Albert-Ludwigs-Universität Freiburg
Lecture: Introduction to Mobile Robotics
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Sheet 2

Topic: Linear Algebra

Submission deadline: May 4, 2010
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Exercise 1: Transformation Matrices

The pose of the robot refers to its position in the $x - y$ plane and its orientation θ, which is commonly written as $\mathbf{x} = (x, y, \theta)^T$.

(a) Express the robot pose $\mathbf{x} = (3, 4, \frac{\pi}{2})^T$ in matrix form relative to the origin $(0, 0, 0)^T$ of the global coordinate system.

(b) Given the two robot poses

$$\mathbf{x}_t = \begin{pmatrix} 2 \\ 2 \\ \frac{\pi}{4} \end{pmatrix}, \quad \mathbf{x}_{t+1} = \begin{pmatrix} 6 \\ 4 \\ 0 \end{pmatrix},$$

find the transformation matrix T that describes the transformation from $\mathbf{x}_t$ to $\mathbf{x}_{t+1}$.

(c) Consider the robotic manipulator depicted in Figure 1. The circles represent rotational joints and the square represents a prismatic joint. Rotational joints rotate around an axis, while prismatic joints extend along an axis. A rotational joint which is rotated by 0 rad points along its x-axis.

The robotic manipulator depicted in Figure 1 has the following properties:

- length of segment s_1 is 0.5 m
- length of segment s_2 is 0.25 m
- segment s_3 is part of the prismatic joint j_3 and its length is therefore variable
- length of segment s_4 is 0.4 m

Suppose that joint j_1 is rotated by 0.2 radians, joint j_2 is rotated by 0.4 radians, the prismatic joint j_3 is extended by 0.3 meters, and joint j_4 is rotated by -0.2 radians.

Use transformation matrices to compute the position of the end effector E with respect to the global coordinate system.

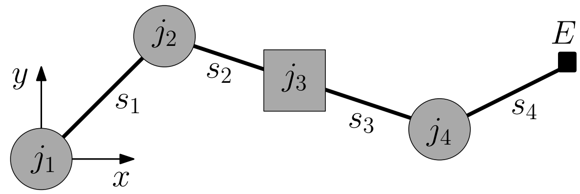

Figure 1: Manipulator with three rotational and one prismatic joint.

Exercise 2: Positive Definite Matrices

(a) Consider the matrices

$$\mathbf{A} = \begin{pmatrix} 0.25 & 0.1 \\ 0.2 & 0.5 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0.25 & -0.3 \\ -0.3 & 0.5 \end{pmatrix}.$$

Are they symmetric positive definite?

(b) For

$$\mathbf{C} = \begin{pmatrix} -3 & 0 \\ 0 & 1 \end{pmatrix},$$

find the smallest value for $\lambda \in \mathbb{R}$ so that $C + \lambda I$ becomes symmetric positive definite.

Exercise 3: Orthogonal Matrices

(a) Write a program in Octave that determines whether a matrix is orthogonal.

(b) Use this program to investigate if

$$\mathbf{D} = \frac{1}{3} \begin{pmatrix} 2 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{pmatrix}$$

is orthogonal.

(c) Construct an arbitrary orthogonal 5x5 matrix which is not the identity matrix.