Albert-Ludwigs-Universität Freiburg — Institut für Informatik
Lecture: Introduction to Mobile Robotics
Summer term 2009 — Prof. Dr. W. Burgard

Sheet 10

Topic: EKF SLAM, FastSLAM, and Data Association

Submission deadline: Tuesday 7.7.2009 (before class)

Exercise 1:

In the lecture the state prediction step of the EKF-framework was presented using the *velocity motion model* as follows:

$$\begin{aligned} \bar{\mu}_t &= g(\mu_{t-1}, u_t) \\ \bar{\Sigma}_t &= G_t \Sigma_{t-1} G_t^T + V_t M_t V_t^T, \end{aligned}$$

where G_t is the Jacobian of the state transition function g and M_t is the covariance matrix of the noise in *control space*. The transformation from the noise in *control space* to *state space* is performed by linear approximation using the Jacobian V_t given by the derivative of the state transition function g with respect to the control parameters (see slide 13 of the Extended Kalman Filter slides).

In this exercise you'll have to specify the following state prediction components of the EKF-framework for the *odometry motion model*.

1. State transition function $\bar{\mu}_t = g(\mu_{t-1}, u_t)$ and its Jacobian G_t.
2. Given the covariance matrix M_t of the noise in *control space*

$$M_t = \begin{pmatrix} \alpha_1|\delta_{\text{rot}_1}| + \alpha_2\delta_{\text{trans}} & 0 & 0 \\ 0 & \alpha_3\delta_{\text{trans}} + \alpha_4(|\delta_{\text{rot}_1}| + |\delta_{\text{rot}_2}|) & 0 \\ 0 & 0 & \alpha_1|\delta_{\text{rot}_2}| + \alpha_2\delta_{\text{trans}} \end{pmatrix},$$

   specify the Jacobian V_t needed by the linear approximation that transforms the noise from *control space* to *state space* (remember that $\alpha_1, \ldots, \alpha_4$ are constant parameters specific to the robot).

Exercise 2:

Each particle in the FastSLAM algorithm is assigned an importance weight based on the likelihood of the current observation z_t and the corresponding map $m = \{l_1, \ldots, l_N\}$ and pose x_t of the particle. Write down the likelihood function for a single measurement $z = \langle d, \varphi \rangle$ with known correspondence to a landmark $l = \langle x_l, y_l \rangle$ and pose $x = \langle x, y, \theta \rangle$.

Exercise 3:

Features extracted from an observation can be interpreted as matches with features in the map, new previously unobserved features, or false alarms (noise).

Consider two features z_t^1 and z_t^2 extracted from an observation z_t, and a map $m_t = \{l_1, l_2\}$ with two features. An assignment ψ associates each observed feature z_i to a map feature l_j, or marks it as a false alarm or as a new feature.

1. Write down all possible assignments for the two observed features z_t^1 and z_t^2, and the two map features l_1 and l_2. Note that in an assignment an observed feature can be associated to one map feature at the most.
2. Suppose now that for a given assignment, every observed feature marked as a new feature is added to the map, and every map feature without an matching observed feature is removed from the map. How many new assignments are generated from the set of assignments computed in the previous exercise if at time $t + 1$ a single feature z_{t+1}^1 is extracted?