# Basic Probability Rules

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## 1 Basic Axioms

$$0 \leq p(x) \leq 1 \tag{1}$$

$$p(\mathit{true}) = 1 \ \text{ and } \ p(\mathit{false}) = 0 \tag{2}$$

$$p(x \vee y) \ = \ p(x) + p(y) - p(x \wedge y) \tag{3}$$

## 2 Negatived Probability

$$p(x) \ = \ 1 - p(\neg x). \tag{4}$$

## 3 Product Rule

The following equation is called the product rule

$$\begin{aligned} p(x, y) &= p(x \mid y) \cdot p(y) && (5) \\ &= p(y \mid x) \cdot p(x). && (6) \end{aligned}$$

## 4 Independence

If $x$ and $y$ are independent, we have

$$p(x, y) \ = \ p(x) \cdot p(y). \tag{7}$$

## 5 Bayes' Rule

The Bayes' rule, which is frequently used in this thesis, is given by

$$p(x \mid y) \ = \ \frac{p(y \mid x) \cdot p(x)}{p(y)}. \tag{8}$$

The denominator is a normalizing constant that ensures that the posterior of the left hand side adds up to 1 over all possible values. Thus, we often write

$$p(x \mid y) \ = \ \eta \cdot p(y \mid x) \cdot p(x). \tag{9}$$

In case the background knowledge $e$ is given, Bayes' rule turns into

$$p(x \mid y, e) \ = \ \frac{p(y \mid x, e) \cdot p(x \mid e)}{p(y \mid e)}. \tag{10}$$

## 6 Marginalization

The marginalization rule is the following equation

$$p(x) \quad = \quad \int_y p(x,y)\, dy. \tag{11}$$

In the discrete case, the integral turns into a sum

$$p(x) \quad = \quad \sum_y p(x,y). \tag{12}$$

## 7 Law of Total Probability

The law of total probability is a variant of the marginalization rule, which can be derived using the product rule

$$p(x) \quad = \quad \int_y p(x \mid y) \cdot p(y)\, dy, \tag{13}$$

and the corresponding sum for the discrete case

$$p(x) \quad = \quad \sum_y p(x \mid y) \cdot p(y). \tag{14}$$

## 8 Markov Assumption

The Markov assumption (also called Markov property) characterizes the fact that a variable $x_t$ depends only on its direct predecessor state $x_{t-1}$ and not on $x_{t'}$ with $t' < t-1$

$$p(x_t \mid x_{1:t-1}) \quad = \quad p(x_t \mid x_{t-1}). \tag{15}$$