Exercise 1: Transformation Matrices

The pose of the robot refers to its position in the $x-y$ plane and its orientation $\theta$, which is commonly written as $\mathbf{x} = (x, y, \theta)^T$.

(a) Express the robot pose $\mathbf{x} = (3, 4, \frac{\pi}{2})^T$ in matrix form relative to the origin $(0, 0, 0)^T$ of the global coordinate system.

(b) Given the two robot poses

$$\mathbf{x}_t = \begin{pmatrix} 2 \\ 2 \\ \frac{\pi}{4} \end{pmatrix}, \quad \mathbf{x}_{t+1} = \begin{pmatrix} 6 \\ 4 \\ 0 \end{pmatrix},$$

find the transformation matrix $T$ that describes the transformation from $\mathbf{x}_t$ to $\mathbf{x}_{t+1}$.

(c) Consider the robotic manipulator depicted in Figure 1. The circles represent rotational joints and the square represents a prismatic joint. Rotational joints rotate around an axis, while prismatic joints extend along an axis. A rotational joint which is rotated by 0 rad points along its $x$-axis.

The robotic manipulator depicted in Figure 1 has the following properties:

- length of segment $s_1$ is 0.5 m
- length of segment $s_2$ is 0.25 m
- segment $s_3$ is part of the prismatic joint $j_3$ and its length is therefore variable
- length of segment $s_4$ is 0.4 m

Suppose that joint $j_1$ is rotated by 0.2 radians, joint $j_2$ is rotated by 0.4 radians, the prismatic joint $j_3$ is extended by 0.3 meters, and joint $j_4$ is rotated by $-0.2$ radians.

Use transformation matrices to compute the position of the end effector $E$ with respect to the global coordinate system.
Figure 1: Manipulator with three rotational and one prismatic joint.

Exercise 2: Positive Definite Matrices

(a) Consider the matrices

\[
A = \begin{pmatrix} 0.25 & 0.1 \\ 0.2 & 0.5 \end{pmatrix}, \quad B = \begin{pmatrix} 0.25 & -0.3 \\ -0.3 & 0.5 \end{pmatrix}.
\]

Are they symmetric positive definite?

(b) For

\[
C = \begin{pmatrix} -3 & 0 \\ 0 & 1 \end{pmatrix},
\]

find the smallest value for \( \lambda \in \mathbb{R} \) so that \( C + \lambda I \) becomes symmetric positive definite.

Exercise 3: Orthogonal Matrices

(a) Write a program in Octave that determines whether a matrix is orthogonal.

(b) Use this program to investigate if

\[
D = \frac{1}{3} \begin{pmatrix} 2 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{pmatrix}
\]

is orthogonal.

(c) Construct an arbitrary orthogonal 5x5 matrix which is not the identity matrix.