Albert-Ludwigs-Universität Freiburg
Institut für Informatik
Lecture: Introduction to Mobile Robotics
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Sheet 9

Topic: Line Fitting

Submission deadline: June 29, 2010

Submit to: `mobilerobotics@informatik.uni-freiburg.de`

General Notice

For the programming exercises on this sheet, you can find stub implementations on the website of the course.

Exercise 1: Split and Merge

In this exercise, you will implement the split and merge algorithm for line extraction. Once you have completed the stubs, you can generate plots by executing `test_split` in *Octave*. You can test your implementation on two datasets. To change the dataset, modify `test_split.m` accordingly.

(a) Complete the first stub in `split_and_merge.m` for the simple threshold based split and merge. As threshold use 0.5.

(b) Implement the second stub in `split_and_merge.m` using residual analysis as splitting criterion (see lecture 14, slide 10).

Exercise 2: Least Squares Fitting

Now fit a line to a point cloud using least squares regression. Use the line representation

$$y_i = ax_i + b.$$

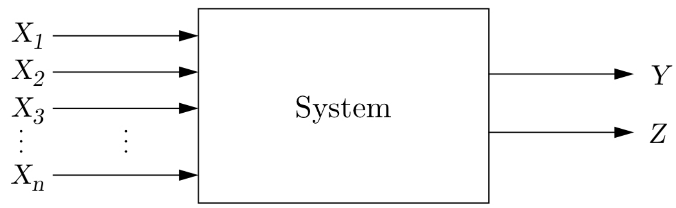
Once you have implemented the stub, generate plots by executing `test_lsq` in *Octave*.

(a) Derive the error sum $S(a, b)$ and its partial derivatives $\frac{\partial S}{\partial a}$ and $\frac{\partial S}{\partial b}$.

(b) Complete the stub in `least_squares_line_fitting.m` using your partial derivatives $\frac{\partial S}{\partial a}$ and $\frac{\partial S}{\partial b}$.

Exercise 3: First-Order Error Propagation

Suppose the general case of a non-linear multi-input multi-output system with n correlated one dimensional input random variables $X_1, ..., X_n$ with $X_i \sim \mathcal{N}(\mu_{X_i}, \sigma^2_{X_i})$ and (without loss of generality) two output random variables Y and Z.



We set $Y = f(X_1, ..., X_n)$ and $Z = g(X_1, ..., X_n)$ and approximate the functions $f(.)$ and $g(.)$ by a first-order Taylor series expansion:

$$Y \approx f(\mu_1, ..., \mu_n) + \sum_{i=1}^{n} \left.\frac{\partial f}{\partial X_i}\right|_{\mu_i,...,\mu_n} (X_i - \mu_i) \quad (1)$$

$$Z \approx g(\mu_1, ..., \mu_n) + \sum_{i=1}^{n} \left.\frac{\partial g}{\partial X_i}\right|_{\mu_i,...,\mu_n} (X_i - \mu_i) \quad (2)$$

Derive the expression for the covariance σ_{YZ} between Y and Z given the rules for the expected value

$$E[a] = a \quad (3)$$

$$E[aX] = aE[X] \quad (4)$$

$$E[X+Y] = E[X] + E[Y] \quad (5)$$

$$E[XY] = E[X]E[Y] \qquad \text{if X and Y are independent} \quad (6)$$

and the following definitions for mean, variance and covariance:

$$\mu_X = E[X] \quad (7)$$

$$\sigma^2_X = E[(X - E[X])^2] \quad (8)$$

$$\sigma_{XY} = E[(X - E[X])(Y - E[Y])] \quad (9)$$

(a) Derive the expression for the covariance σ_{YZ} between Y and Z.

(b) Simplify this expression assuming stochastic independence of $X_1, ..., X_n$.