

## Sheet 13

Topic: Exploration, SLAM

Submission deadline: Tuesday 28.7.2009

### Exercise 1: Pursuit Evasion Problem

Suppose a certain number of robots are chasing a moving intruder through a known, bounded environment. The robots have omni-vision and can detect the intruder at any distance if the intruder is in the line-of-sight. Can you draw an environment where  $k$  robots can succeed in finding the intruder in finite time, but  $k - 1$  robots cannot? Draw such an environment for  $k = 2$ ,  $k = 3$ , and  $k = 4$  robots. Describe the successful search strategy for  $k$  robots and explain why  $k - 1$  robots could not accomplish the task.

### Exercise 2: Entropy

1. Compute the entropy  $H(p)$  in bits (therefore use  $\log_2$ ) of the following discrete distribution  $p$ :

$$\frac{p(x_1) \quad p(x_2) \quad p(x_3) \quad p(x_4)}{0.04 \quad 0.06 \quad 0.2 \quad 0.7}$$

2. Prove that the entropy of a grid map cell  $m_{x,y}$  is maximal for  $p(m_{x,y}) = 0.5$ .
3. Consider a discrete uniform distribution of a random variable with  $n$  possible outcomes. Prove that the entropy of the distribution decreases if you change the distribution by increasing the probability of a single event and accordingly reducing the probability of another event.

### Exercise 3: Factoring the SLAM posterior

The full SLAM posterior can be written in the factored form:

$$p(x_{1:t}, m | z_{1:t}, u_{0:t-1}) = p(x_{1:t} | z_{1:t}, u_{0:t-1}) \prod_{n=1}^N p(m_n | x_{1:t}, z_{1:t}) \quad (1)$$

In the second factor of the factorization, the landmarks are supposed to be independent given the complete trajectory  $x_{1:t}$  and the observations  $z_{1:t}$ . Is it possible to condition the map on the most recent pose  $x_t$  only? That is:

$$p(x_{1:t}, m|z_{1:t}, u_{0:t-1}) = p(x_{1:t}|z_{1:t}, u_{0:t-1}) \prod_{n=1}^N p(m_n|x_t, z_{1:t}) \quad (2)$$

Explain your answer.