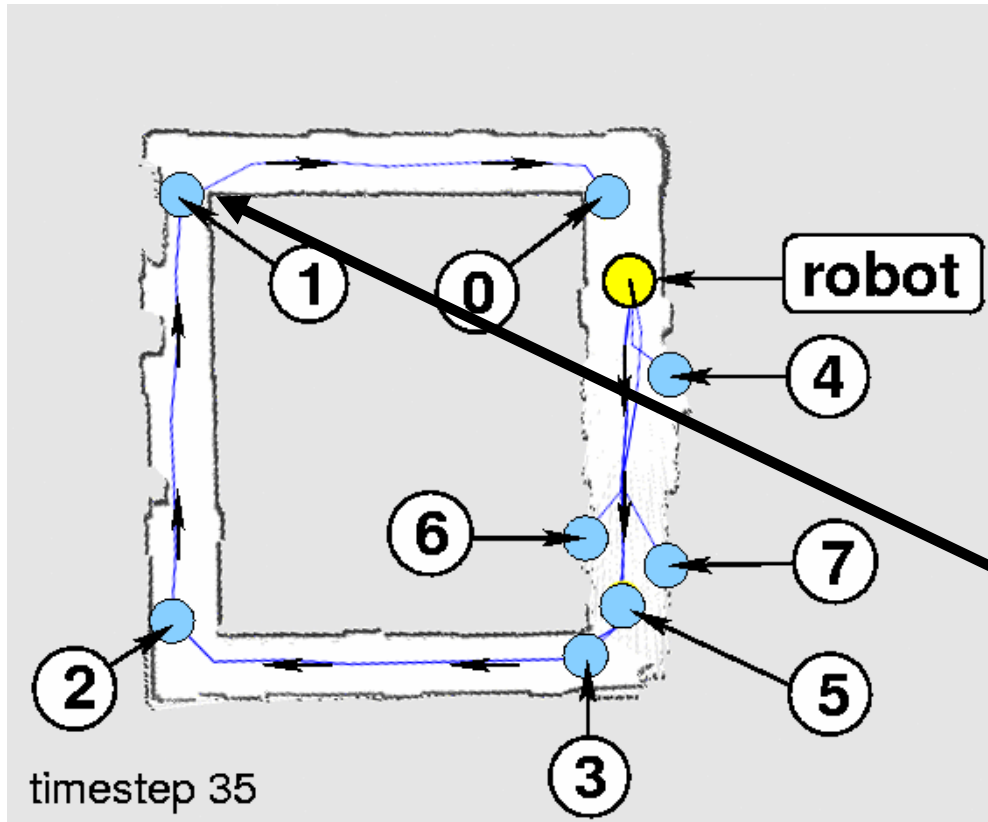


high pose uncertainty

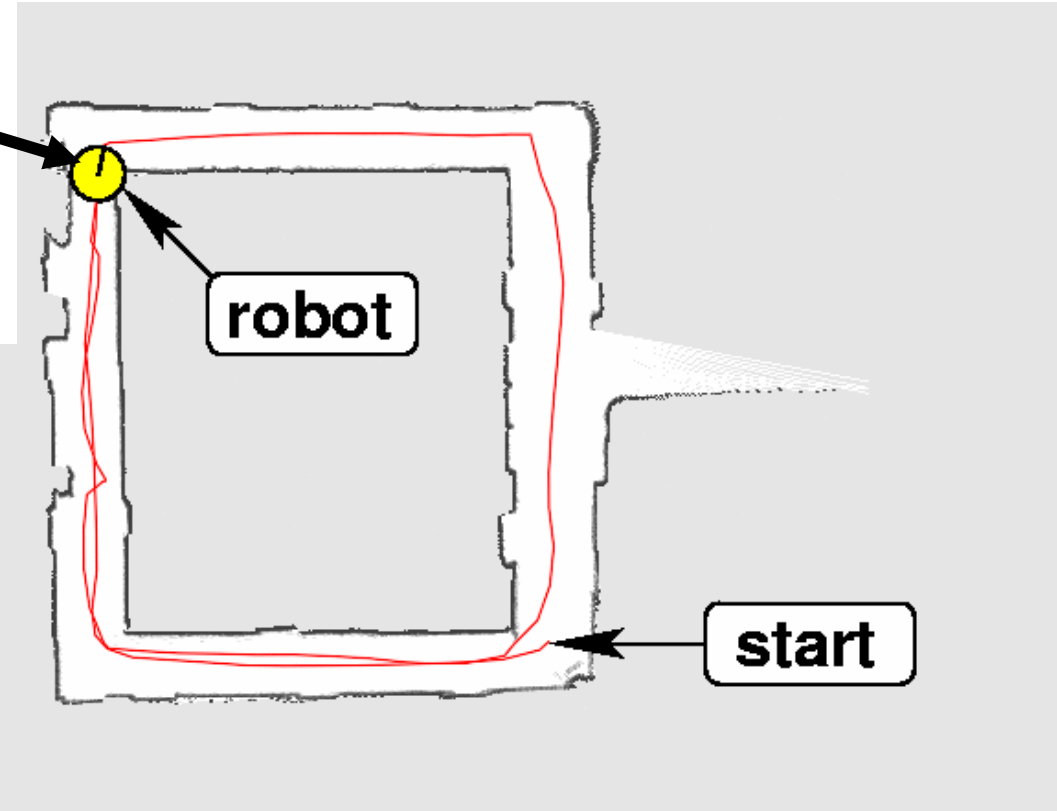
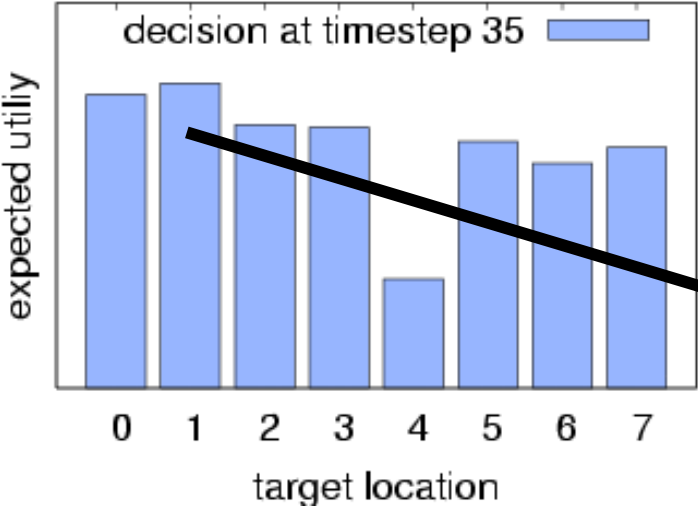
Example: Possible Targets



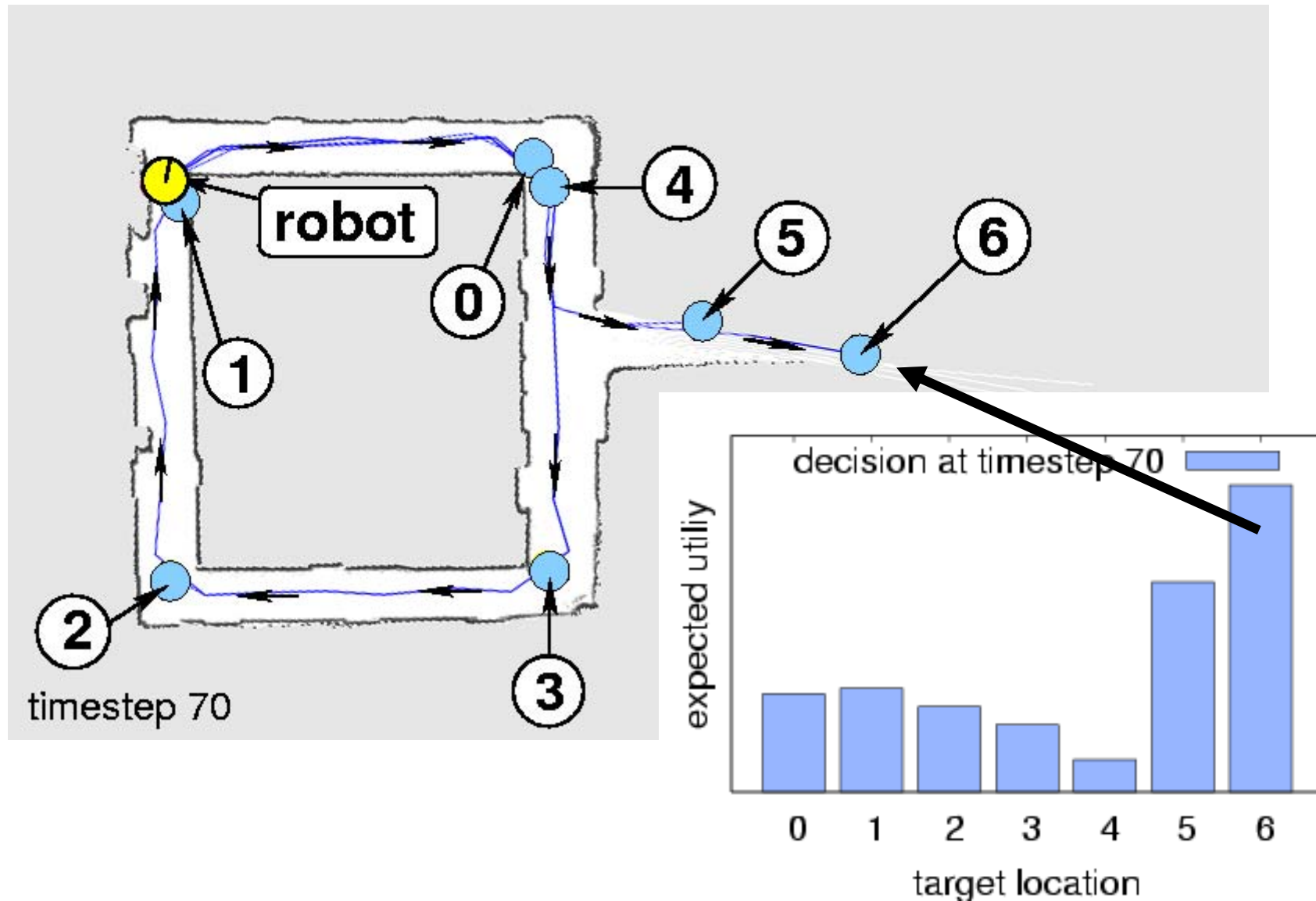
Example: Evaluate Targets



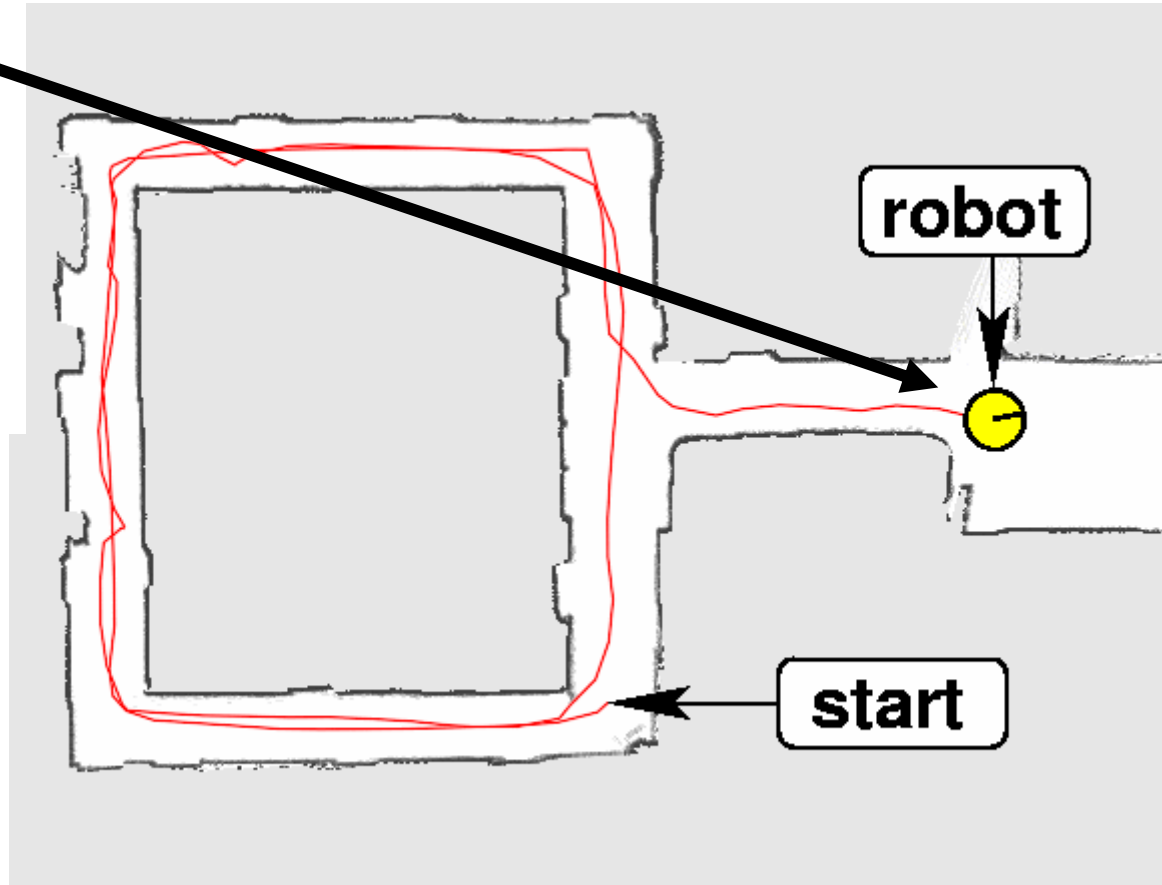
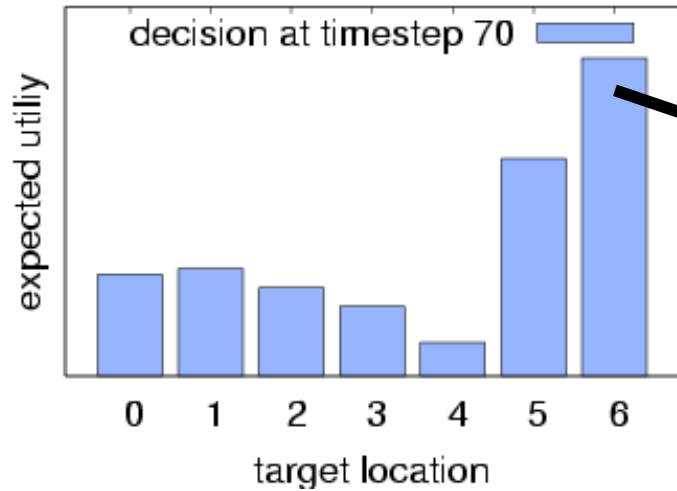
Example: Move Robot to Target



Example: Evaluate Targets

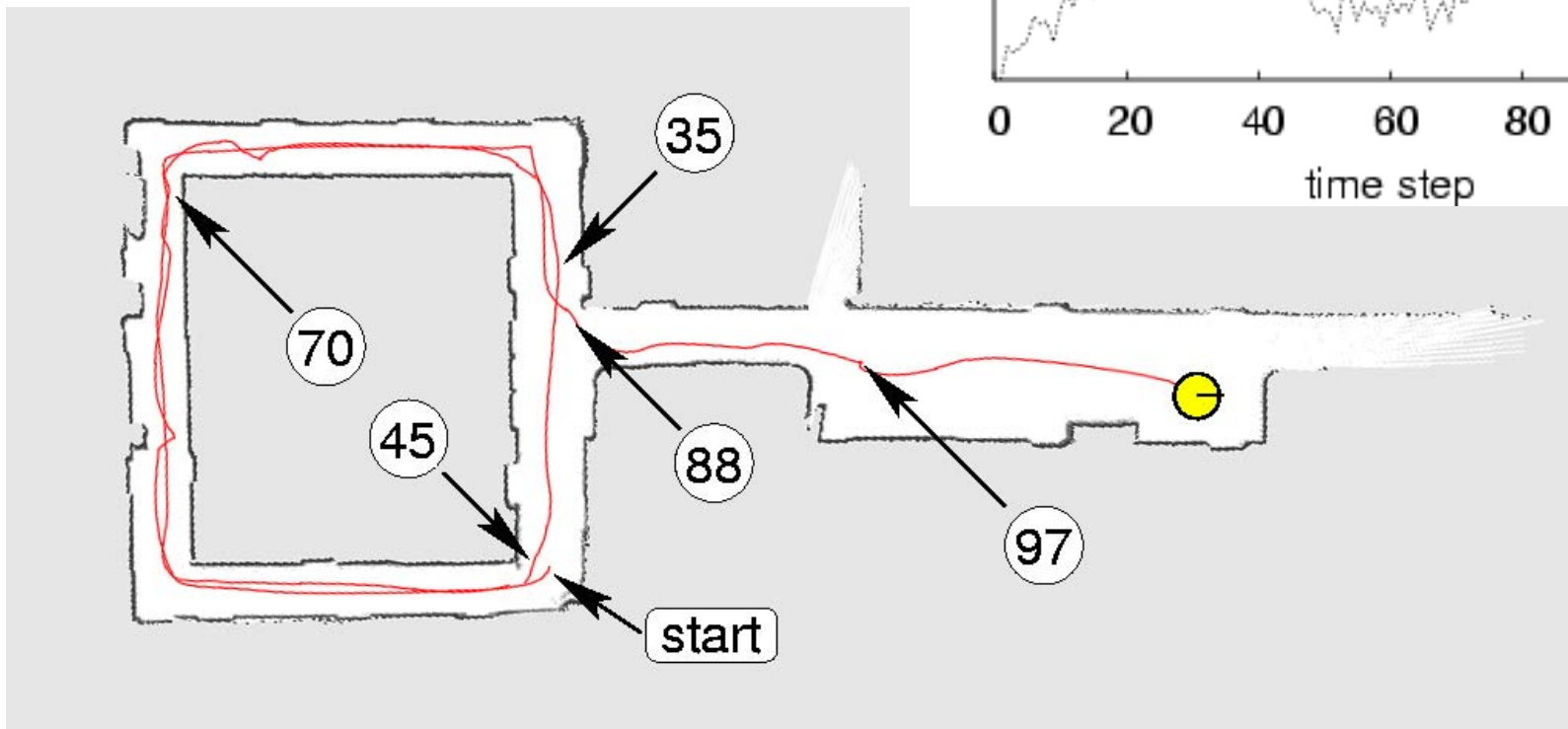


Example: Move Robot



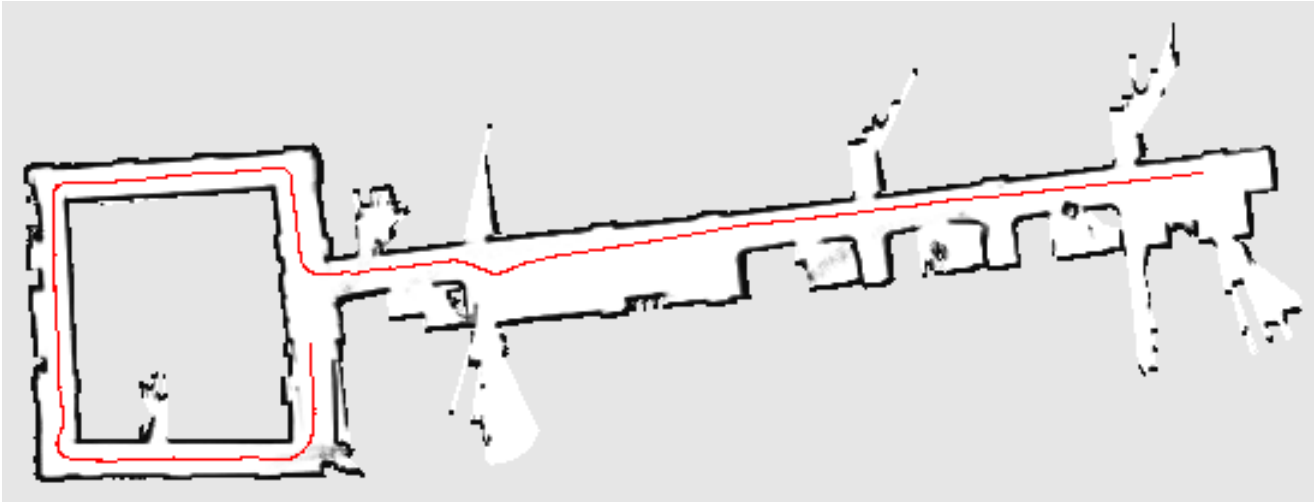
... continue ...³²

Example: Entropy Evolution

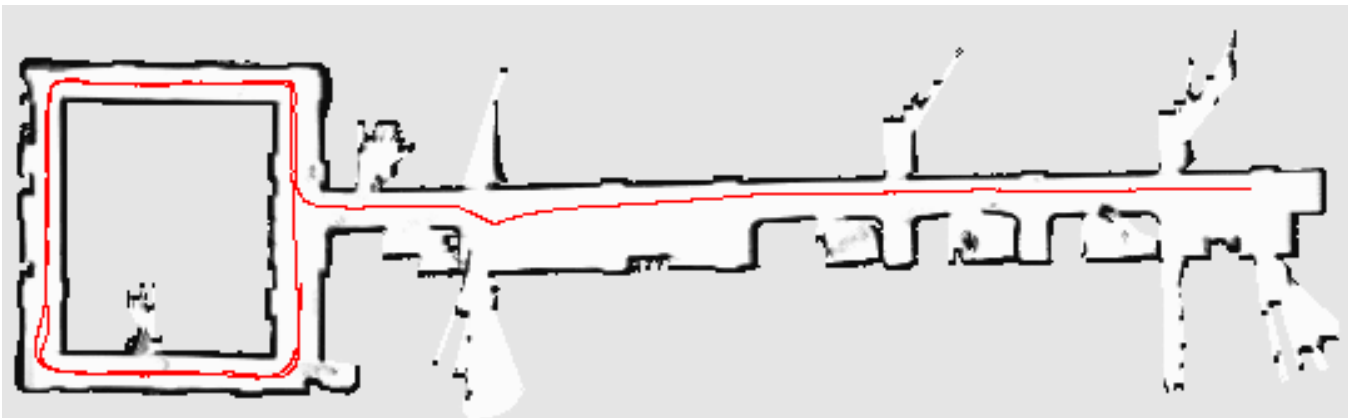


Comparison

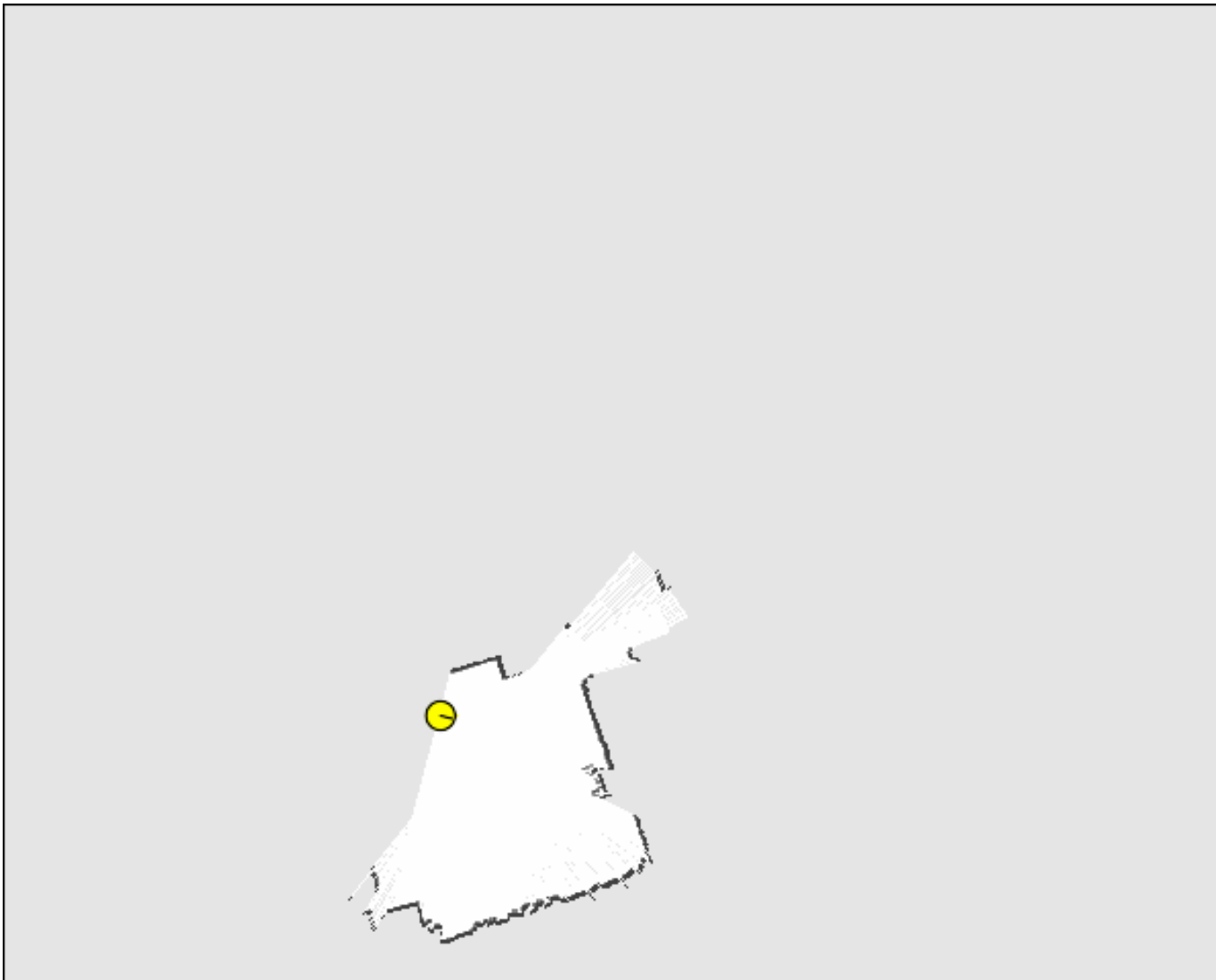
Map uncertainty only:



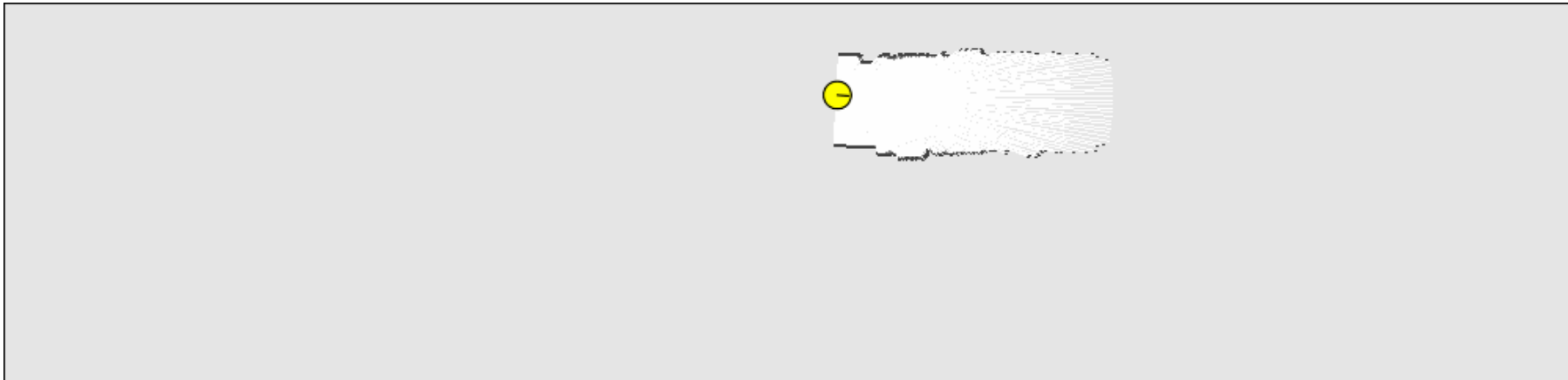
After loop closing action:



Real Exploration Example



Corridor Exploration



Summary

- A decision-theoretic approach to exploration in the context of RBPF-SLAM
- The approach utilizes the factorization of the Rao-Blackwellization to efficiently calculate the expected information gain
- Reasons about measurements obtained along the path of the robot
- Considers a reduced action set consisting of exploration, loop-closing, and place-revisiting actions
- Experimental results demonstrate the usefulness of the overall approach