Albert-Ludwigs-Universität Freiburg — Institut für Informatik
Lecture: Introduction to Mobile Robotics
Summer term 2009 — Prof. Dr. W. Burgard

Sheet 7

Topic: Kalman Filter

Submission deadline: Tuesday 16.6.2009 (before class)

Figure 1: The blimp is equipped with downward-looking ultrasound sensor for measuring the vertical distance to the underlying obstacles.

Exercise 1:

Consider an flying vehicle, like the blimp in Figure 1, equipped with a sensor to measure the height h of the vehicle above the ground. The state x_t of the vehicle is characterized only by its height h_t. The controls u_t describe the desired change in the height Δh_t from state x_{t-1} to x_t.

1. Specify the components ($A_t, B_t, C_t, \varepsilon_t$ and δ_t) of a Kalman filter for estimating the state x_t of the vehicle.
2. Let the process and measurement noise be $\sigma_u^2 = 0.25$ and $\sigma_z^2 = 0.1$ respectively. Estimate the state of the system after the following sequence of controls and measurements:

$$u_1 = 0.5, z_1 = 3.6, u_2 = 0.25, z_2 = 3.7\,.$$

   Indicate the values for $\mu_t, \sigma_t^2, \bar{\mu}_t, \bar{\sigma}_t^2$ and K_t for each time step.

   Assume the initial belief is $\text{bel}(x_0) \sim \mathcal{N}(\mu_0 = 3, \sigma_0^2 = 0.5)$. Assume also that the components of the filter are static, that is, they do not change in time ($A_t = A_{t-1}$).

Exercise 2:

Let the state x_t of the above mentioned vehicle be characterized by the its height h_t and vertical velocity v_t. As in the previous exercise, the controls u_t describe the desired change in the height Δh_t of the vehicle from state x_{t-1} to x_t. Assuming a constant velocity motion model:

1. Specify the components ($A_t, B_t, C_t, \varepsilon_t$ and δ_t) of a Kalman filter for estimating the state x_t of the vehicle.

2. Write a program in *Octave* that implements a Kalman filter for the above mentioned system and use it to estimate the state of the blimp. Use the following process and measurement noise

$$R = \begin{pmatrix} 0.25 & 0 \\ 0 & 0.5 \end{pmatrix} \text{ and } Q = (0.05) \ ,$$

   initial belief $\text{bel}(x_0) \sim \mathcal{N}(\mu_0, \Sigma_0)$ with

$$\mu_0 = \begin{pmatrix} 0.20 \\ 0.05 \end{pmatrix} \text{ and } \Sigma_0 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.2 \end{pmatrix} ,$$

   and the sequence of controls and measurements listed in the file *log3.log* (available for downloading on the web page). Plot the estimated height and the ± 2 sigma error bounds (2 times the estimated standard deviation of the height) as a function of time. In the same diagram, plot the observations as a function of time.

3. Repeat the experiment using different process and measurement noise:

   (a) plot the results for a case when the process noise is considerably smaller than the measurements noise,

   (b) and for the case when the measurement noise is considerably smaller than the process noise.

   Explain how different values for the process and measurement noise affect state estimation referencing the resulting diagrams.

4. Plot the vertical velocity estimate v_t from exercise 2.2 as a function of time. In the same diagram, plot the vertical velocity computed as $\Delta h / \Delta t$. What are the fundamental differences?