Exercise 1: Distance-Only Sensor

In this exercise, you try to locate your friend using her cell phone signals. Suppose that in the map of Freiburg, the campus of the University of Freiburg is located at $m_0 = (10, 8)^T$, and your friend’s home is situated at $m_1 = (6, 3)^T$. You have access to the data received by two cell towers, which are located at the positions $x_0 = (12, 4)^T$ and $x_1 = (5, 7)^T$, respectively. The distance between your friend’s cell phone and the towers can be computed from the intensities of your friend’s cell phone signals. These distance measurements are disturbed by zero-mean Gaussian noise with variances $\sigma^2_0 = 1$ for tower 0 and $\sigma^2_1 = 1.5$ for tower 1. You recieve the distance measurements $d_0 = 3.9$ and $d_1 = 4.5$ from the two towers.

(a) At which of the two places is your friend more likely to be? Explain your calculations.

(b) Implement a function in Octave which generates a 3D-plot of the likelihood function which you used in a). Furthermore, mark $m_0$, $m_1$, $x_0$ and $x_1$ in the plot. Is the likelihood function which you plotted a probability density function? Give a reason for your answer.

(c) Now, suppose you have prior knowledge about your friend’s habits which suggests that your friend currently is at home with probability $P(\text{at home}) = 0.7$, at the university with $P(\text{at university}) = 0.3$, and at any other place with $P(\text{other}) = 0$. Use this prior knowledge to recalculate the likelihoods of a).

Exercise 2: Sensor Model

Remark: This exercise is to be solved without Octave.

Assume you have a robot equipped with a sensor capable of measuring the distance and bearing to landmarks. The sensor furthermore provides you with the identity of the observed landmarks.
A sensor measurement $z = (z_r, z_\theta)^T$ is composed of the measured distance $z_r$ and the measured bearing $z_\theta$ to the landmark $l$. Both the range and the bearing measurements are subject to zero-mean Gaussian noise with variances $\sigma_r^2$, and $\sigma_\theta^2$, respectively. The range and the bearing measurements are independent of each other. A sensor model
\[
p(z \mid x, l)
\]
models the probability of a measurement $z$ of landmark $l$ observed by the robot from pose $x$.

Design a sensor model $p(z \mid x, l)$ for this type of sensor. Furthermore, explain your sensor model.

**Exercise 3: Sensor Model Implementation**

Write an Octave function which implements the sensor model you specified in Exercise 2. Instead of computing a probability it is sufficient to compute a likelihood. The function should have the following signature
\[
\text{function likelihood = landmark_sensor_model(z, x, l)}
\]
where $z$ is the measured range and bearing to the observed landmark, $x$ is the current pose of the robot, and $l$ is the actual position of the observed landmark.

Evaluate your sensor model using the following data:

\[
x = \begin{pmatrix} 2 \\ 3 \\ \frac{\pi}{4} \end{pmatrix} \quad l = \begin{pmatrix} 2 \\ 8 \end{pmatrix} \quad \sigma_r^2 = 0.25 \quad \sigma_\theta^2 = 0.01
\]

\[
z_0 = \begin{pmatrix} 5.0 \\ \frac{\pi}{4} \end{pmatrix} \quad z_1 = \begin{pmatrix} 5.0 \\ 0.6 \end{pmatrix} \quad z_2 = \begin{pmatrix} 4.5 \\ \frac{\pi}{4} \end{pmatrix} \quad z_3 = \begin{pmatrix} 5.5 \\ 0.9 \end{pmatrix}
\]