Introduction to Mobile Robotics

EKF Localization

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Localization

“Using sensory information to locate the robot in its environment is the most fundamental problem to providing a mobile robot with autonomous capabilities.” [Cox ’91]

- **Given**
  - Map of the environment.
  - Sequence of sensor measurements.

- **Wanted**
  - Estimate of the robot’s position.

- **Problem classes**
  - Position tracking
  - Global localization
  - Kidnapped robot problem (recovery)
Landmark-based Localization

**EKF Localization:** Basic Cycle
Landmark-based Localization

EKF Localization: Basic Cycle
Landmark-based Localization

**EKF Localization: Basic Cycle**

- **Odometry or IMU**: encoder measurements
- **State Prediction**: predicted state
- **Measurement Prediction**: predicted measurements in sensor coordinates
- **Data Association**: innovation from matched landmarks
- **Update**: posterior state
- **Map**: landmarks in global coordinates
- **Feature/Landmark Extraction**: raw sensory data
- **Sensors**: landmarks

**Flowchart Diagram**
Landmark-based Localization

State Prediction (Odometry)

\[
\hat{x}_k = f(x_{k-1}, u_k) \\
\hat{C}_k = F_x C_k F_x^T + F_u U_k F_u^T
\]

Control \( u_k \): wheel displacements \( s_l, s_r \)

\[
u_k = (s_l \ s_r)^T \quad \quad U_k = \begin{bmatrix} \sigma_l^2 & 0 \\ 0 & \sigma_r^2 \end{bmatrix}
\]

Error model: linear growth

\[
\sigma_l = k_l |s_l| \\
\sigma_r = k_r |s_r|
\]

Nonlinear process model \( f \):

\[
x_k = \begin{bmatrix} x_{k-1} \\ y_{k-1} \\ \theta_{k-1} \end{bmatrix} + \begin{bmatrix} \frac{b}{2} \frac{s_l + s_r}{s_r - s_l} \left( -\sin \theta_{k-1} + \sin(\theta_{k-1} + \frac{s_r - s_l}{b}) \right) \\ \frac{b}{2} \frac{s_l + s_r}{s_r - s_l} \left( \cos \theta_{k-1} - \cos(\theta_{k-1} + \frac{s_r - s_l}{b}) \right) \\ \frac{b}{2} \frac{s_r - s_l}{s_r - s_l} \end{bmatrix}
\]
Landmark-based Localization

**State Prediction (Odometry)**

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**Nonlinear** process model \( f \):

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x_k = \begin{bmatrix} x_{k-1} \\ y_{k-1} \\ \theta_{k-1} \end{bmatrix} + \begin{bmatrix} \frac{b}{2} & \frac{b}{2} & \frac{b}{2} & \frac{b}{2} \\ \frac{b}{2} & \frac{b}{2} & \frac{b}{2} & \frac{b}{2} \end{bmatrix} \begin{bmatrix} -\sin \theta_{k-1} + \sin(\theta_{k-1} + \frac{s_r - s_l}{b}) \\ \cos \theta_{k-1} - \cos(\theta_{k-1} + \frac{s_r - s_l}{b}) \end{bmatrix}
\]
Landmark-based Localization

Landmark Extraction (Observation)

Raw laser range data

Extracted lines

Extracted lines in model space

Hessian line model

\[ x \cos(\alpha) + y \sin(\alpha) - r = 0 \]
Landmark-based Localization

Measurement Prediction

- ...is a coordinate frame transform world-to-sensor
- Given the predicted state (robot pose), predicts the location $\hat{z}_k$ and location uncertainty $H \hat{C}_k H^T$ of expected observations in sensor coordinates

$$\hat{z}_k = h(\hat{x}_k, m)$$
Data Association (Matching)

- Associates predicted measurements $\hat{z}^i_k$ with observations $z^j_k$
  
  $\nu^{ij}_k = z^j_k - \hat{z}^i_k$
  $S^{ij}_k = R^j_k + H^i_k \hat{C}_k H^i_k T$

- Innovation $\nu^{ij}_k$ and innovation covariance $S^{ij}_k$

- Matching on significance level alpha

Green: observation
Magenta: measurement prediction
Landmark-based Localization

Update

- Kalman gain
  \[ K_k = \hat{C}_k H^T S_k^{-1} \]

- State update (robot pose)
  \[ x_k = \hat{x}_k + K_k \nu_k \]

- State covariance update
  \[ C_k = (I - K_k H) \hat{C}_k \]

Red: posterior estimate
Landmark-based Localization

• EKF Localization with Point Features
1. **EKF_localization** $(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, m)$:

**Prediction:**

2. $G_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}} = \begin{pmatrix}
\frac{\partial x'}{\partial \mu_{t-1,x}} & \frac{\partial x'}{\partial \mu_{t-1,y}} & \frac{\partial x'}{\partial \mu_{t-1,\theta}} \\
\frac{\partial y'}{\partial \mu_{t-1,x}} & \frac{\partial y'}{\partial \mu_{t-1,y}} & \frac{\partial y'}{\partial \mu_{t-1,\theta}} \\
\frac{\partial \theta'}{\partial \mu_{t-1,x}} & \frac{\partial \theta'}{\partial \mu_{t-1,y}} & \frac{\partial \theta'}{\partial \mu_{t-1,\theta}}
\end{pmatrix}$ Jacobian of $g$ w.r.t location

3. $V_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial u_t} = \begin{pmatrix}
\frac{\partial x'}{\partial v_t} & \frac{\partial x'}{\partial \omega_t} \\
\frac{\partial y'}{\partial v_t} & \frac{\partial y'}{\partial \omega_t} \\
\frac{\partial \theta'}{\partial v_t} & \frac{\partial \theta'}{\partial \omega_t}
\end{pmatrix}$ Jacobian of $g$ w.r.t control

4. $M_t = \begin{pmatrix}
(\alpha_1 | v_t | + \alpha_2 | \omega_t |)^2 & 0 \\
0 & (\alpha_3 | v_t | + \alpha_4 | \omega_t |)^2
\end{pmatrix}$ Motion noise

5. $\bar{\mu}_t = g(u_t, \mu_{t-1})$ Predicted mean

6. $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + V_t M_t V_t^T$ Predicted covariance
1. **EKF_localization** $(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, m)$:

**Correction:**

2. $\hat{z}_t = \left( \sqrt{(m_x - \mu_{t,x})^2 + (m_y - \mu_{t,y})^2} \right.$

\[= \arctan \left( \frac{m_y - \mu_{t,y}}{m_x - \mu_{t,x} - \mu_{t,\theta}} \right) \]  

**Predicted measurement mean**

3. $H_t = \frac{\partial h(\mu, m)}{\partial x_i} = \begin{pmatrix} \frac{\partial r_i}{\partial \mu_{t,x}} & \frac{\partial r_i}{\partial \mu_{t,y}} & \frac{\partial r_i}{\partial \mu_{t,\theta}} \\ \frac{\partial \phi_i}{\partial \mu_{t,x}} & \frac{\partial \phi_i}{\partial \mu_{t,y}} & \frac{\partial \phi_i}{\partial \mu_{t,\theta}} \end{pmatrix}$  

**Jacobian of $h$ w.r.t location**

4. $Q_t = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_r^2 \end{pmatrix}$

5. $S_t = H_t \bar{\Sigma}_t H_t^T + Q_t$  

**Innovation covariance**

6. $K_t = \bar{\Sigma}_t H_t^T S_t^{-1}$  

**Kalman gain**

7. $\mu_t = \mu_t + K_t (z_t - \hat{z}_t)$  

**Updated mean**

8. $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$  

**Updated covariance**
EKF Prediction Step
EKF Observation Prediction Step

- Diagrams illustrating the prediction step in Extended Kalman Filter (EKF) for tracking and prediction.
EKF Correction Step
Estimation Sequence (1)
Estimation Sequence (2)
Comparison to GroundTruth
EKF Summary

- **Highly efficient**: Polynomial in measurement dimensionality $k$ and state dimensionality $n$:
  $$O(k^{2.376} + n^2)$$

- **Not optimal**!
- **Can diverge** if nonlinearities are large!
- **Works very well** even when all assumptions are violated!
EKF Localization Example

- [Arras et al. 98]:
  - Laser range-finder and vision
  - High precision (<1cm accuracy)

Courtesy of K. Arras
EKF Localization Example

• Line and point landmarks
EKF Localization Example

- Line and point landmarks
EKF Localization Example

• **Expo.02:** Swiss National Exhibition 2002
• Pavilion "Robotics"
• 11 fully autonomous robots
• tour guides, entertainer, photographer
• 12 hours per day
• 7 days per week
• 5 months

• **3,316** km travel distance
• almost **700,000** visitors
• 400 visitors per hour

• Localization method: **Line-Based, EKF**

• Still the *biggest project in mobile robotics* of its kind!
EKF Localization Example

“Robotics”

Expo.02 Switzerland

May 15th - October 20th, 2002

Autonomous Systems Lab

EPFL

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