

Bayes' Rule Example

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1 What does the test t for an illness i tell me?

Given we have:

- 1% of the population is ill: $p(i) = 0.01$
- Given an ill person, the test is positive in 90% of the cases: $p(t | i) = 0.9$
- Given a person that is not ill, the test is positive in 20% of the cases: $p(t | \neg i) = 0.2$

What is the probability of being ill given a positive test?

$$p(i | t) = \frac{p(t | i)p(i)}{p(t)} \quad (1)$$

$$= \frac{p(t | i)p(i)}{\sum_i p(t | i)p(i)} \quad (2)$$

$$= \frac{p(t | i)p(i)}{p(t | i)p(i) + p(t | \neg i)p(\neg i)} \quad (3)$$

$$= \frac{p(t | i)p(i)}{p(t | i)p(i) + p(t | \neg i)(1 - p(i))} \quad (4)$$

$$= \frac{0.9 \times 0.01}{0.9 \times 0.01 + 0.2 \times 0.99} \quad (5)$$

$$= \frac{0.009}{0.207} \quad (6)$$

$$\approx 0.043 \quad (7)$$

$$\approx 4\% \quad (8)$$

The probability of being ill is only 4%!