Albert-Ludwigs-Universität Freiburg — Institut für Informatik
Lecture: Introduction to Mobile Robotics
Summer term 2009 — Prof. Dr. W. Burgard

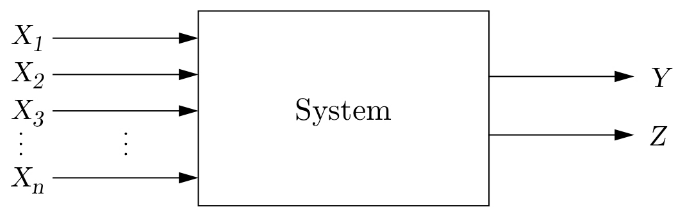
# Sheet 8

## Topic: Error Propagation

Submission deadline: Tuesday 23.6.2009 (before class)

**Exercise 1: First-Order Error Propagation**

Suppose the general case of a non-linear multi-input multi-output system with $n$ correlated input random variables $X_1, ..., X_n$ with $X_i \sim \mathcal{N}(\mu_{X_i}, \sigma^2_{X_i})$ and (without loss of generality) two output random variables $Y$ and $Z$.




We set $Y = f(X_1, ..., X_n)$ and $Z = g(X_1, ..., X_n)$ and approximate the functions $f(.)$ and $g(.)$ by a first-order Taylor series expansion:

$$Y \approx f(\mu_1, ..., \mu_n) + \sum_{i=1}^{n} \left.\frac{\partial f}{\partial X_i}\right|_{\mu_i,...,\mu_n} (X_i - \mu_i) \tag{1}$$

$$Z \approx g(\mu_1, ..., \mu_n) + \sum_{i=1}^{n} \left.\frac{\partial g}{\partial X_i}\right|_{\mu_i,...,\mu_n} (X_i - \mu_i) \tag{2}$$

Derive the expression for the covariance $\sigma_{YZ}$ between $Y$ and $Z$ given the rules for the expected value

$$E[a] = a \tag{3}$$

$$E[aX] = aE[X] \tag{4}$$

$$E[X+Y] = E[X] + E[Y] \tag{5}$$

$$E[XY] = E[X]E[Y] \qquad \text{if X and Y are independent} \tag{6}$$

and the following definitions for mean, variance and covariance:

$$\mu_X = E[X] \tag{7}$$

$$\sigma^2_X = E[(X - E[X])^2] \tag{8}$$

$$\sigma_{XY} = E[(X - E[X])(Y - E[Y])] \tag{9}$$

**Exercise 2: Extended Kalman Filter**

Consider an electric toy-train on a circular track with unknown radius $r$. It moves with unknown but constant translational velocity $v$ along the track. Assume a coordinate system that is placed with its origin in the center of the track circle, and let $\theta$ be the unknown angle from the x-axis to the head of the train, as shown in Fig. 1. A wall is located parallel to the y-axis with its shortest distance from the train track being $d\!=\!3m$.

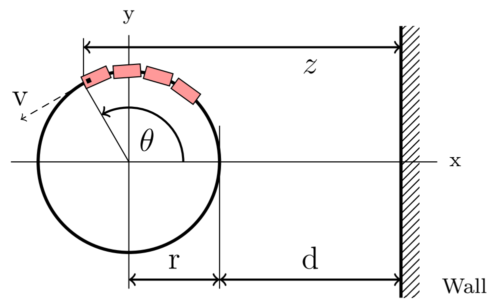



Figure 1: Toy-train with constant translational velocity $v$ on circular track

Suppose a rough estimate made by a human observer for the initial state at $t\!=\!0$, given by $\theta\!=\!\frac{\pi}{2}$, $v\!=\!1\frac{m}{s}$ and $r\!=\!1m$. A range sensor measures the shortest distance $d$ from the head of the train to the wall in meters, with gaussian sensor noise $(\mu=0,\;\; \sigma=0.2m)$. We consider the state of the system as a vector of the three aforementioned unknowns, $x_t = \langle\theta_t, v_t, r_t\rangle^T$ at time step $t$, and want to estimate these values over time using the state mean $\mu_t$ and state covariance $\Sigma_t$.

The Extended Kalman Filter (EKF) can be used to solve this problem, as it uses first order error propagation to approximate the uncertainty in the state and measurement predictions when dealing with non-linear systems.

1. Specify the state transition function $\mu_t\!=\!g(\mu_{t-1})$ and the measurement function $\hat{z} = h(\mu_t)$ of the system, along with their Jacobians $G$ and $H$. *Hints:* since $\theta_t$ measures the angle in radians, the difference in theta is equal to the travelled distance along the track. The definition of a Jacobian can be found on error propagation slides.

2. Determine the measurement covariance matrix $Q$, and choose reasonable values for the initial state covariance matrix $\Sigma_0$ and the state transition covariance matrix $R$. Consider that the radius of the track circle and the translational velocity of the train is constant, and that the initial state estimate is prone to higher errors.

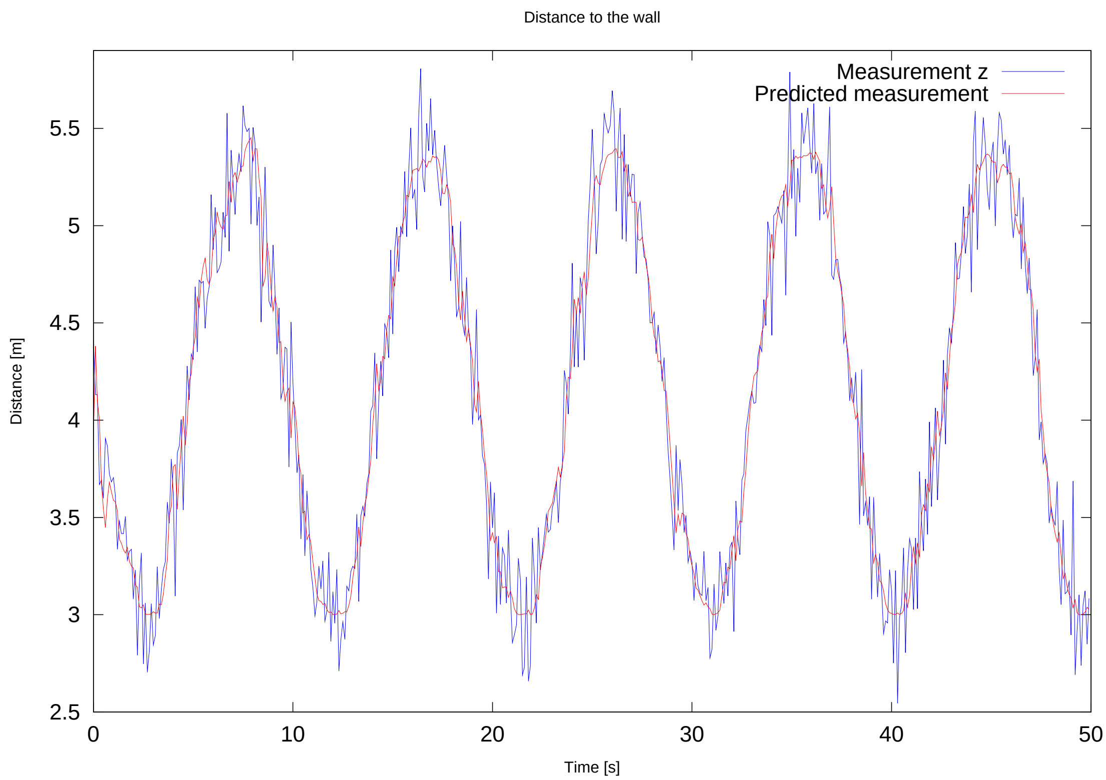
3. Implement the EKF for this exercise using *Octave*. Use the source code stub provided on the web page: it loads the data file, visualizes the data and gives further guidance. Test your program with the data file **log4.log** provided on the web page.

4. Refine your noise matrices to achieve results similar to the plots in Fig. 2 at the end of the document.

**Exercise 3: Landmark-Based SLAM**

Consider a mobile robot that has to map office and outdoor environments using a landmark-based SLAM approach. For both environments, think of five different types of landmarks that might be well suited. How useful are your landmarks in combination with the following sensors: sonar, laser, monocular vision, stereo vision? Please arrange your ratings[1] ("++" very useful, "+" useful, "−" not useful, "−−" not possible) in a $10 \times 4$ table and give short explanations for not-obvious ratings.

[1]Please note, that different ratings might be possible under different assumptions (e.g. lighting-conditions). The optimal choice of landmarks is still an open research question.



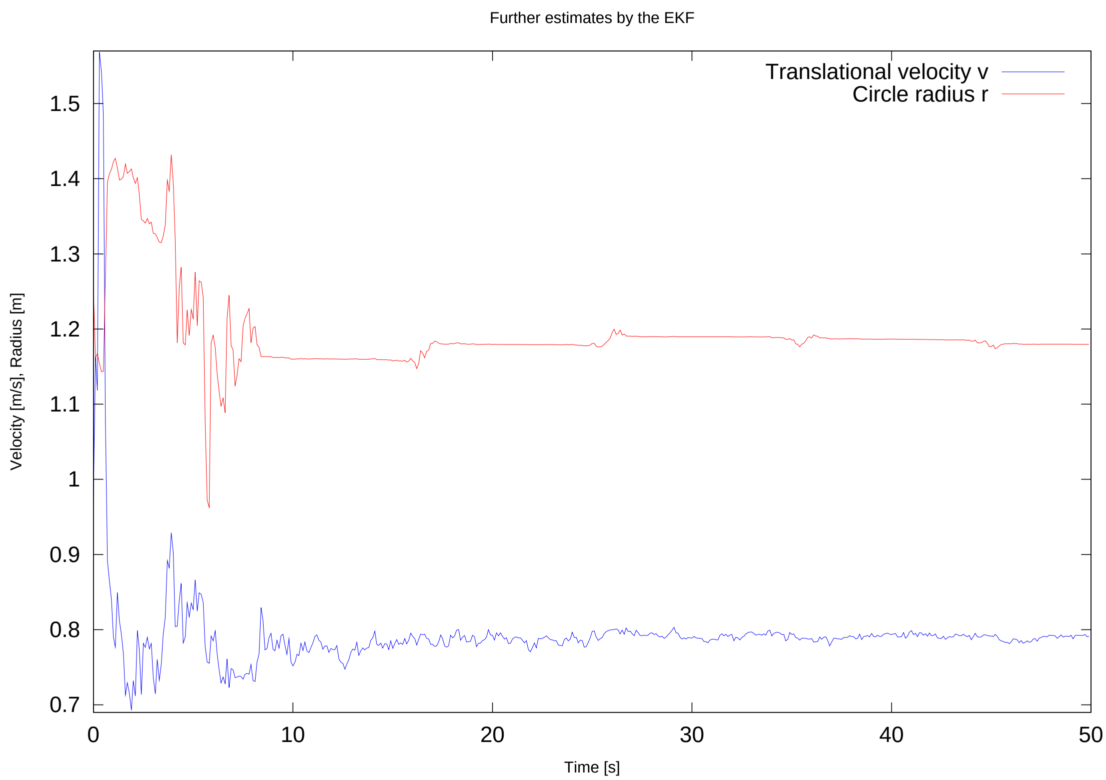




Figure 2: Estimates of $x = \langle \theta, v, r \rangle$ over time.