Albert-Ludwigs-Universität Freiburg | Institut für Informatik
Lecture: Introduction to Mobile Robotics
Summer term 2009 | Prof. Dr. W. Burgard

Sheet 6

Topic: Particle Filter and Monte Carlo Localization, Kalman Filter

Submission deadline: Tuesday 9.6.2009 (before class)

Exercise 1:

Consider a robot in a corridor like the one illustrated in Figure 1.

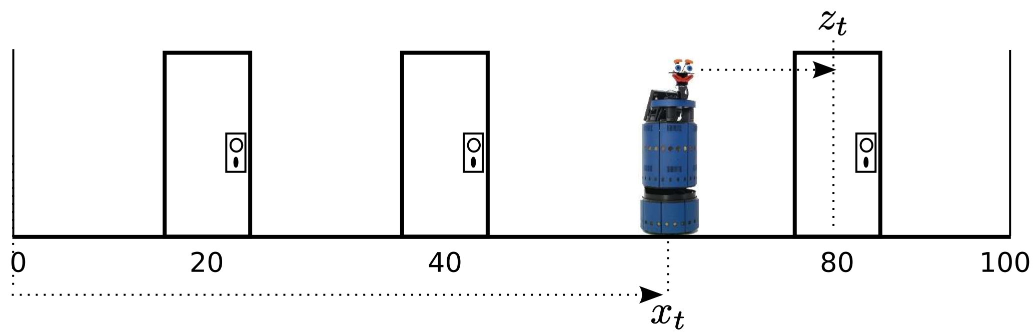

Figure 1: The state x_t of the robot corresponds to the distance of the robot to the start of the corridor. Each sensor measurement z_t is a single range reading that corresponds to the distance between the robot and the nearest door.

Write a program in *Octave* that implements a particle filter to estimate the state x_t of the robot from a sequence $u_0, z_1, u_1, z_2, \ldots, u_{t-1}, z_t$ of odometry commands and sensor measurements.

- The robot is equipped with a sensor that measures the distance to the nearest door in the corridor. Each sensor measurement is a single range reading z_t that corresponds to the distance to the nearest door. Every reading is corrupted only with Gaussian noise $\mathcal{N}(\mu, \sigma)$ with $\mu = 0$ and $\sigma = 0.5m$.
- The robot is also equipped with wheel encoders that produce time-discrete odometry measurements $u_t = \langle \delta_{trans} \rangle$. Here too, every measurement is corrupted only with Gaussian noise $\mathcal{N}(\mu, \sigma)$ with $\mu = 0$ and $\sigma = 2m$. (This motion model is the same motion model presented in the lecture but without the rotational components and in the x dimension only)

To implement the particle filter you will need to:

- Write a method for sampling a new pose x_t given an old pose x_t and an odometry command $u_t = \langle \delta_{trans} \rangle$. Implement the motion model described above.

- Write a function that, given a range measurement z_t, a pose x_t and a map m, computes the likelihood $p(z \mid x, m)$ of the measurement. The map m consists of a sequence of distances each one corresponding to the distance between a door and the start of the corridor. To compute the likelihood $p(z \mid x, m)$ of the measurement implement the observation model described above.
- Write a function to generate a particle set with N particles that is uniformly distributed throughout the state space.
- Write a method to resample the particles according to their weights. Concretely, implement the stochastic universal resampling algorithm presented in the lecture.

Use your implementation of the particle filter on the following data. The map is the one illustrated in Figure 1. The length of the corridor is 100 meters. There are three doors at the following positions: 20.0, 40.0, and 80.0 meters.

You have no prior knowledge about the position of the robot at first. Then, you obtain the following odometry commands and observation pairs:

1. $u_0 = 10m, z_1 = 10m$
2. $u_1 = 20m, z_2 = 10m$
3. $u_2 = 20m, z_3 = 10m$
4. $u_3 = 10m, z_4 = 20m$

Use 10 particles and plot the position of the particles in the map after integrating each pair of odometry command and observation.

Repeat the exercise using 250 particles.

Exercise 2:

1. Particle filters use a set of weighted state hypotheses or particles to approximate the true state x_t of the robot at every time step t. Describe three different techniques to obtain a single state estimate $\bar{x}_t$ out of a set of N weighted samples $S_t = \{\langle x_t^{[i]}, w_t^{[i]} \rangle \mid i = 1, \ldots, N\}$.
2. How does the computational cost of the particle filter scale with the number of particles and the number of dimensions in the state vector of the particles? Why can a large dimensionality be a problem for practice filters?

Exercise 3:

1. Write down the analytical equations to compute the determinant for a 2×2 and a 3×3 matrix.

2. Write down the analytical equation to compute the inverse of a 2×2 matrix and a 3×3 matrix.

3. Using the analytical equation above, invert the following matrix showing all intermediate computation and results:

$$\mathbf{C} = \begin{pmatrix} 0.25 & 0.1 \\ 0.20 & 0.5 \end{pmatrix}$$

4. Explain briefly, in your own words, two techniques to invert an $N \times N$ matrix for large values of N.

Exercise 4:

1. Write down the equation that corresponds to the state transition probability $p(x_t \mid u_t, x_{t-1})$ within the Kalman filter framework. Explain each term in the equation, including the dimension of the vectors and matrices.

2. Write down the equation corresponding to the observation probability $p(z_t \mid x_t)$ within the Kalman filter framework. Explain each term in the equation, including the dimension of the vectors and matrices.