Sheet 3

Topic: State Estimation

Submission deadline: Tuesday 12.5.2009 (before class)

Exercise 1:

1. Prove the conditionalized version of the general product rule:

$$P(A, B \mid E) = P(A \mid B, E) \cdot P(B \mid E)$$

2. Prove the conditionalized version of Bayes' rule:

$$P(A \mid B, C) = \frac{P(B \mid A, C) \cdot P(A \mid C)}{P(B \mid C)}$$

Exercise 2:

Suppose you are a witness to a nighttime hit-and-run accident involving a taxi in Athens. All taxi cars in Athens are blue or green. You swear, under oath, that the taxi was blue. Extensive testing shows that, under the dim lighting conditions, discrimination between blue and green is 75% reliable. Is it possible to calculate the most likely color for the taxi? (Hint: distinguish carefully between the proposition that the taxi is blue and the proposition that the taxi appears blue.) What is your resulting estimate, given that 9 out of 10 Athenian taxis are green?

Exercise 3:

Consider a robot that can be in any of 10 possible states ($x = 1, \ldots, x = 10$). The robot is equipped with a sensor that indicates the state x the robot is in. This sensor, however, is not perfect and indicates the correct state only 90% of the cases. In 10% of the cases, the sensor indicates an incorrect state with equal probability.

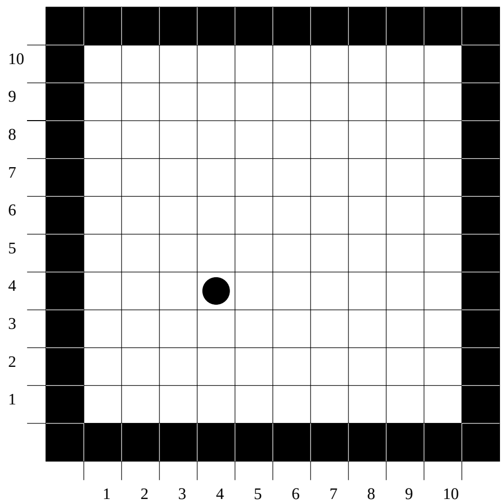
1. Compute the probability $P(x = 5|z = 5)$ for the robot of being in state 5 given that the sensor indicates that the robot is in state 5, if the robot could be in any state before the first sensor indication. What is the probability $P(x = 4|z = 5)$ for the robot of being in state 4?

2. Assuming that a second indication $z = 5$ is obtained, what would be the probability of being is state 5?

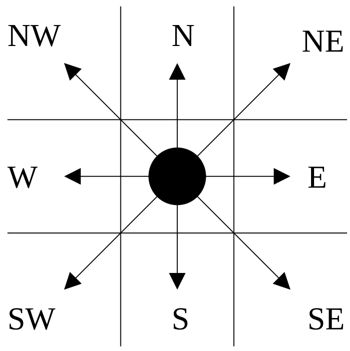
3. What is the *belief state* (the probability $P(x)$ for all possible values of x) for the sequence of sensor indications: $z_1 = 4$, $z_2 = 5$, and $z_3 = 5$, if the robot could be in any state before the first sensor indication.

Exercise 4:

Consider a robot within a grid as the one illustrated in Figure 1(a). The position of the robot is specified by the cell $< i, j >$ it occupies, and assume it can move to any of the 8 neighboring empty cells in a single movement as illustrated in Figure 1(b). However, due to imperfections robot actuation, the robot ends up in the destination cell in 80%. In the remaining 20% of the cases, the robot doesn't move at all, and remains in the same cell.



(a) Representation of the state space of a robot using an occupancy grid. Cells can be occupied (black-colored cells) or not (white-colored cells). The state x of the robot is specified by the cell it occupies.



(b) Possible single-step movements of a robot in a simple grid world.

1. Write an *Octave* program that given an grid map, a belief state corresponding to the position of the robot in the grid and a motion command, computes the new belief state based on the described motion model.
2. Assuming the robot is located in cell $< 5, 1 >$ in the grid map shown in Figure 1(a), compute the belief state of the robot after executing 9 movements in the N direction (see Figure 1(b)).
3. Repeat Exercise 2.2 but assuming the robot could be located in any cell in the bottom row $< 1, 1 >, < 2, 1 >, \ldots, < 10, 1 >$ of the grid.
4. Repeat Exercise 2.3 but now assuming that cells $< 4, 5 >, < 5, 5 >, \ldots, < 8, 5 >$ are occupied.

Exercise 5:

Assume that a proximity sensor is installed on the robot in the previous exercise. The sensor measures the Manhattan distance between the current cell of the robot and cell $< 5, 5 >$ in the grid map. The sensor indicates the correct distance d 80% of the time; 10% of the time the reading is $d - 1$, and $d + 1$ the other 10%.

1. Write an *Octave* program that given a grid map, a belief state corresponding to the position of the robot in the grid and a distance measurement d obtained from the above described sensor, computes the new belief state integrating the measurement using the described sensor model.

2. Assuming the robot is located in any cell in the grid shown in Figure 1(a) with equal probability, compute the belief state of the robot after obtaining each of the following measurements: $z_1 = 4$, $z_2 = 4$, $z_3 = 3$, and $z_4 = 4$

3. Integrate the program written for Exercise 2.1 and Exercise 3.1 into a single *Octave* program capable of handling both motion commands and sensor measurements, and compute the belief state of the robot after sequentially processing the following sequence of motion commands and observations:

   $z_1 = 4, u_1 = \text{N}, z_2 = 3, u_2 = \text{N}, z_3 = 3, u_3 = \text{N}, z_4 = 2, u_4 = \text{N}, z_5 = 1$

   Since nothing is said about the initial location of the robot, assume that it could be located in any empty cell in the grid.