Sheet 9

Topic: Mapping with Known Poses, Landmark-based SLAM

Submission deadline: Tuesday 29.6.2009 (before class)

Exercise 1:

Two dimensional evidence grids are one of the most popular representations of the environment in mobile robotics. In this representation, the environment is subdivided into a grid of rectangular cells and to each cell an occupancy probability is associated indicating whether the cell is occupied or not. A probability of 0 means that the corresponding cell is most likely unoccupied while a probability of 1 means that the cell is most likely occupied.

There exists many methods for updating the occupancy probability of the cells as new observations become available. One of them is the *simple counting* method presented in the lecture.

1. Using the source code stub provided on the web page, implement the *simple counting* method to update the occupancy values of a grid map. The provided coded loads the data, displays the map as its being build, and saves a couple of screen-shots of the map-building process. Please include the generated images in your solution.
2. An alternative technique for building an occupancy grid would be to use only the end points of the range measurements. A cell is considered occupied if one or more range measurements ended in that cell. Using the source code stub provided on the web page, implement the *end-point* method to update the occupancy values of a grid map. Please include the generated images in your solution.
3. Discuss the advantages and disadvantages of the *simple counting* method and the *end-point*.

Exercise 2:

At time index t a robot performs a measurement $z_t = \langle d, \phi \rangle$ of a single unique landmark l, where d denotes the distance to the landmark and ϕ the bearing angle with respect to the robot's coordinate system. The robot's poses from which the measurements were performed are known and given by $\vec{x_t} = \langle x, y, \theta \rangle$ in world coordinates. The measurements are distorted by Gaussian noise, with $\sigma_d = 1$ and $\sigma_\phi = 0.1$.

The positions $\mu_t = \langle x_l, y_l \rangle$ of the landmarks expressed in world coordinates and the corresponding uncertainties Σ_t are integrated into the map and updated for each measurement using one Extended Kalman Filter (EKF) per landmark. Since the landmarks are assumed to be stationary, the prediction step of the EKF is omitted.

When a landmark is perceived for the first time, the corresponding measurement and the measurement noise matrix are transformed into world coordinates to get the initial estimates $\mu_0 = \bar{h}(z)$ and $\Sigma_0 = \bar{H}Q\bar{H}'$ where $\bar{h}$ is the inverse of h, and $\bar{H}$ the Jacobian of $\bar{h}$.

This exercise comprises calculations that you can do on paper (or using any tool like Octave). Please provide formulas and numbers, no matter how you do it. Use $\Delta x = x_l - x$ and $\Delta y = y_l - y$.

1. Determine the measurement function h, its linearization H (Jacobian), and the measurement noise matrix Q. This might help:

$$\frac{\partial \text{atan2}(y,x)}{\partial x} = \frac{-y}{x^2+y^2} \quad , \quad \frac{\partial \text{atan2}(y,x)}{\partial y} = \frac{x}{x^2+y^2}$$

2. Derive the inverses $\bar{h}$ and $\bar{H}$ and compute μ_0 and Σ_0 using z_0 and $\vec{x}_0$.

3. Calculate μ_t and Σ_t for $t = 1, \ldots, 3$. Draw all poses, measurements and landmark position estimates into a diagram (axes from 0 to 4), or plot them using octave/gnuplot.

4. Visualize the uncertainties with ellipses, either as a rough sketch in the diagram or, if you use Octave, by calling drawCov(μ, Σ). The drawCov function is provided on the web page. Just as an explanation: the eigenvalues of a covariance matrix are proportional to the length of the semi-axes of the ellipse, and the eigenvectors denote the direction of the semi-axes. If the eigenvalues are scaled with $\chi^2_{2,0.05} = 5.991464$, the true value is in the ellipse with a probability of 95%.

t	x	y	θ	d	ϕ
0	0	0	$0°$	2	$44°$
1	1	0.1	$10°$	4	$48°$
2	2	0	$-10°$	1	$102°$
3	3	-0.1	$-5°$	0.5	$114°$

Exercise 3:

No calculations for this exercise, only descriptions!

1. Suppose there are multiple landmarks that are not unique but indistinguishable. What extra steps have to be performed? Sketch an approach to this problem.

2. Suppose the true pose of the robot is not known, but estimated using an EKF. How can you include the pose uncertainties into the mapping process?