Introduction to Mobile Robotics

Bayes Filter – Particle Filter and Monte Carlo Localization

Wolfram Burgard, Cyrill Stachniss, Maren Bennewitz, Giorgio Grisetti, Kai Arras

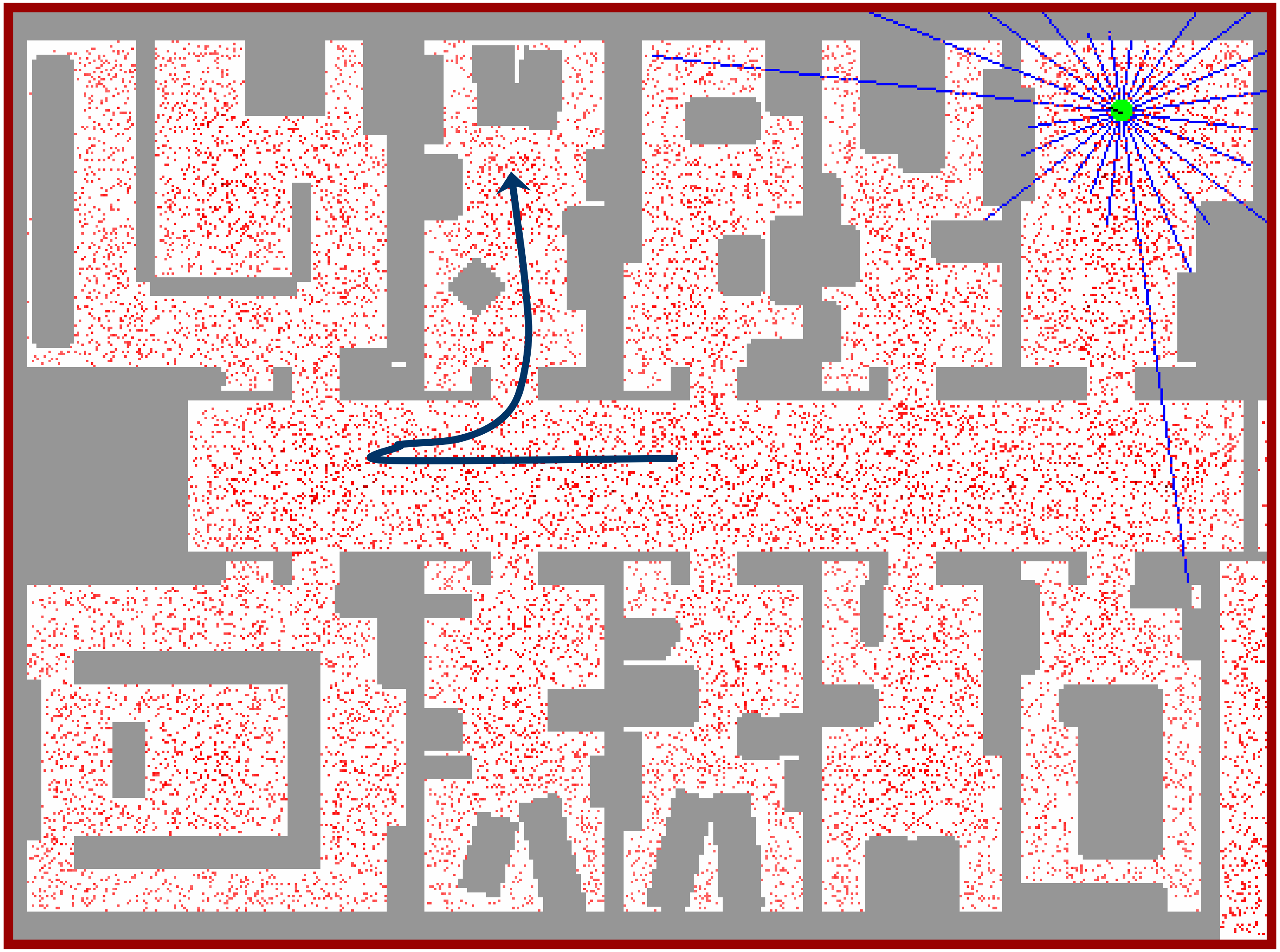
Motivation

- Recall: Discrete filter
  - Discretize the continuous state space
  - High memory complexity
  - Fixed resolution (does not adapt to the belief)

- Particle filters are a way to **efficiently** represent **non-Gaussian distribution**

- Basic principle
  - Set of state hypotheses (“particles”)
  - Survival-of-the-fittest

Sample-based Localization (sonar)



Mathematical Description

- Set of weighted samples

$$S = \left\{ \left\langle s^{[i]}, w^{[i]} \right\rangle \mid i = 1, \ldots, N \right\}$$

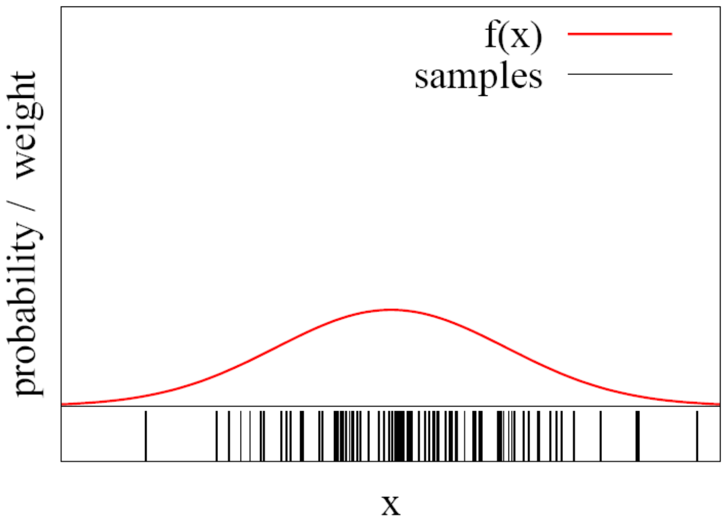
State hypothesis   Importance weight

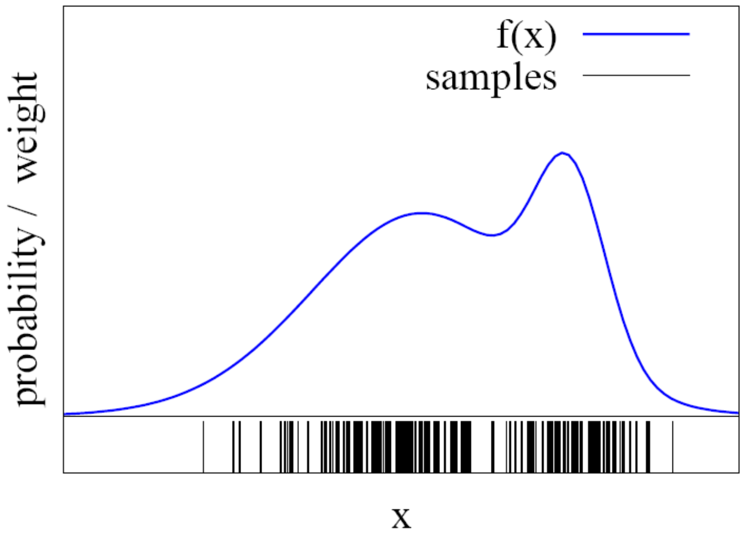
- The samples represent the posterior

$$p(x) = \sum_{i=1}^{N} w_i \cdot \delta_{s^{[i]}}(x)$$

Function Approximation

- Particle sets can be used to approximate functions



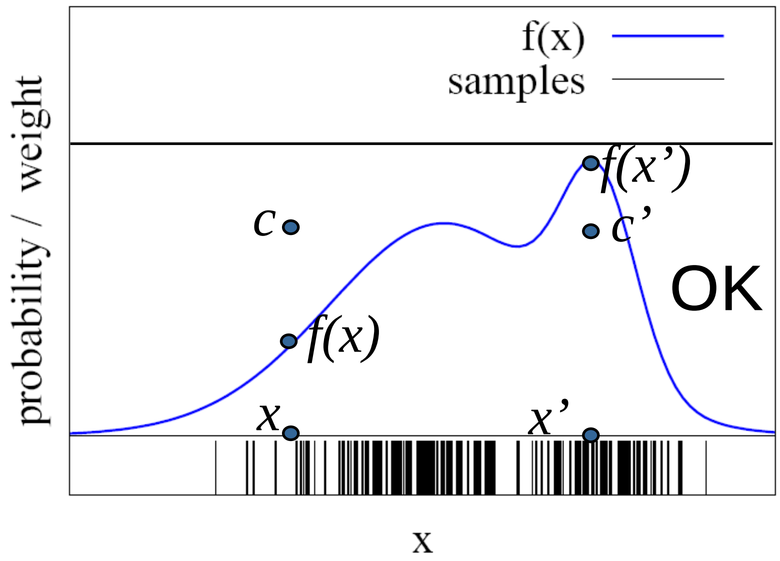


- The more particles fall into an interval, the higher the probability of that interval

- How to draw samples form a function/distribution?

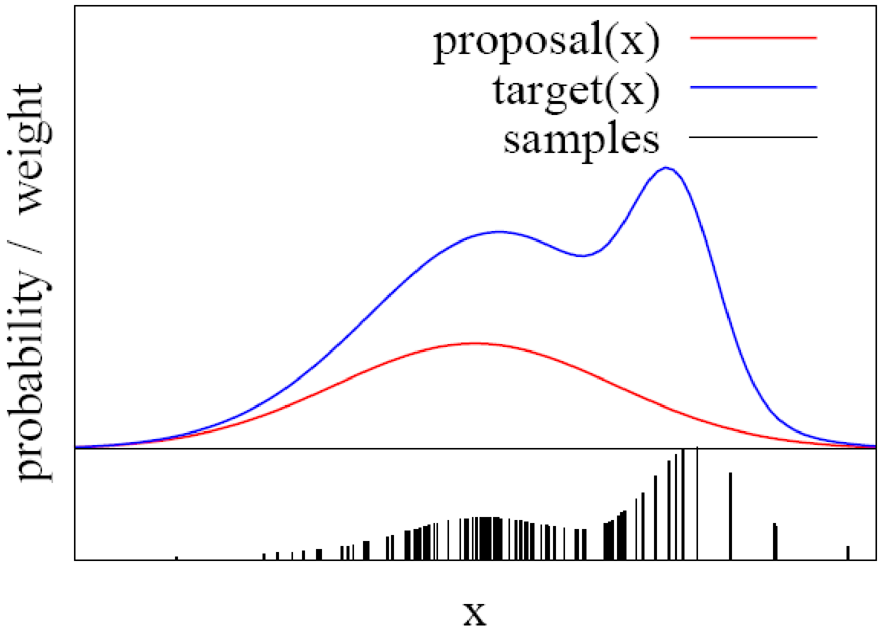
Rejection Sampling

- Let us assume that $f(x)<1$ for all x
- Sample x from a uniform distribution
- Sample c from [0,1]
- if $f(x) > c$ keep the sample
  otherwise reject the sampe



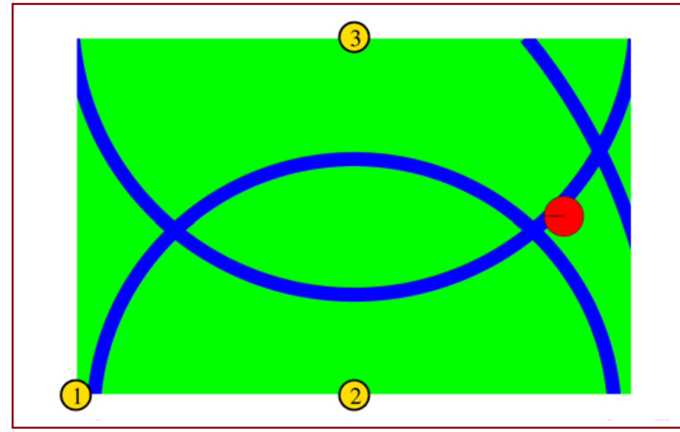
Importance Sampling Principle

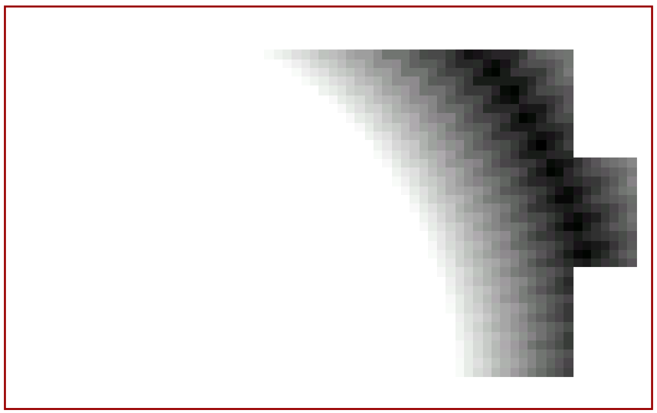
- We can even use a different distribution g to generate samples from f
- By introducing an importance weight w, we can account for the "differences between g and f"
- $w = f / g$
- f is often called target
- g is often called proposal
- Pre-condition: $f(x)>0 \rightarrow g(x)>0$

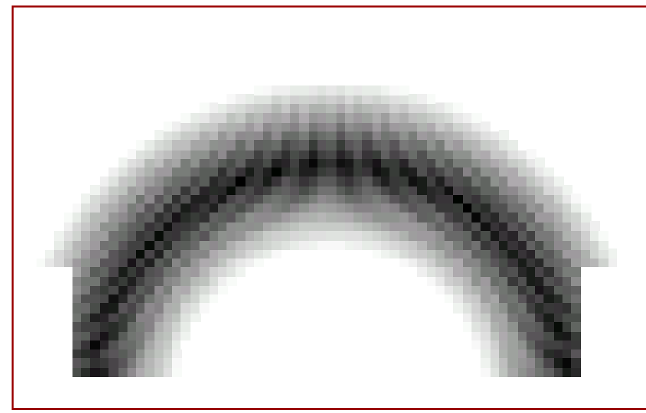


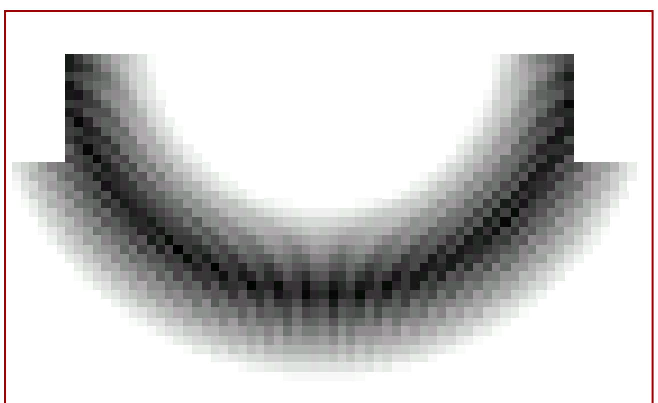
Importance Sampling with Resampling: Landmark Detection Example

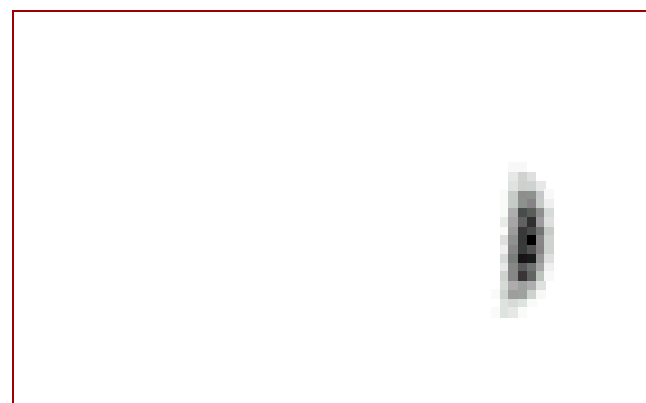
Distributions



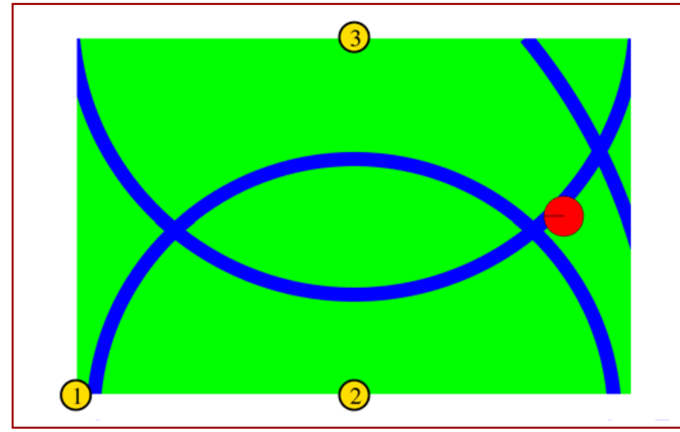


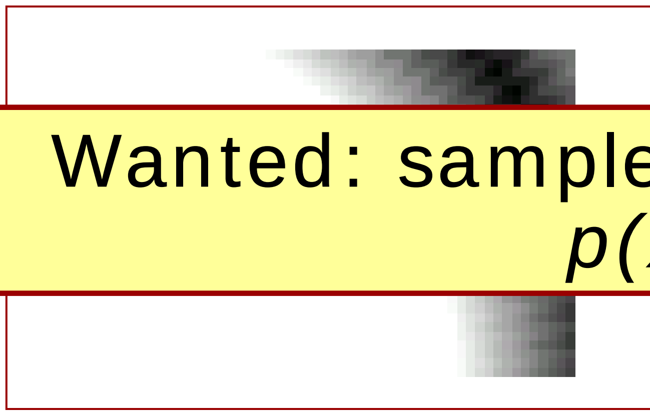


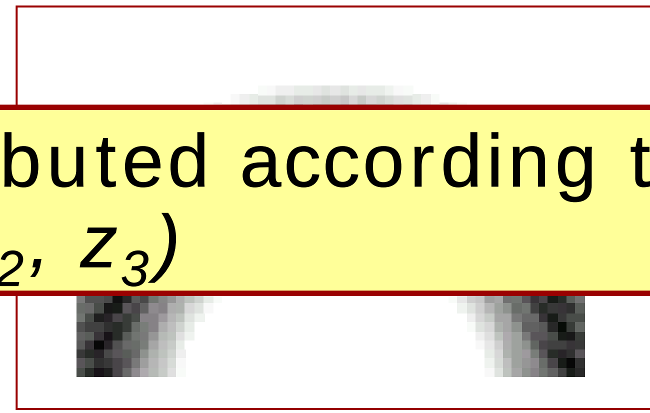




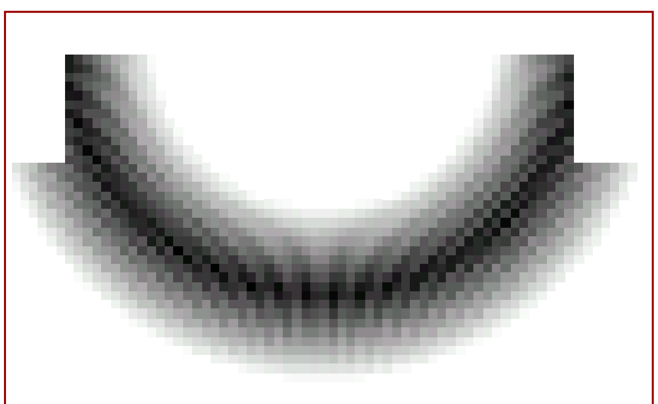
Distributions

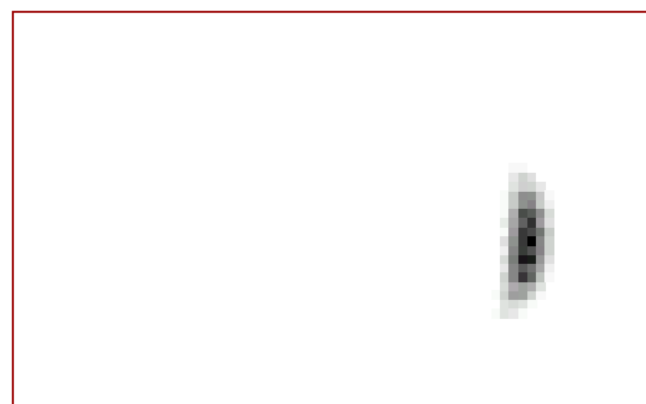






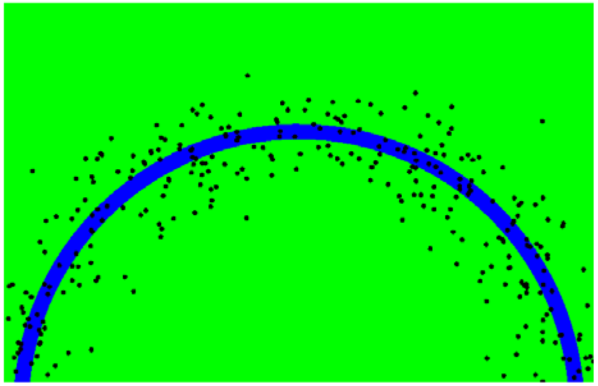
Wanted: samples distributed according to $p(x|\ z_1, z_2, z_3)$

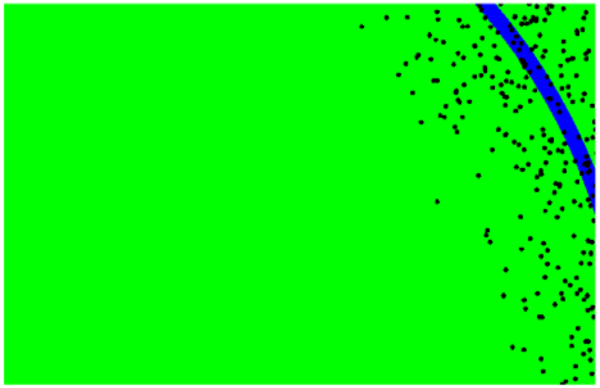


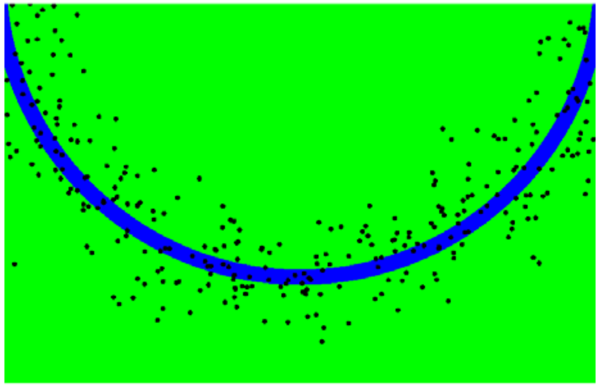


This is Easy!

We can draw samples from $p(x|z_l)$ by adding noise to the detection parameters.







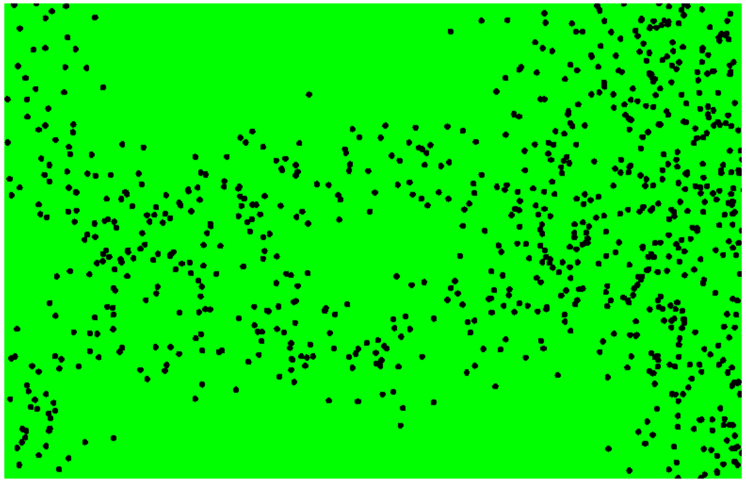
Importance Sampling

$$\text{Target distribution f}: p(x \mid z_1, z_2, \ldots, z_n) = \frac{\prod_k p(z_k \mid x) \quad p(x)}{p(z_1, z_2, \ldots, z_n)}$$

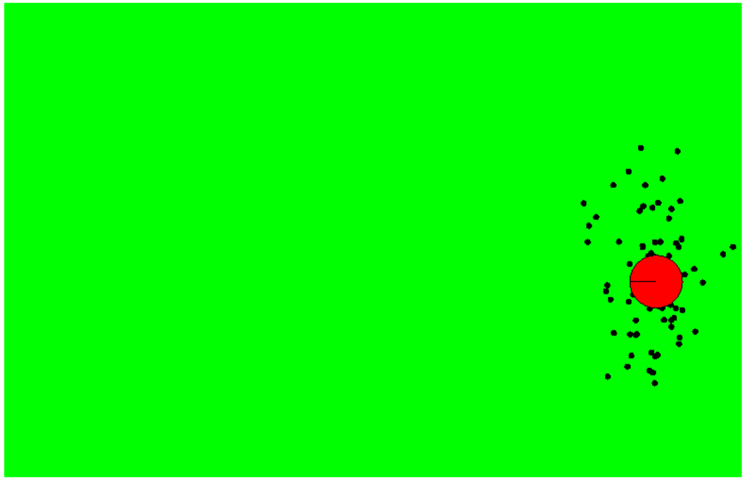
$$\text{Sampling distribution g}: p(x \mid z_l) = \frac{p(z_l \mid x) p(x)}{p(z_l)}$$

$$\text{Importance weights w}: \frac{f}{g} = \frac{p(x \mid z_1, z_2, \ldots, z_n)}{p(x \mid z_l)} = \frac{p(z_l) \prod_{k \neq l} p(z_k \mid x)}{p(z_1, z_2, \ldots, z_n)}$$

Importance Sampling with Resampling

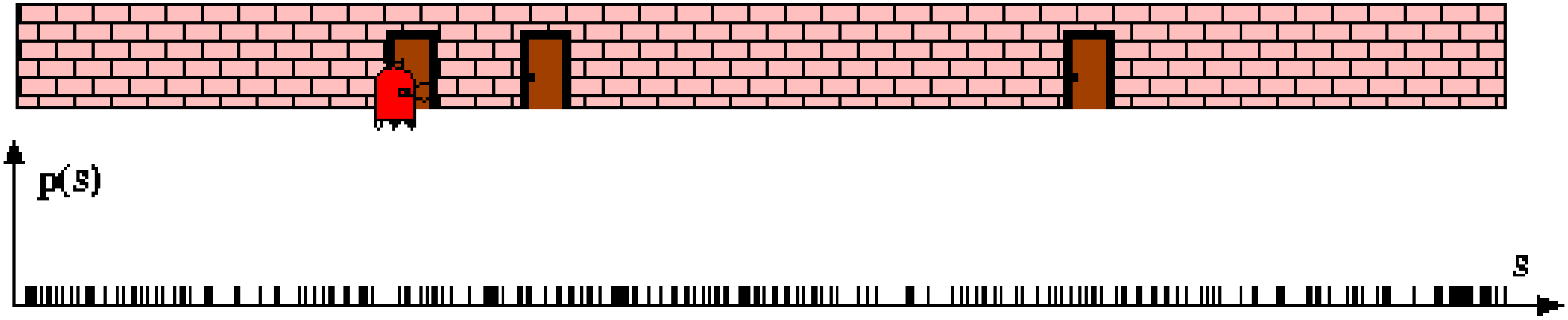


Weighted samples



After resampling

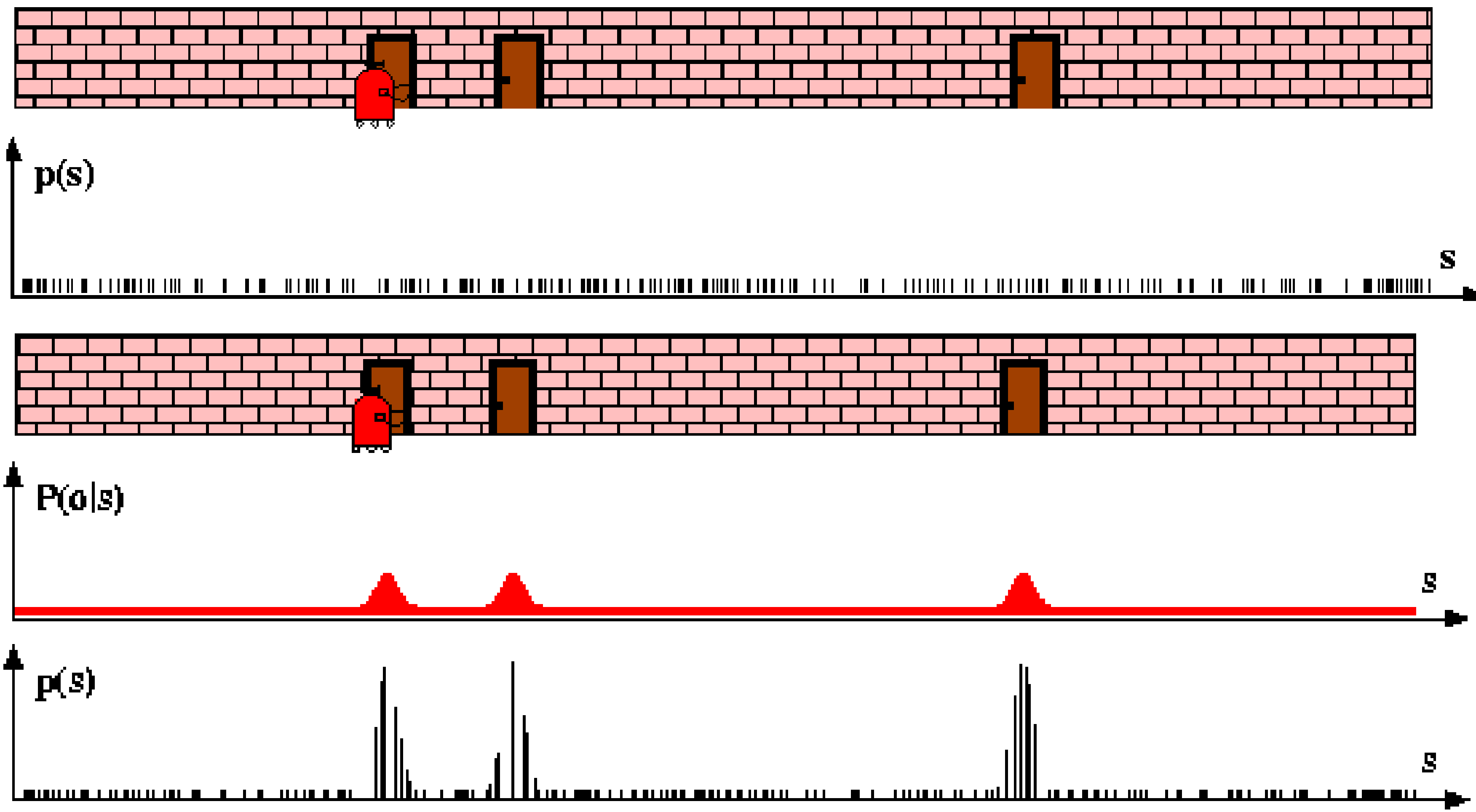
Particle Filters



Sensor Information: Importance Sampling

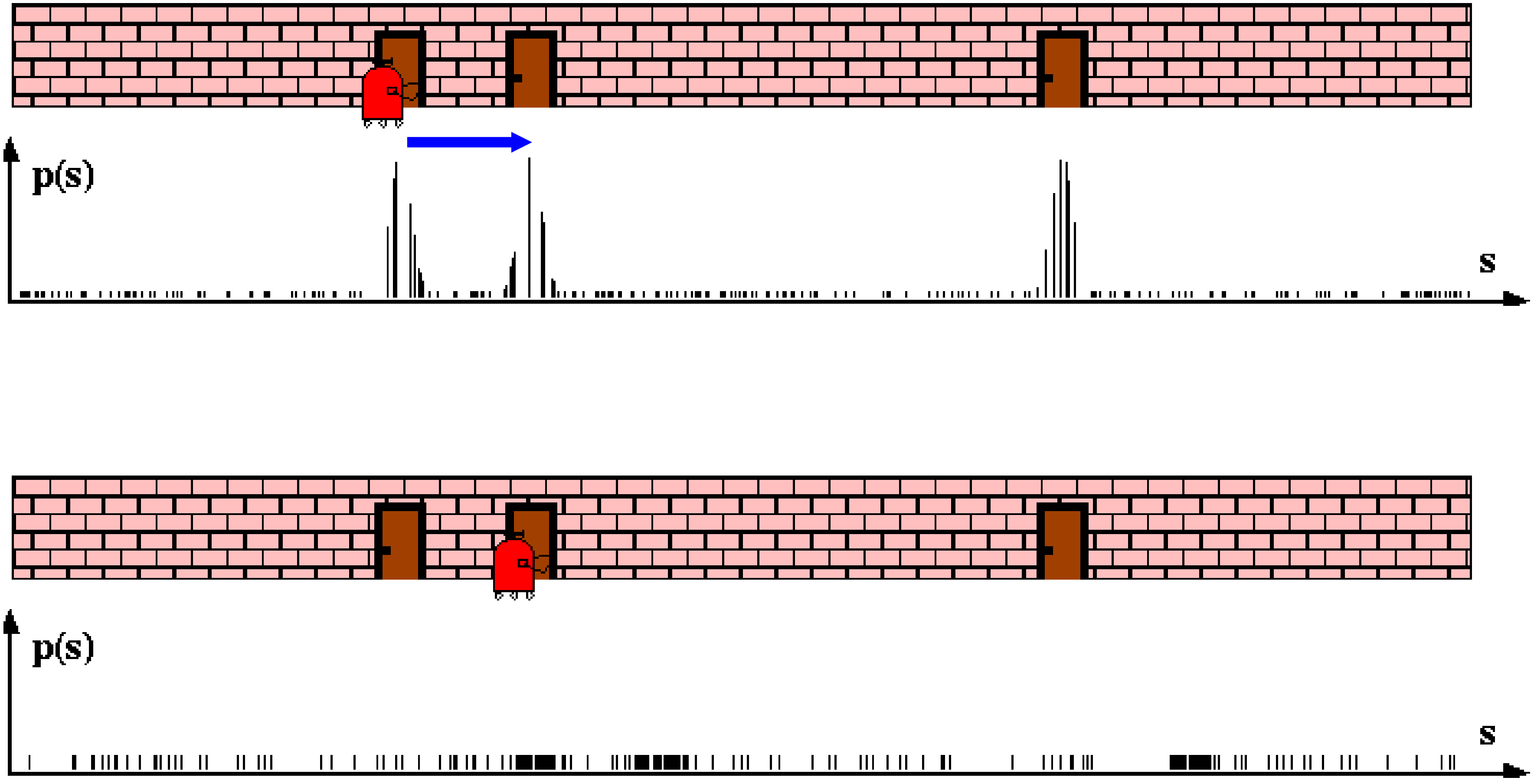
$$Bel(x) \leftarrow \alpha\; p(z \mid x)\, Bel^{-}(x)$$

$$w \leftarrow \frac{\alpha\; p(z \mid x)\, Bel^{-}(x)}{Bel^{-}(x)} = \alpha\; p(z \mid x)$$



Robot Motion

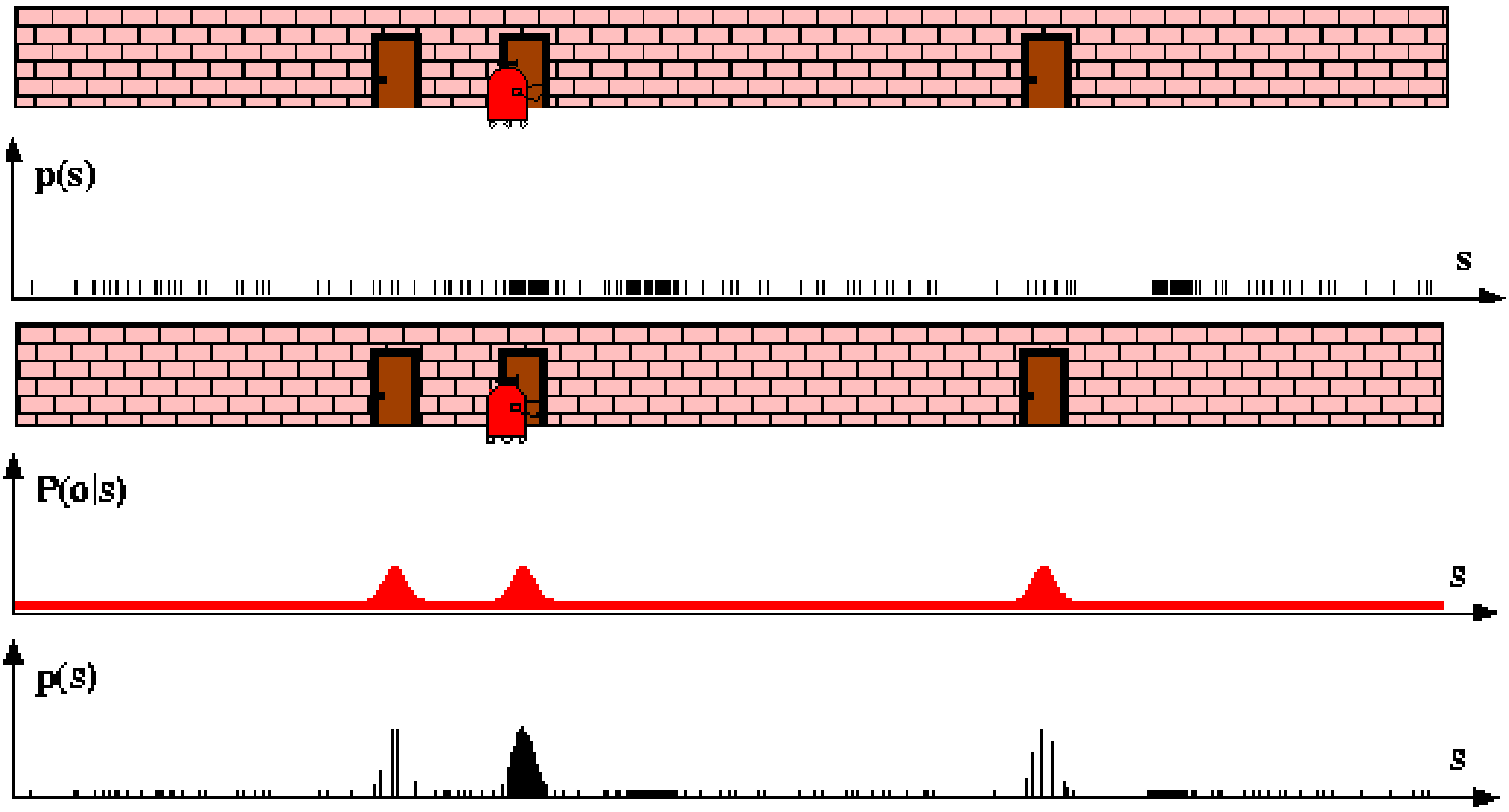
$$Bel^{-}(x) \leftarrow \int p(x \mid u, x')\, Bel(x')\, \mathrm{d}x'$$



Sensor Information: Importance Sampling

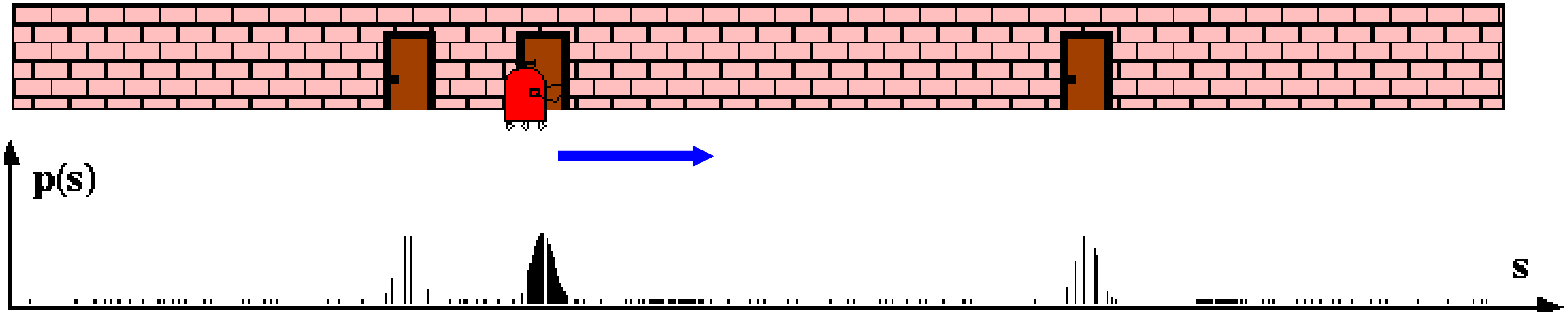
$$Bel(x) \quad \leftarrow \quad \alpha\ p(z\,|\,x)\ Bel^{-}(x)$$

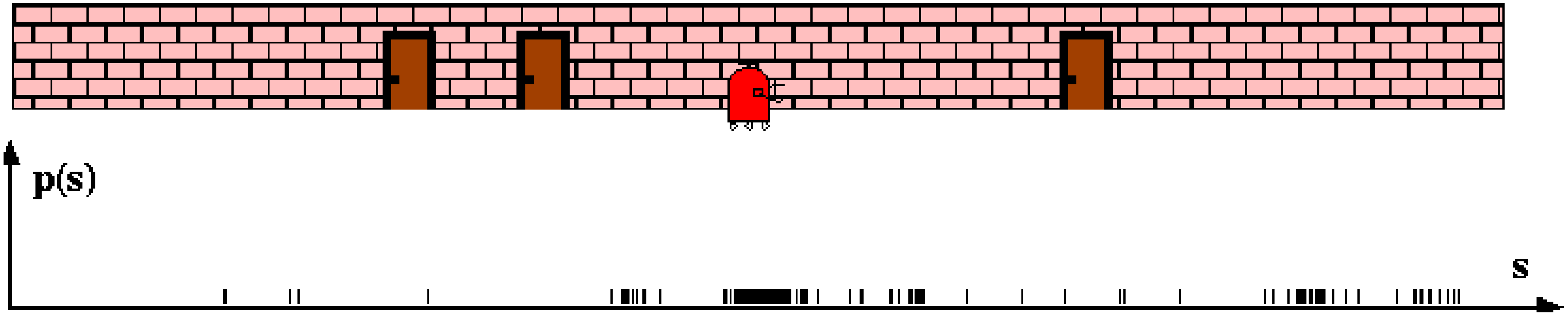
$$w \quad \leftarrow \quad \frac{\alpha\ p(z\,|\,x)\ Bel^{-}(x)}{Bel^{-}(x)} \quad = \quad \alpha\ p(z\,|\,x)$$



Robot Motion

$$Bel^{-}(x) \quad \leftarrow \quad \int p(x \mid u, x')\, Bel(x') \;\mathrm{d}\, x'$$





Particle Filter Algorithm

- Sample the next generation for particles using the proposal distribution

- Compute the importance weights :

  $$weight = target\ distribution\ /\ proposal\ distribution$$

- Resampling: “Replace unlikely samples by more likely ones”

- [Derivation of the MCL equations on the blackboard]

Particle Filter Algorithm

1. Algorithm **particle_filter**(S_{t-1}, u_{t-1} z_t):
2. $S_t = \emptyset, \quad \eta = 0$
3. **For** $i = 1 \ldots n$ ***Generate new samples***
4. Sample index *j(i)* from the discrete distribution given by w_{t-1}
5. Sample x_t^i from $p(x_t \mid x_{t-1}, u_{t-1})$ using $x_{t-1}^{j(i)}$ and u_{t-1}
6. $w_t^i = p(z_t \mid x_t^i)$ ***Compute importance weight***
7. $\eta = \eta + w_t^i$ ***Update normalization factor***
8. $S_t = S_t \cup \{< x_t^i, w_t^i >\}$ ***Insert***
9. **For** $i = 1 \ldots n$
10. $w_t^i = w_t^i / \eta$ ***Normalize weights***

Particle Filter Algorithm

$$Bel(x_t) = \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_{t-1})\, Bel(x_{t-1})\, dx_{t-1}$$

draw x^i_{t-1} from $Bel(x_{t-1})$

draw x^i_t from $p(x_t \mid x^i_{t-1}, u_{t-1})$

Importance factor for x^i_t:

$$\begin{aligned}
w^i_t &= \frac{\text{target distribution}}{\text{proposal distribution}} \\
&= \frac{\eta\, p(z_t \mid x_t)\, p(x_t \mid x_{t-1}, u_{t-1})\, Bel(x_{t-1})}{p(x_t \mid x_{t-1}, u_{t-1})\, Bel(x_{t-1})} \\
&\propto p(z_t \mid x_t)
\end{aligned}$$

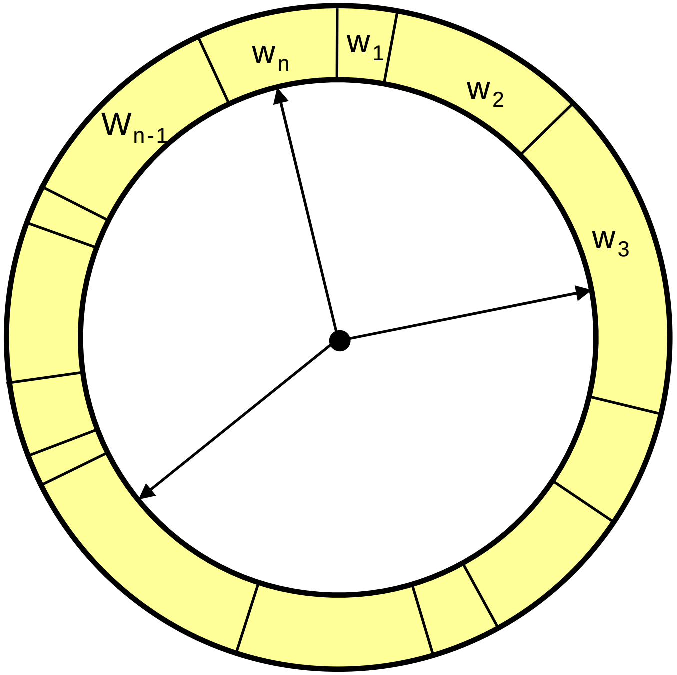
Resampling

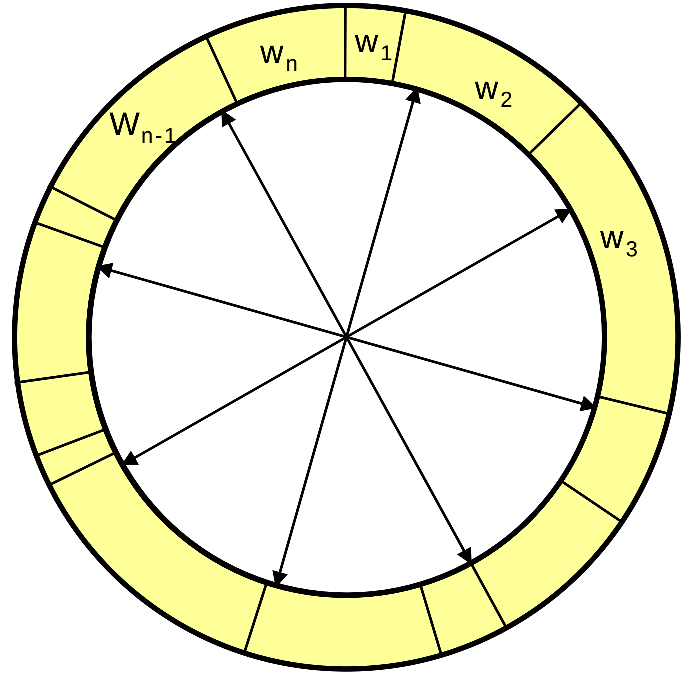
- **Given**: Set S of weighted samples.

- **Wanted** : Random sample, where the probability of drawing x_i is given by w_i.

- Typically done n times with replacement to generate new sample set S'.

Resampling





- Roulette wheel
- Binary search, n log n

- Stochastic universal sampling
- Systematic resampling
- Linear time complexity
- Easy to implement, low variance

Resampling Algorithm

1. Algorithm **systematic_resampling**(S,n):

2. $S' = \emptyset, c_1 = w^1$
3. **For** $i = 2 \ldots n$ ***Generate cdf***
4. $c_i = c_{i-1} + w^i$
5. $u_1 \sim U]0, n^{-1}], i = 1$ ***Initialize threshold***

6. **For** $j = 1 \ldots n$ ***Draw samples …***
7. **While** $(u_j > c_i)$ ***Skip until next threshold reached***
8. $i = i + 1$
9. $S' = S' \cup \{ < x^i, n^{-1} > \}$ ***Insert***
10. $u_{j+1} = u_j + n^{-1}$ ***Increment threshold***

11. **Return** S'

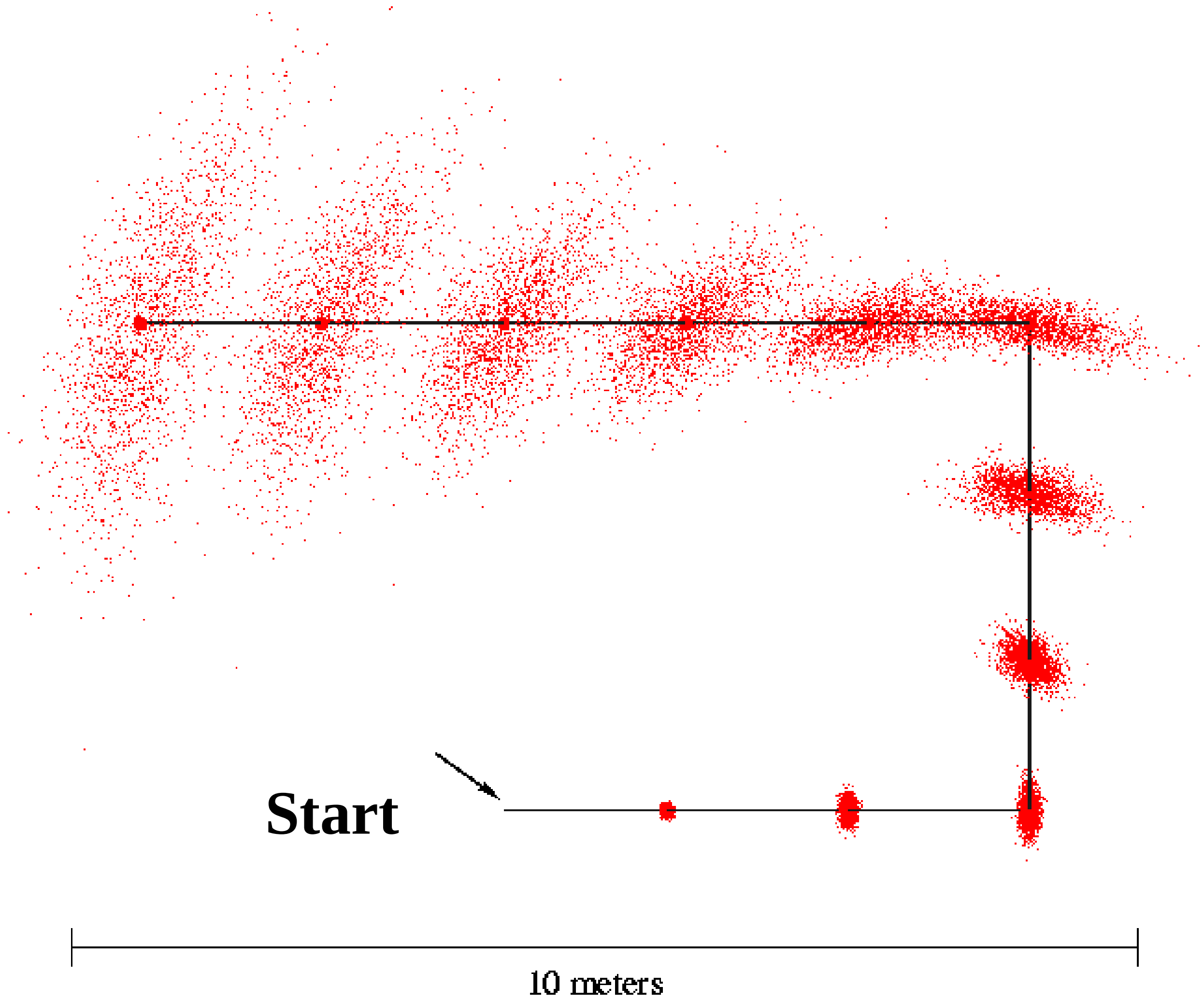
Also called **stochastic universal sampling**

Mobile Robot Localization

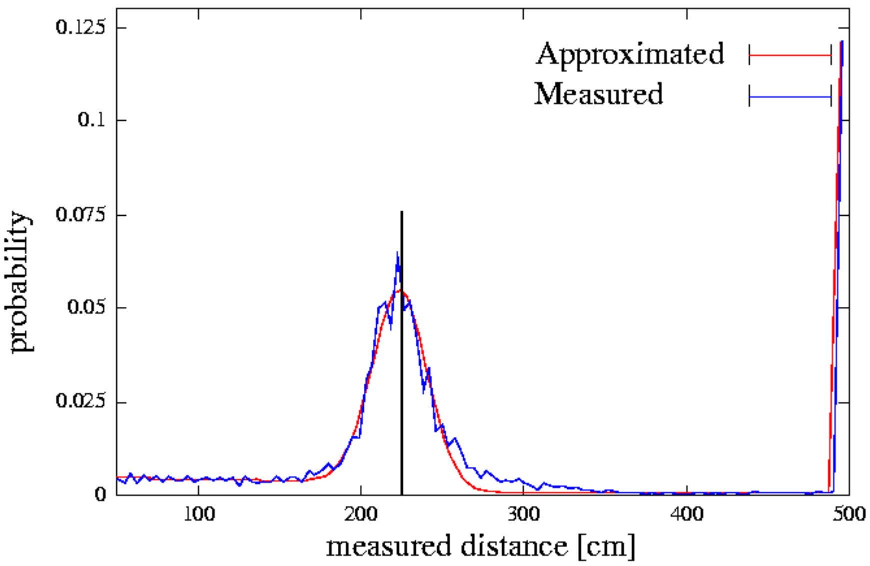
- Each particle is a potential pose of the robot
- Proposal distribution is the motion model of the robot (prediction step)
- The observation model is used to compute the importance weight (correction step)

[For details, see PDF file on the lecture web page]

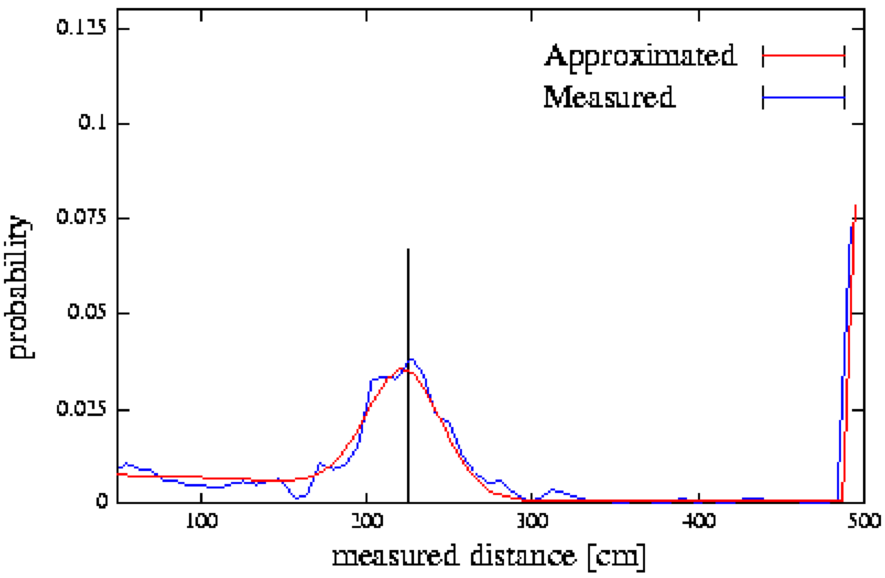
Motion Model Reminder



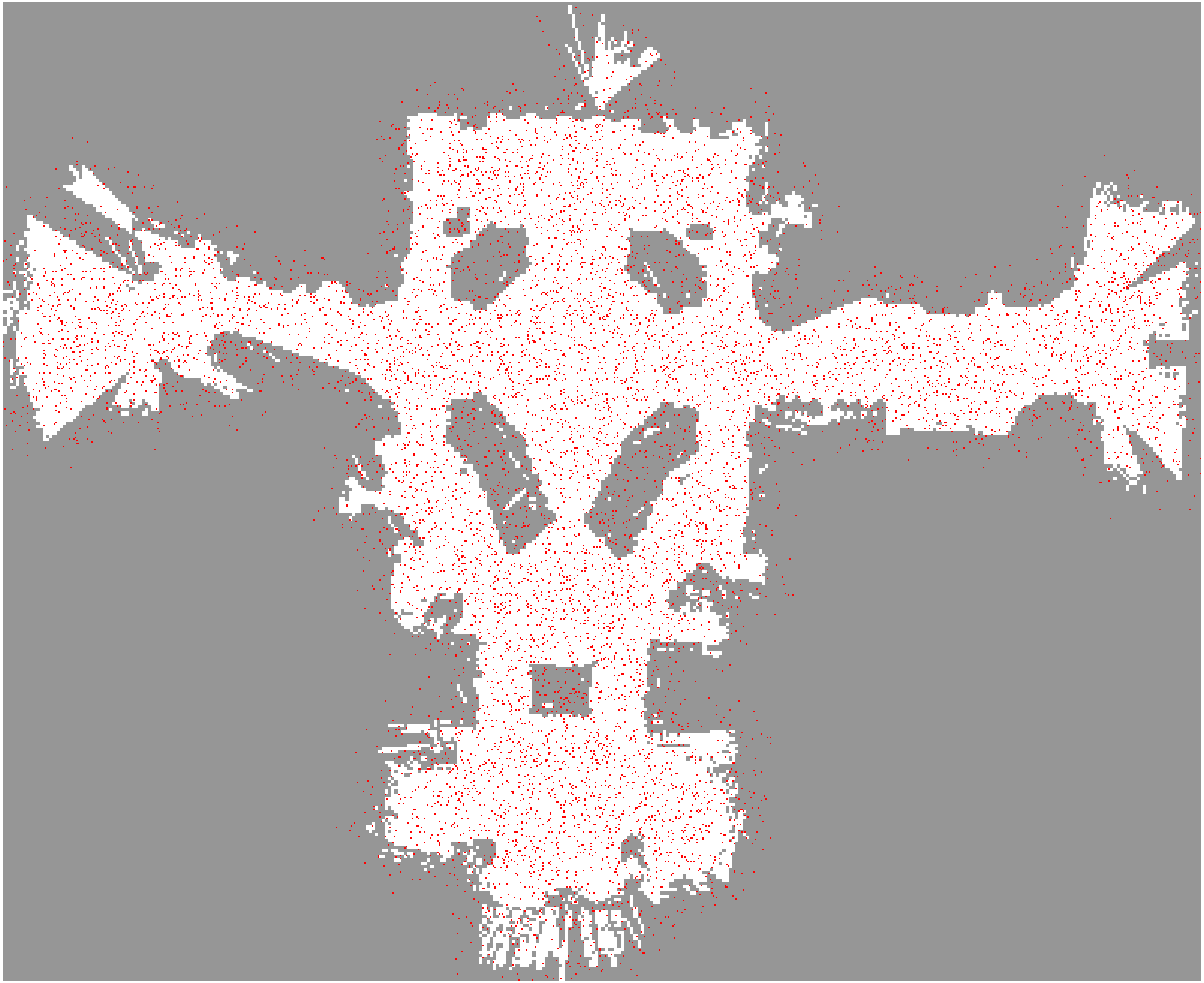
Proximity Sensor Model Reminder

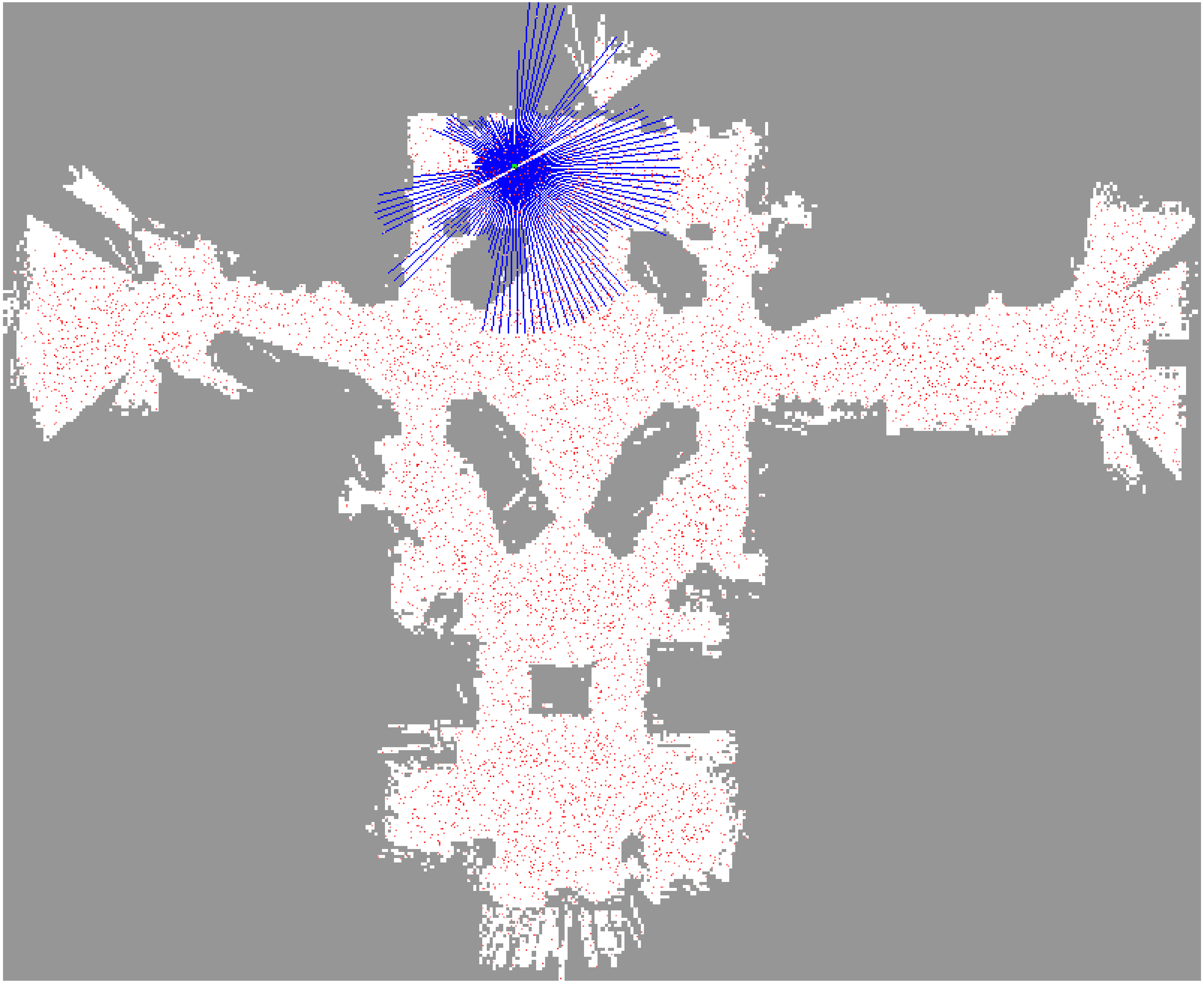


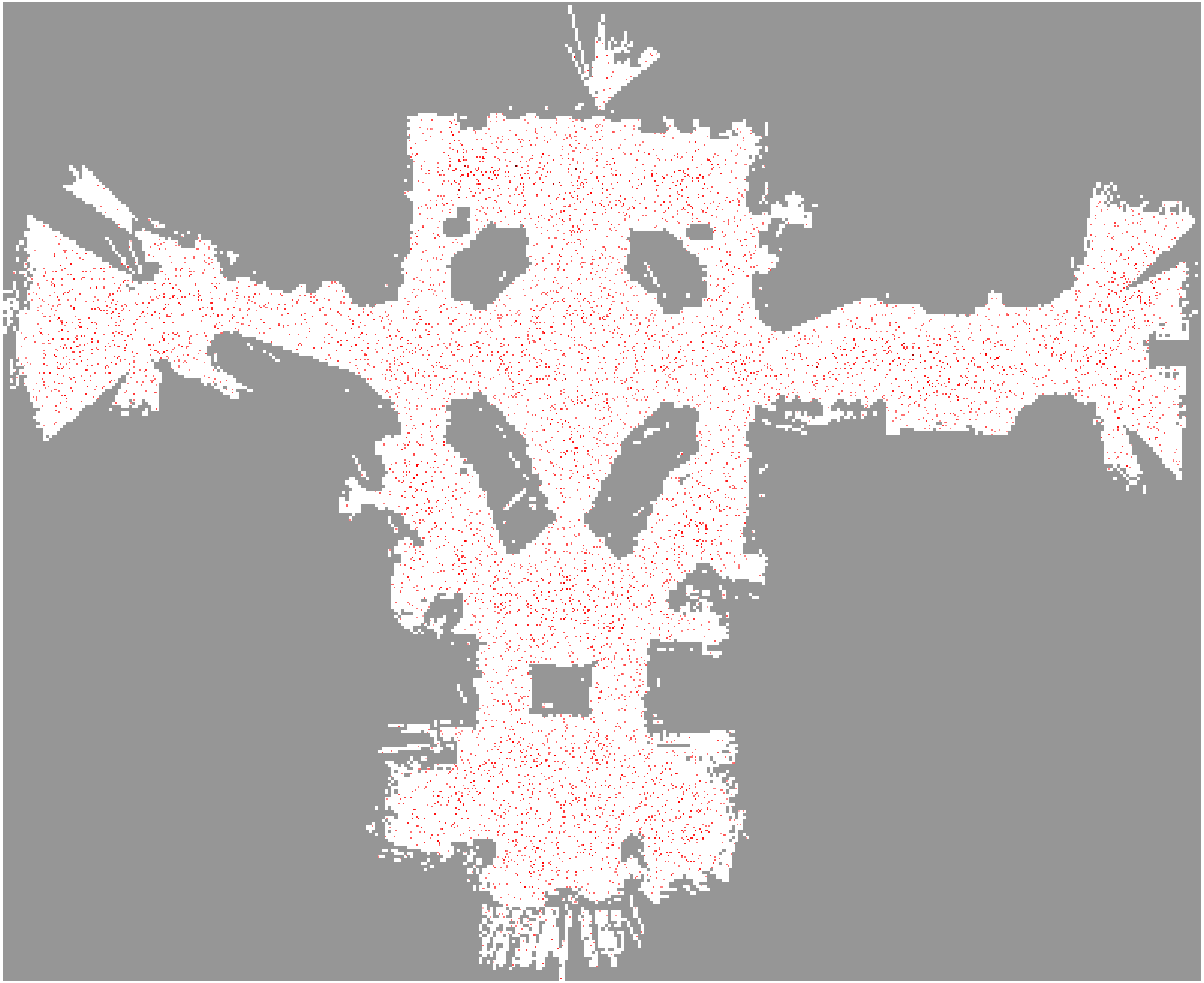
Laser sensor

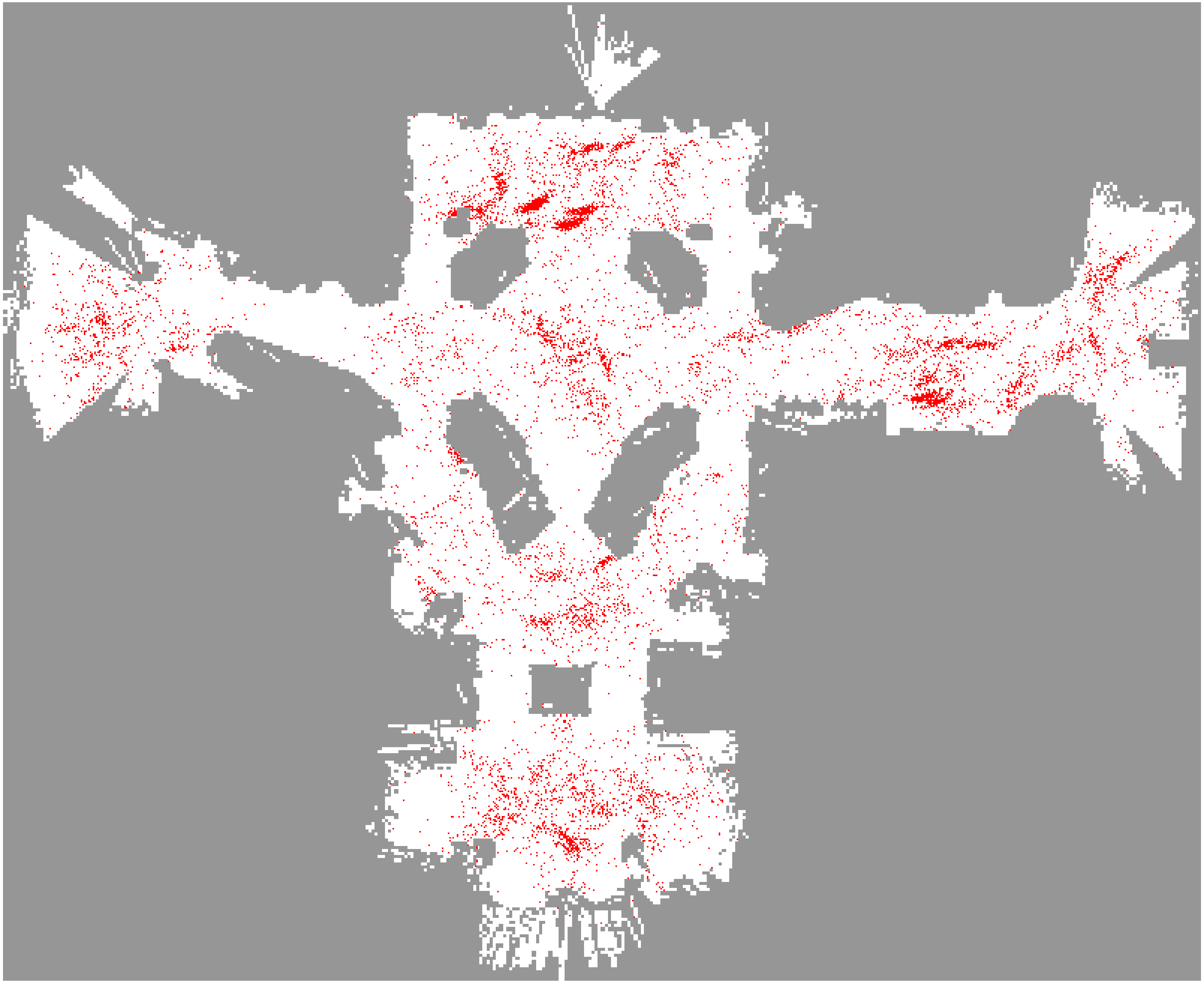


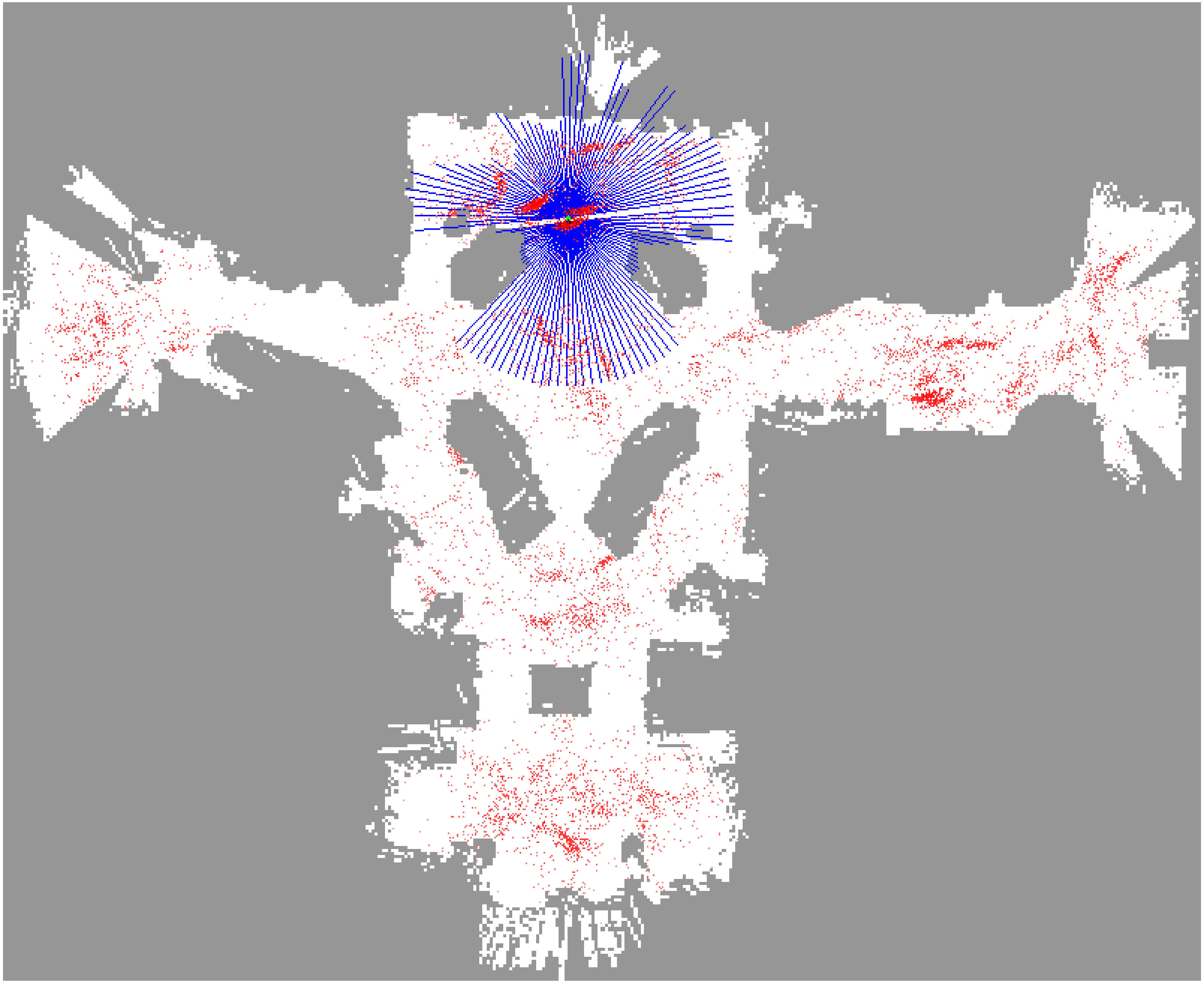
Sonar sensor

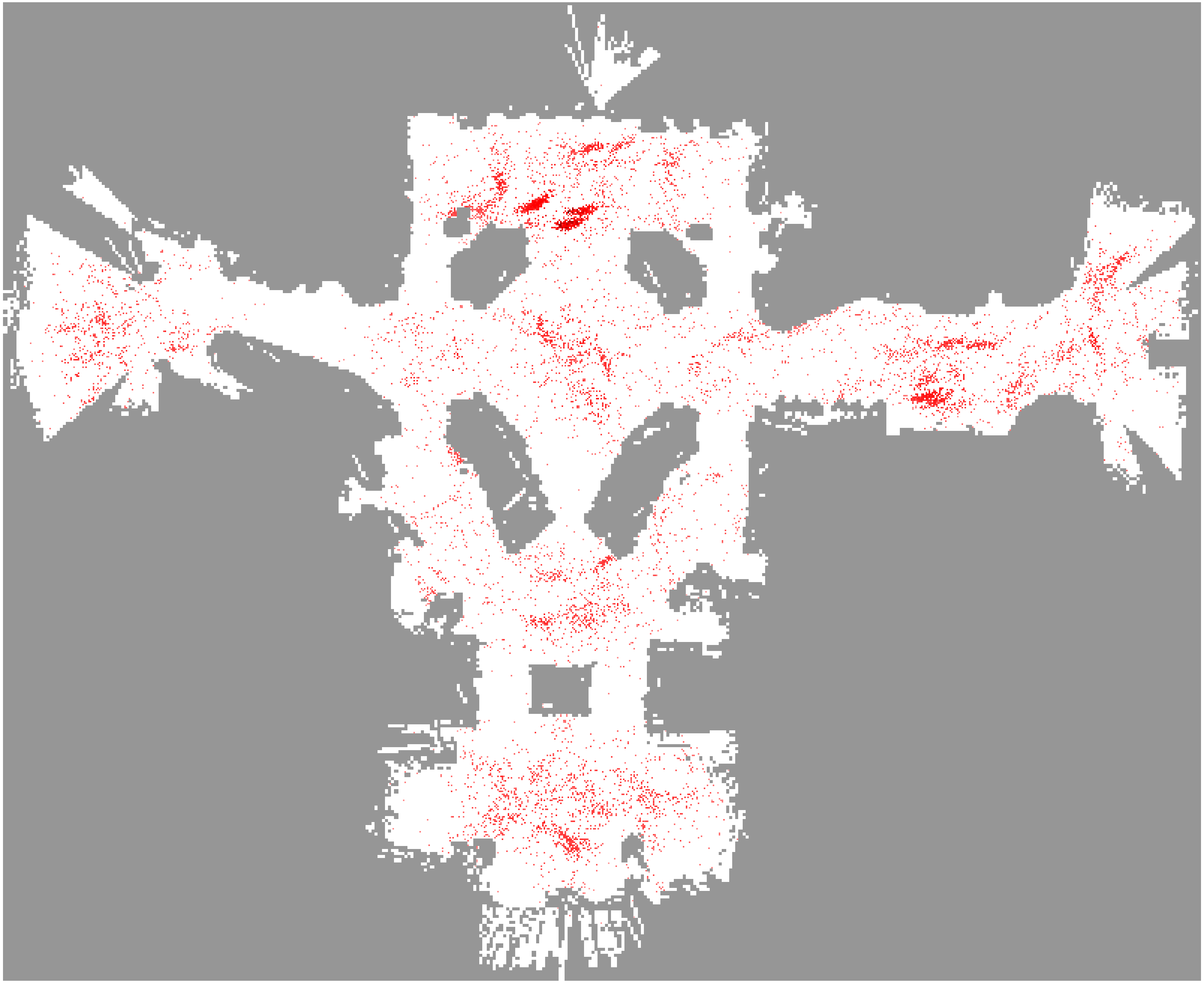


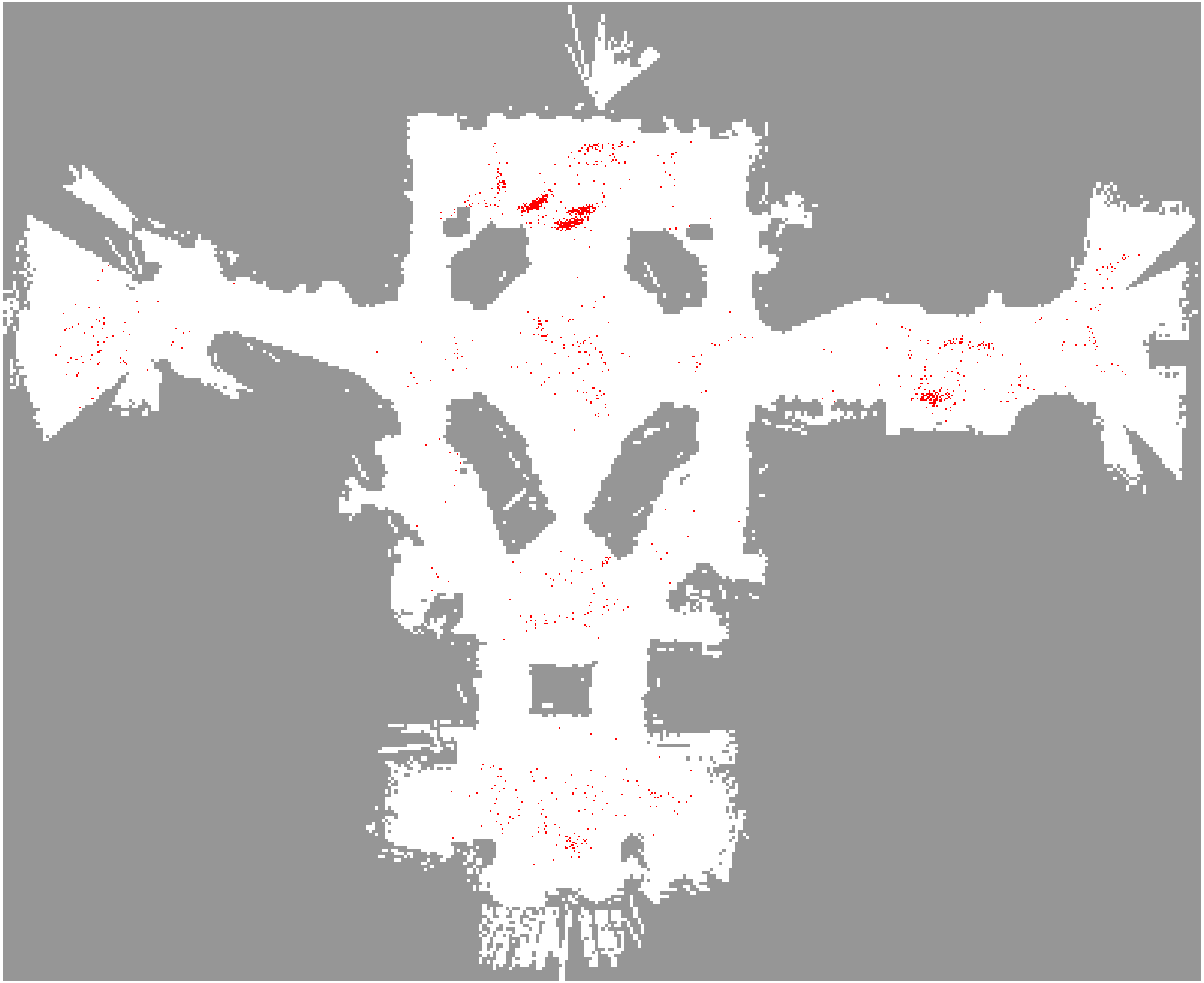


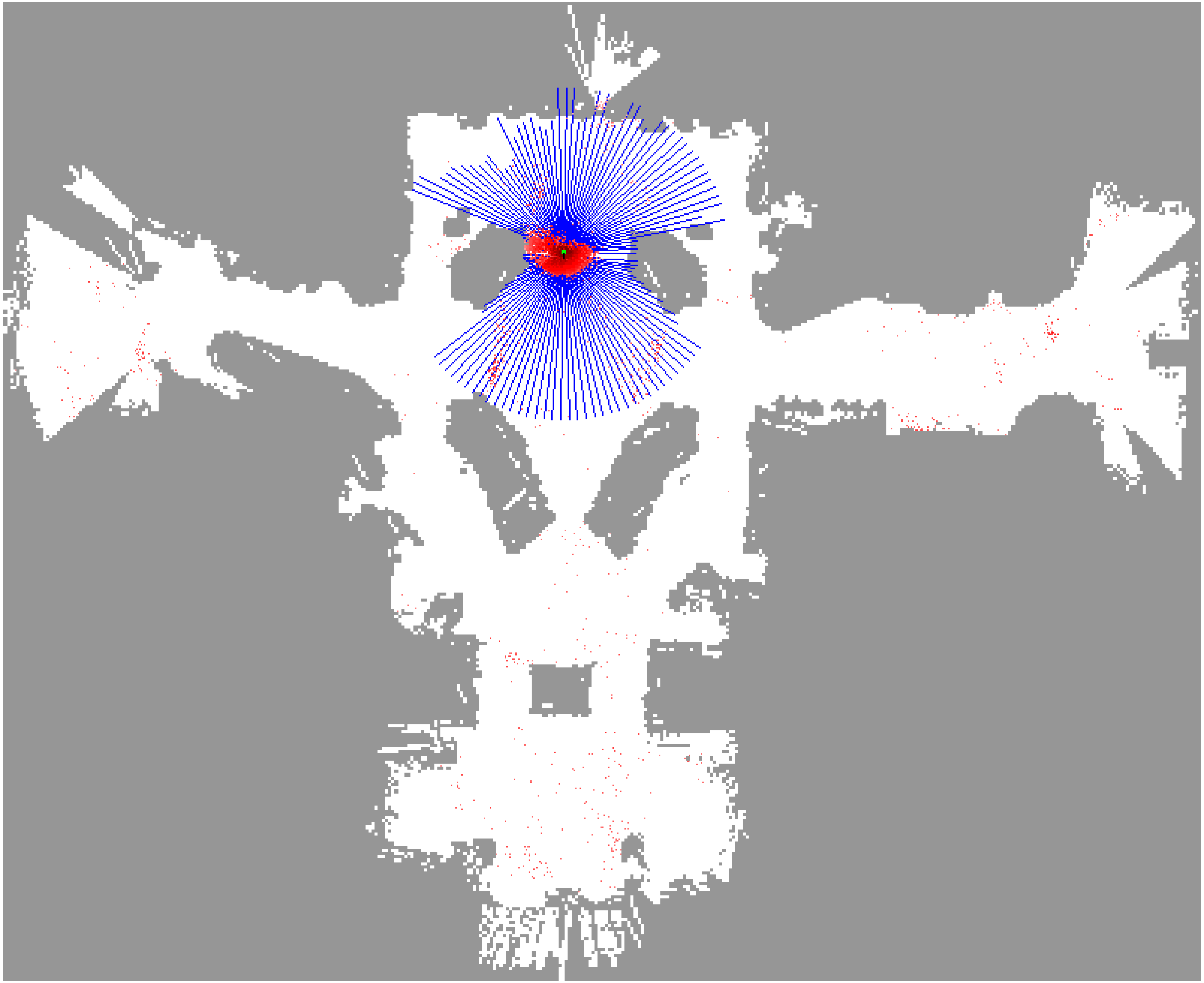


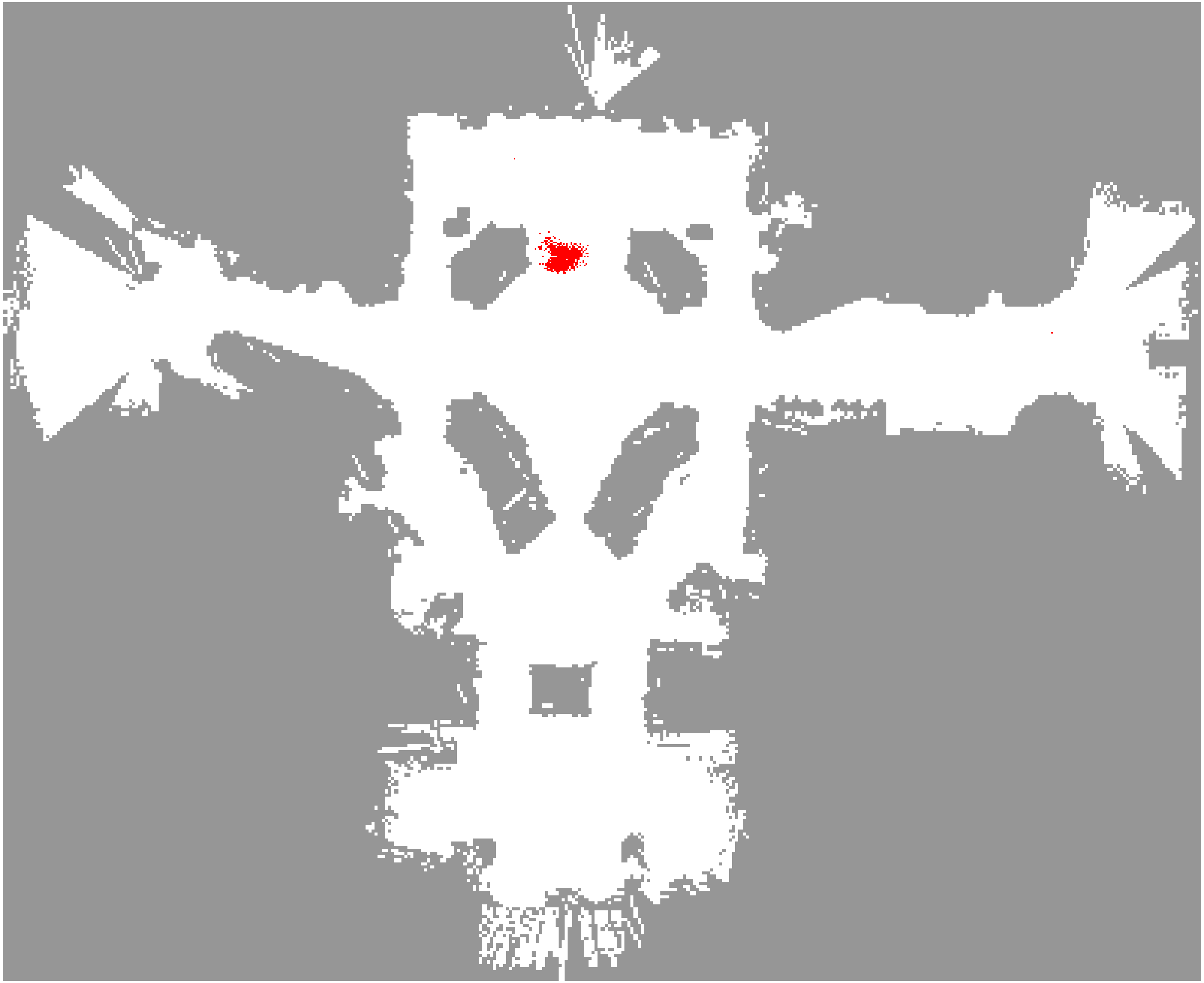


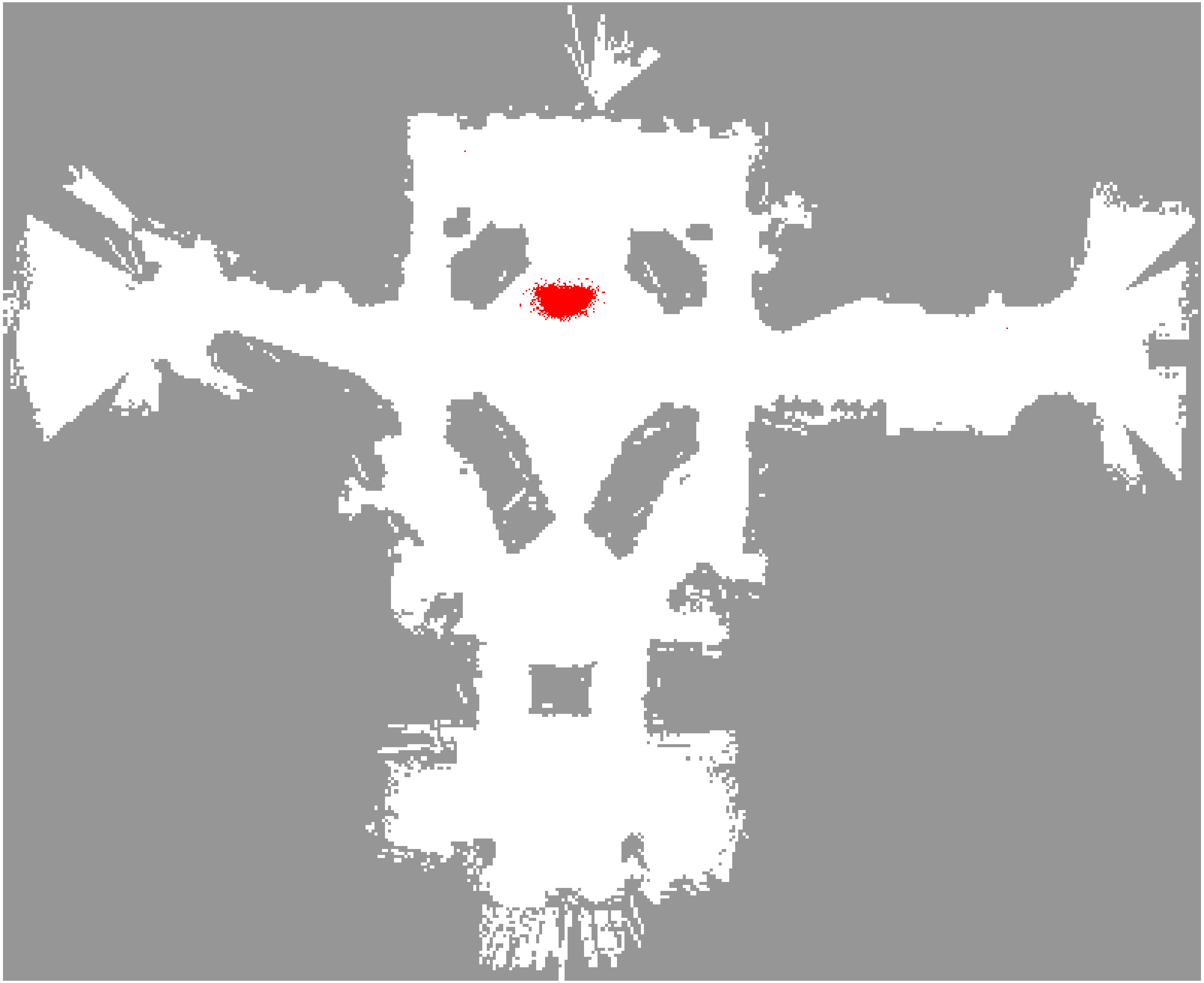


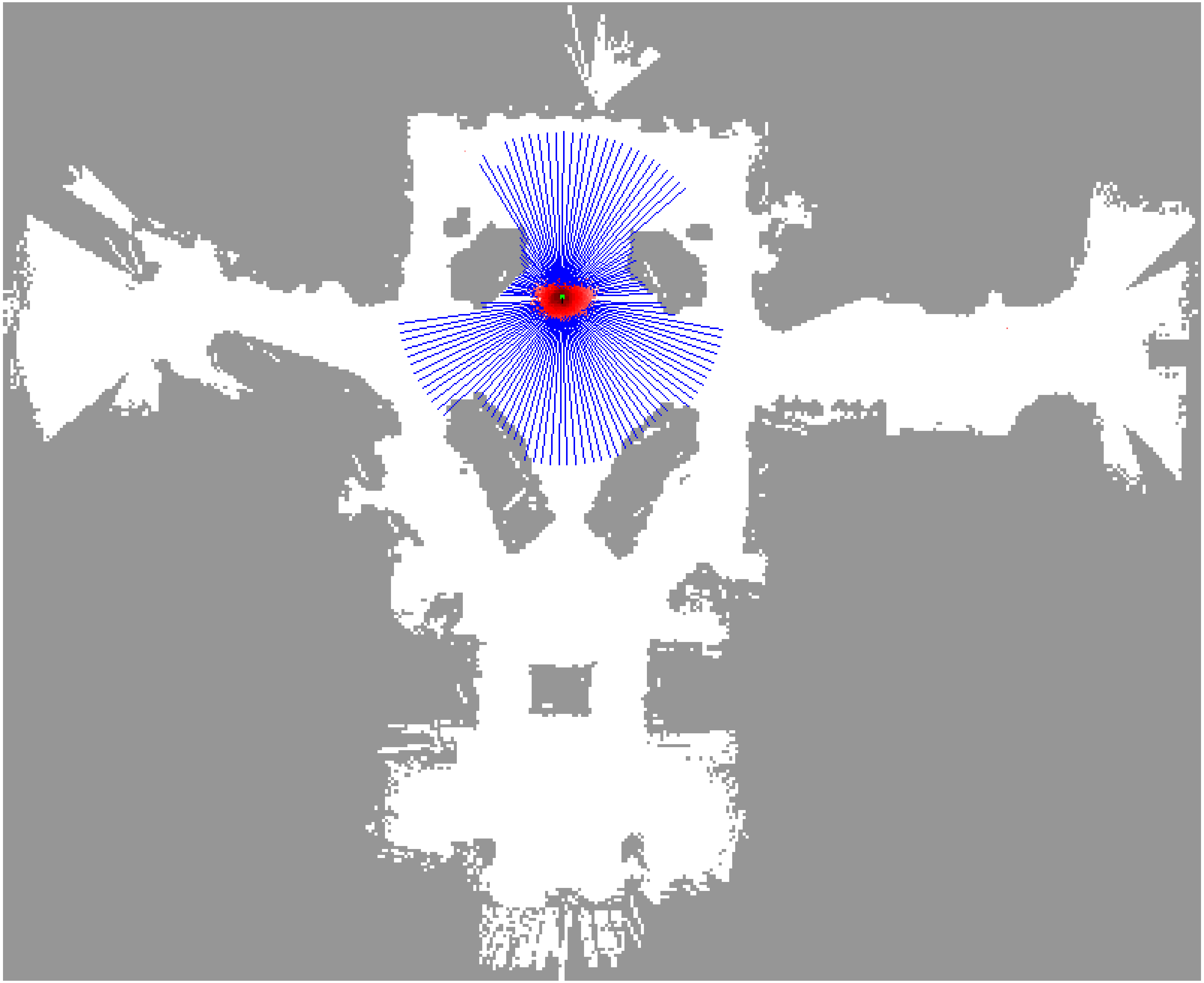


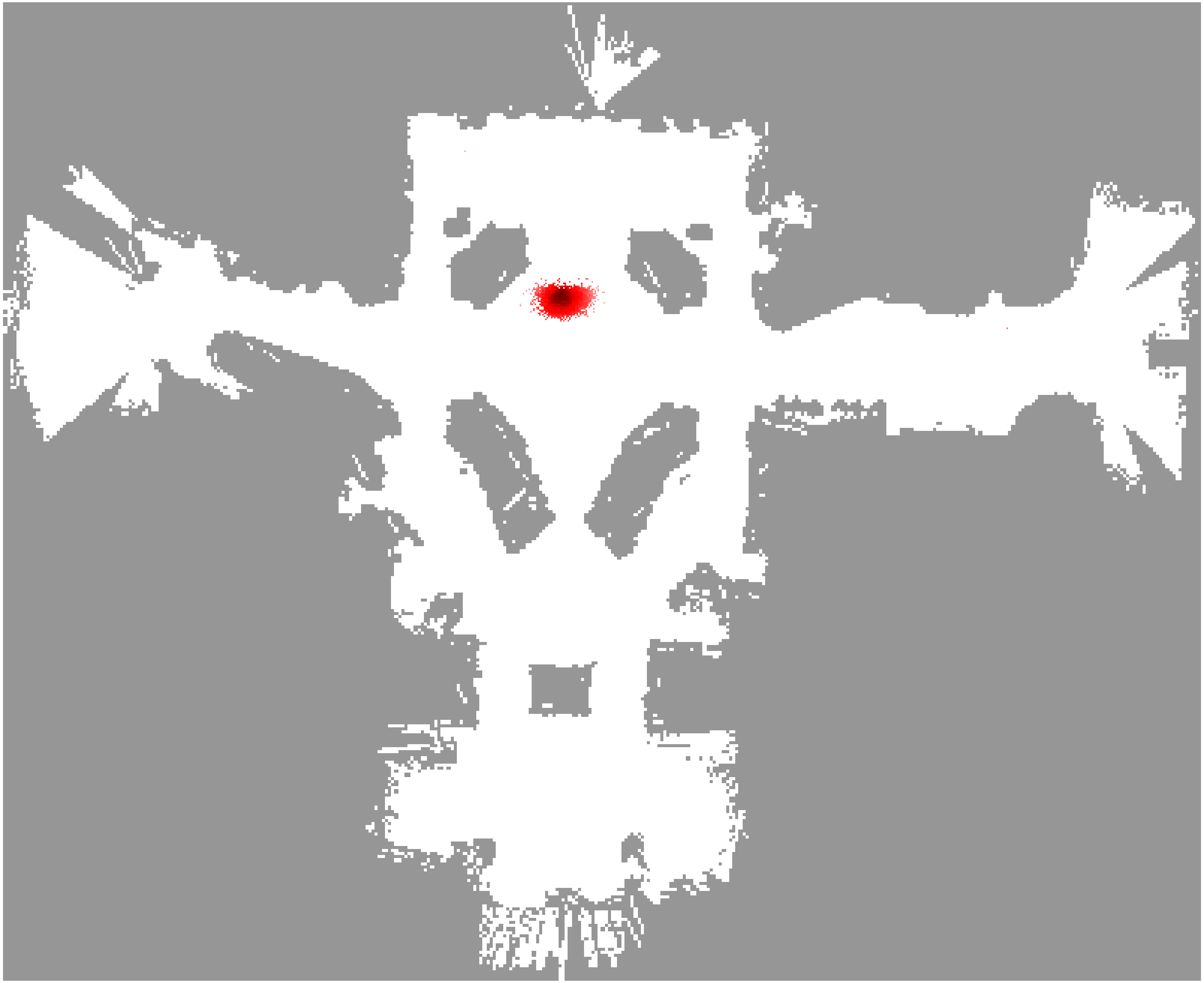


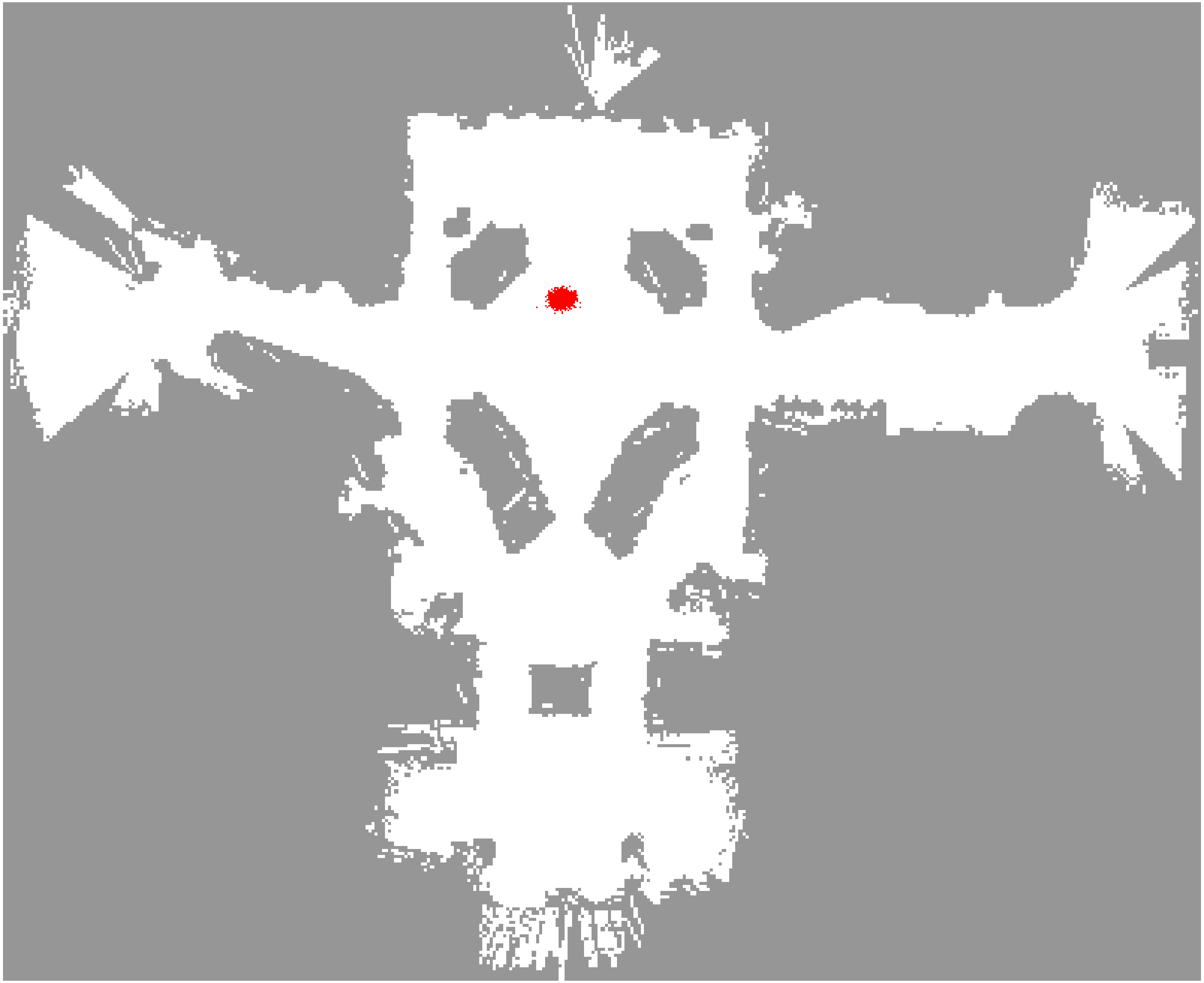


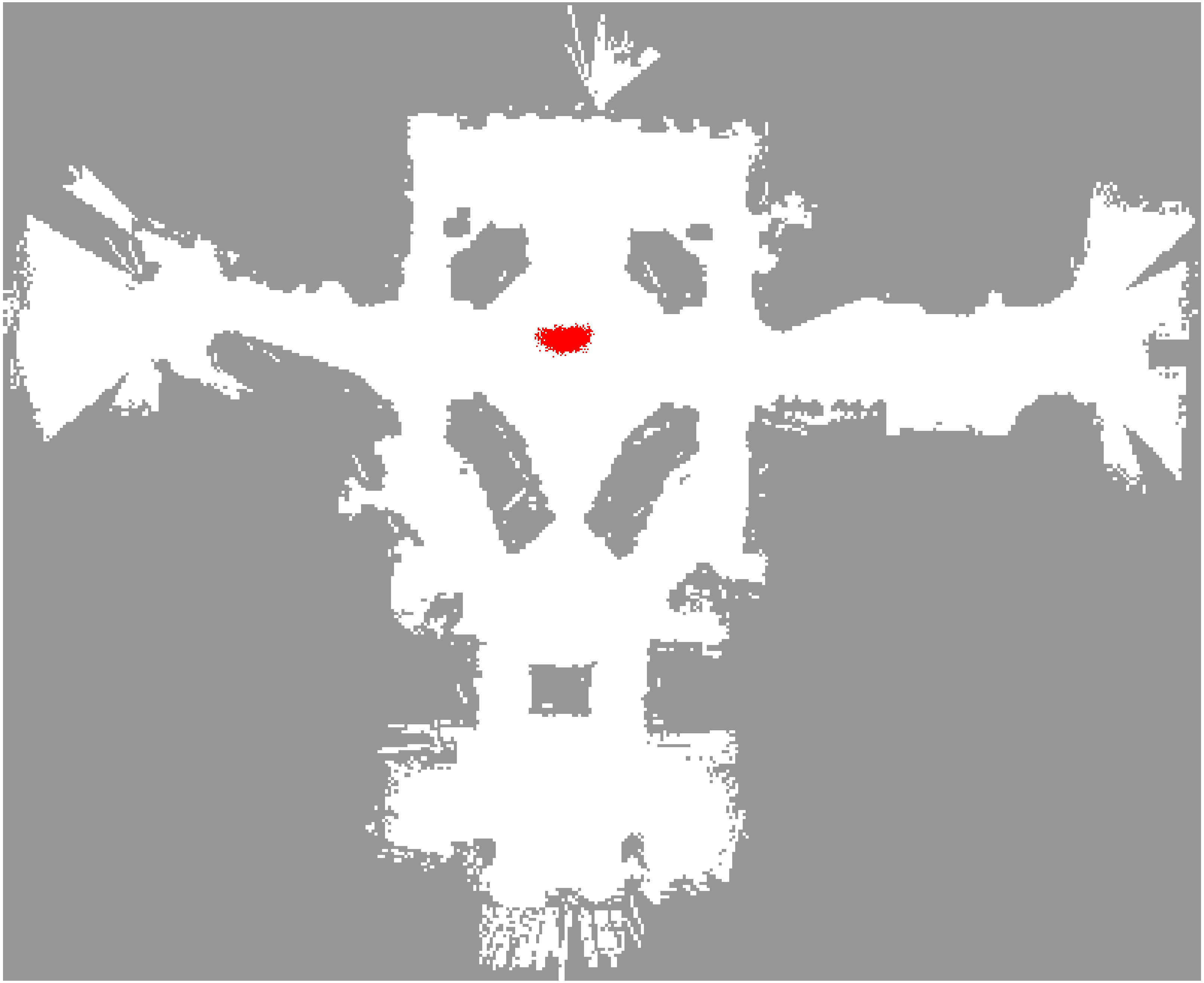


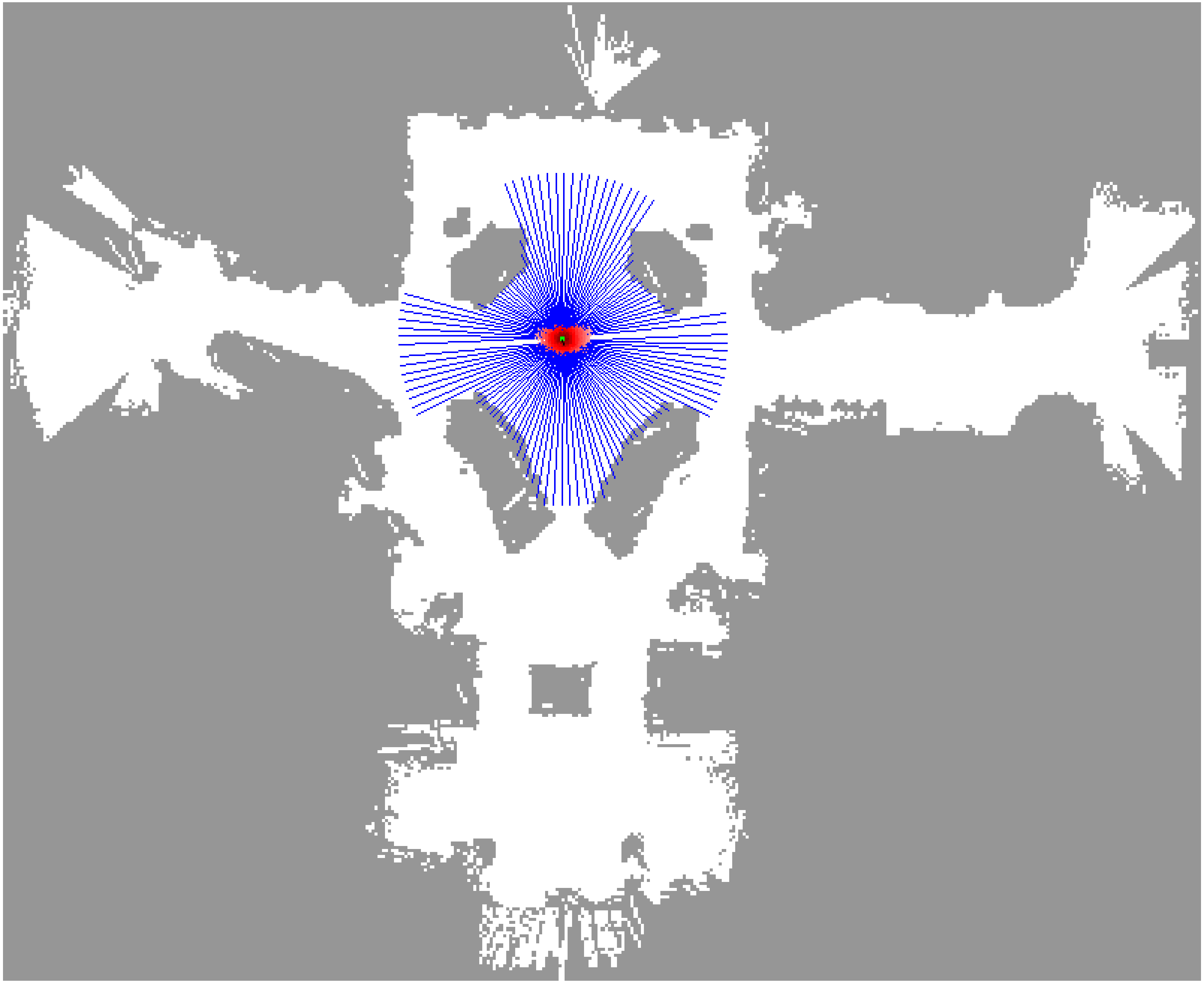


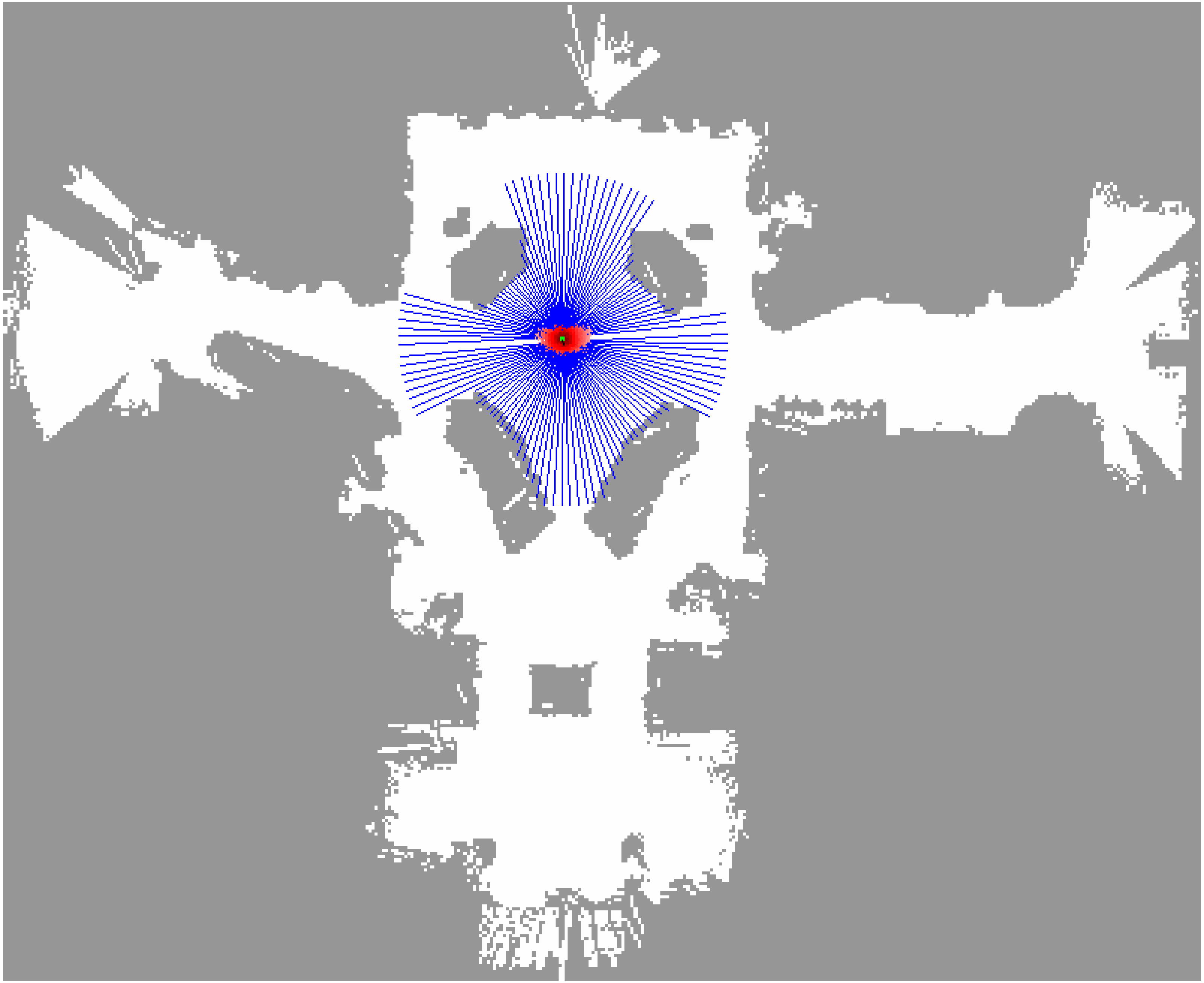




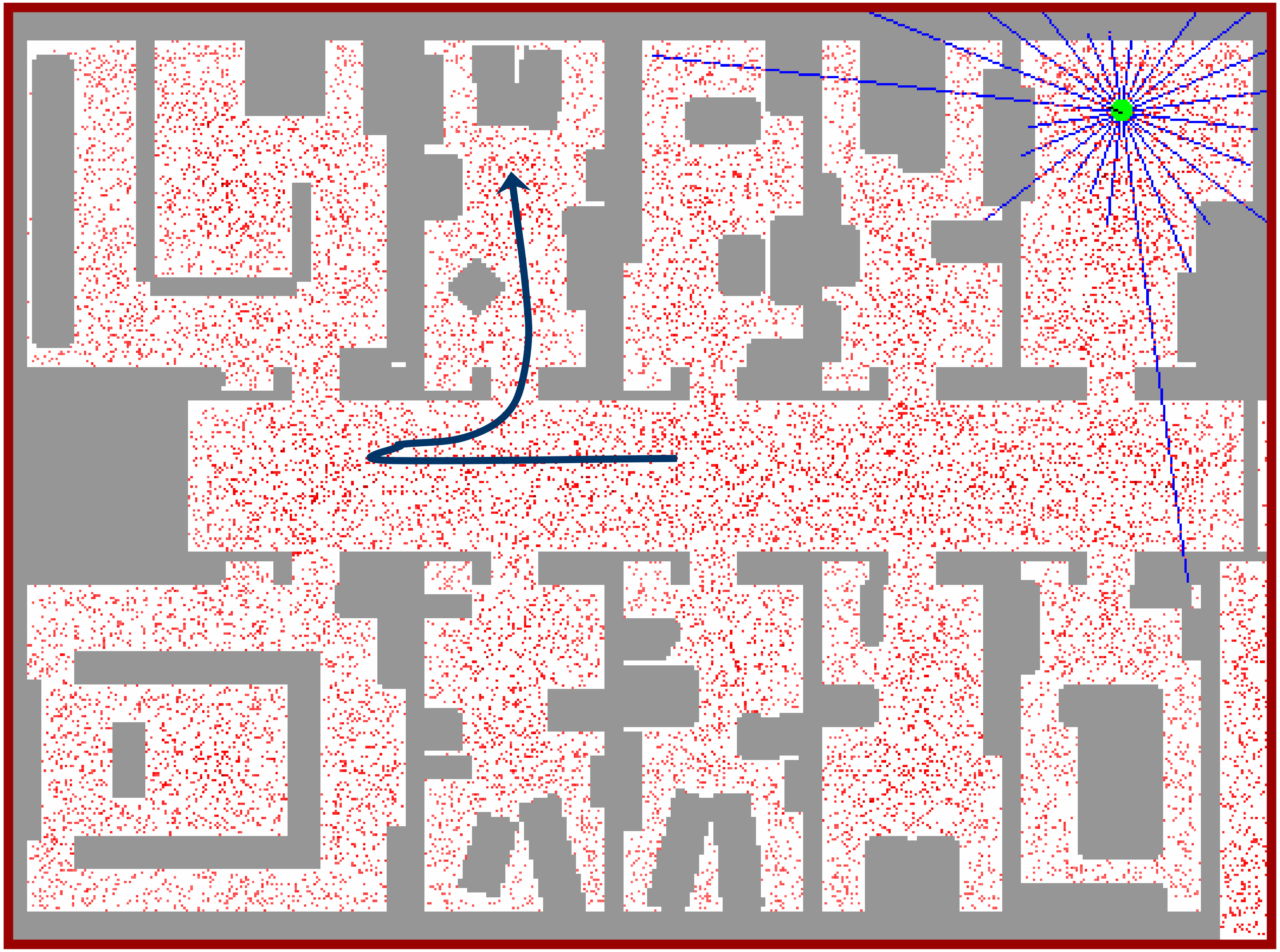




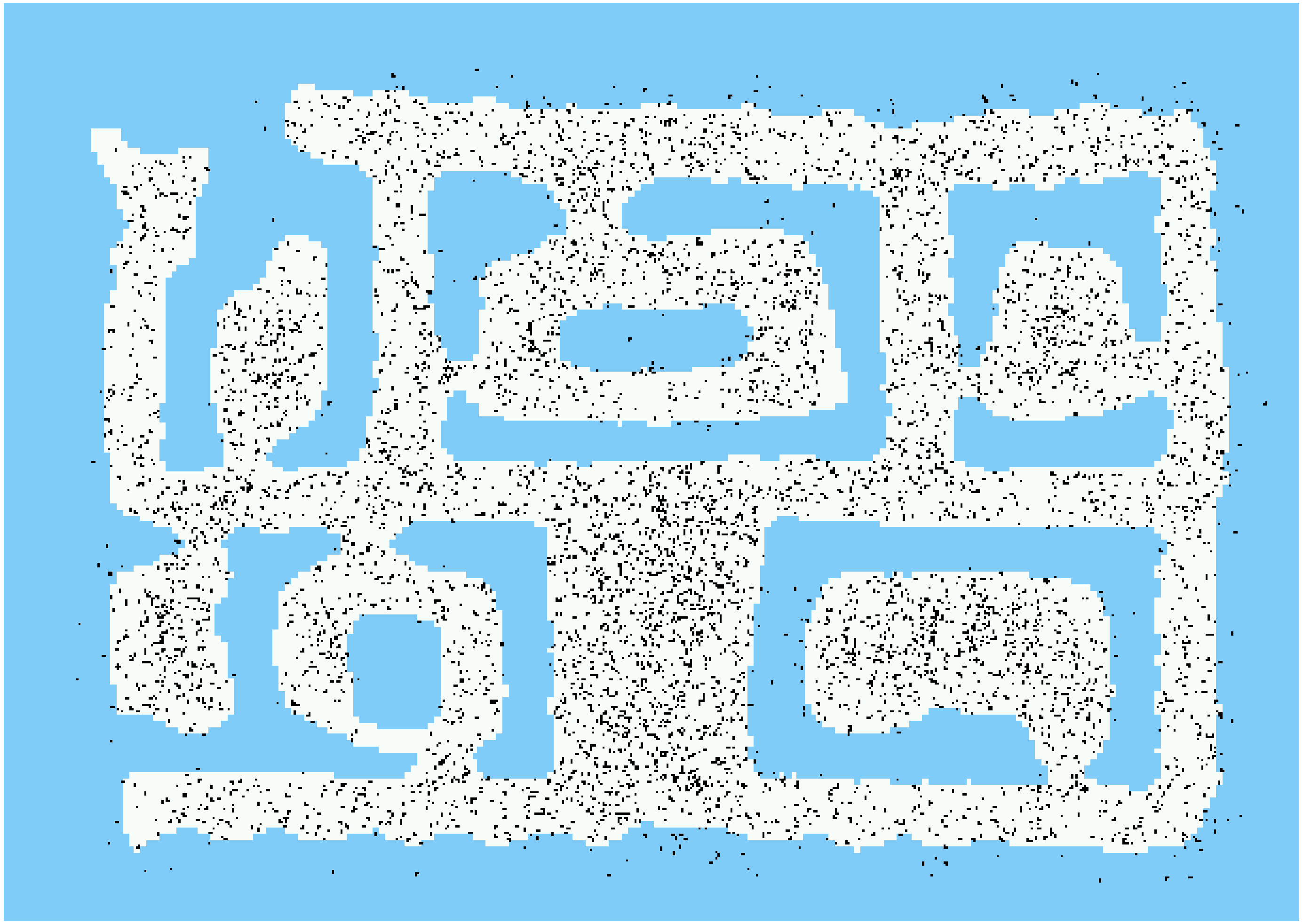




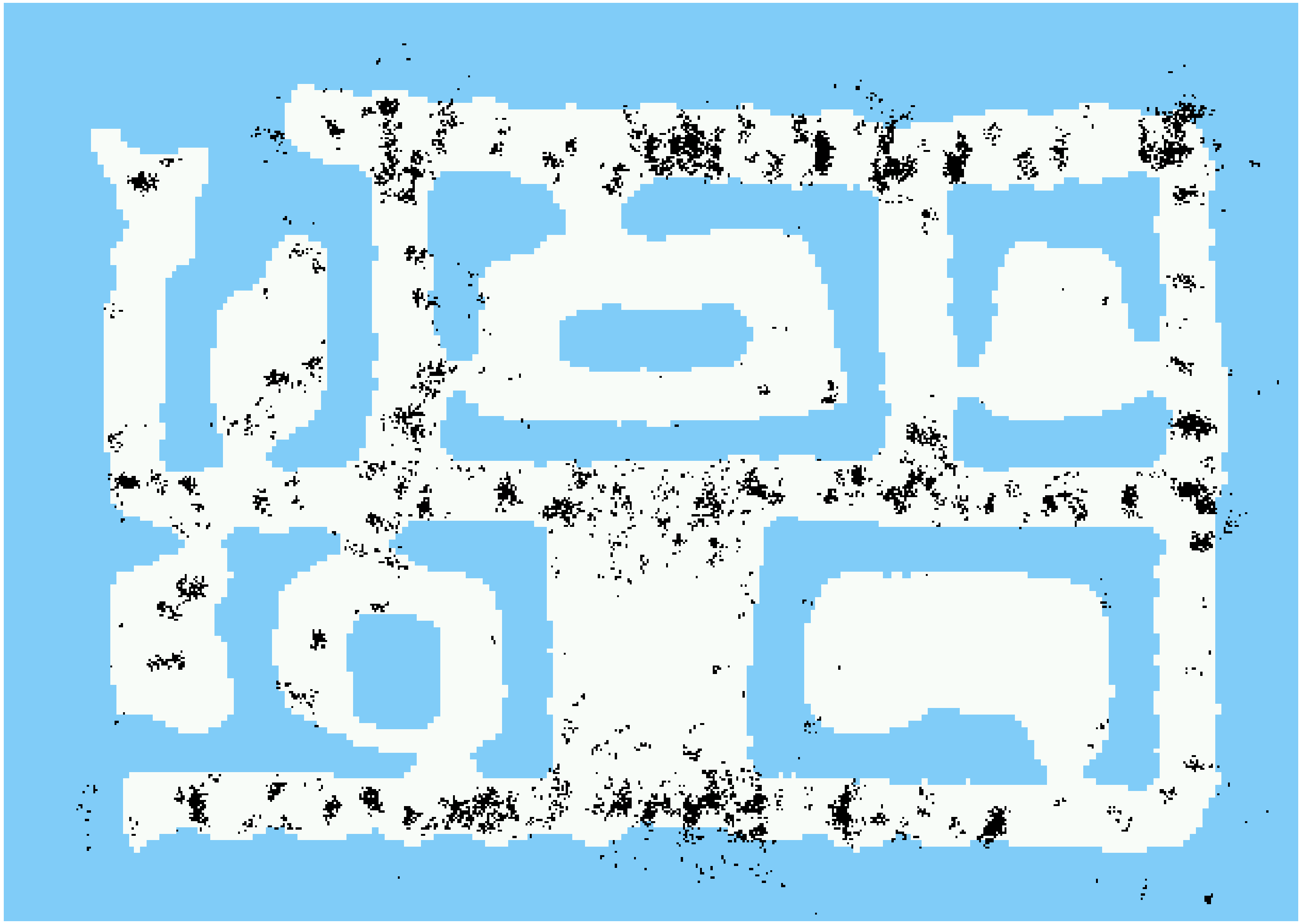
Sample-based Localization (sonar)



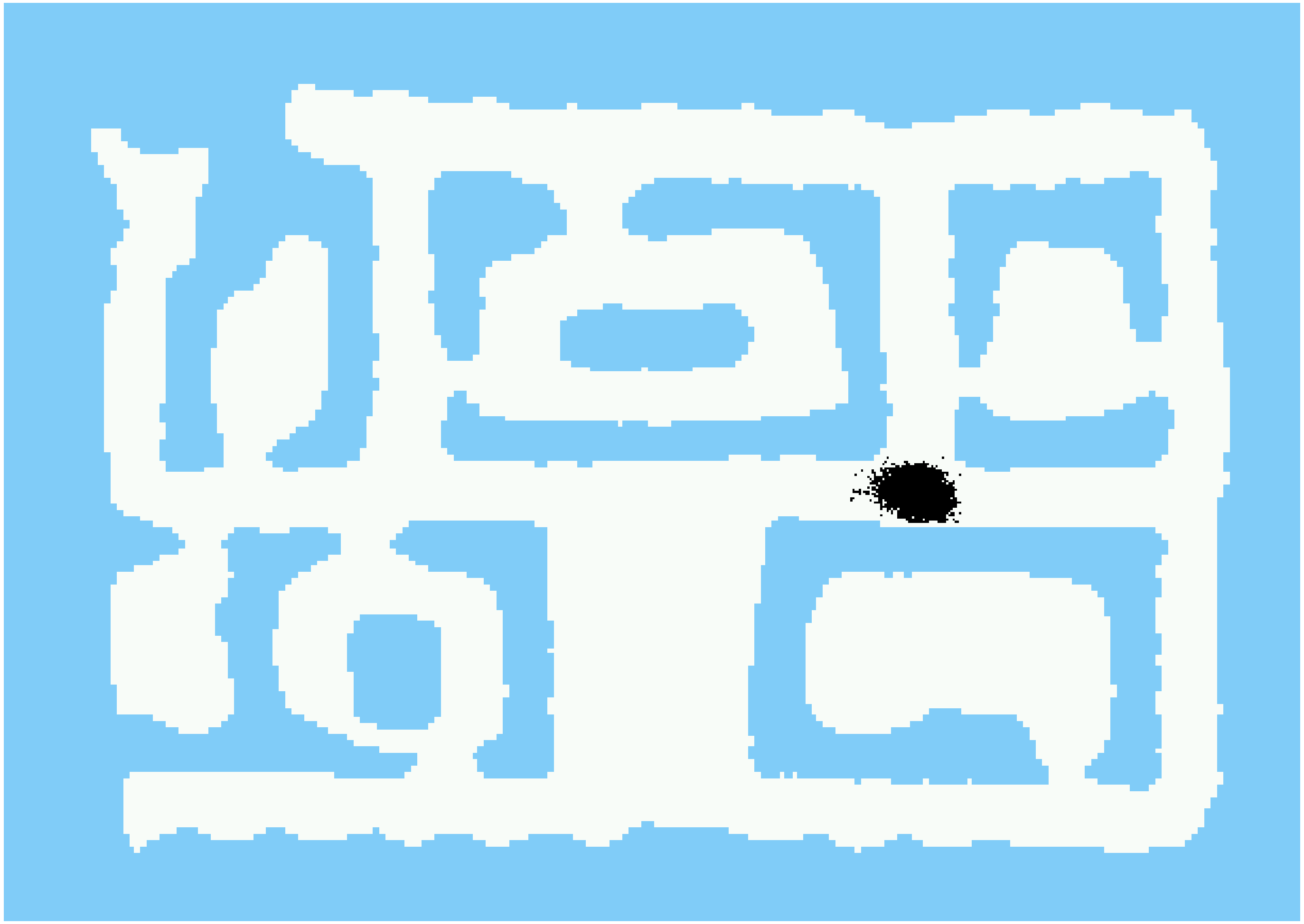
Initial Distribution



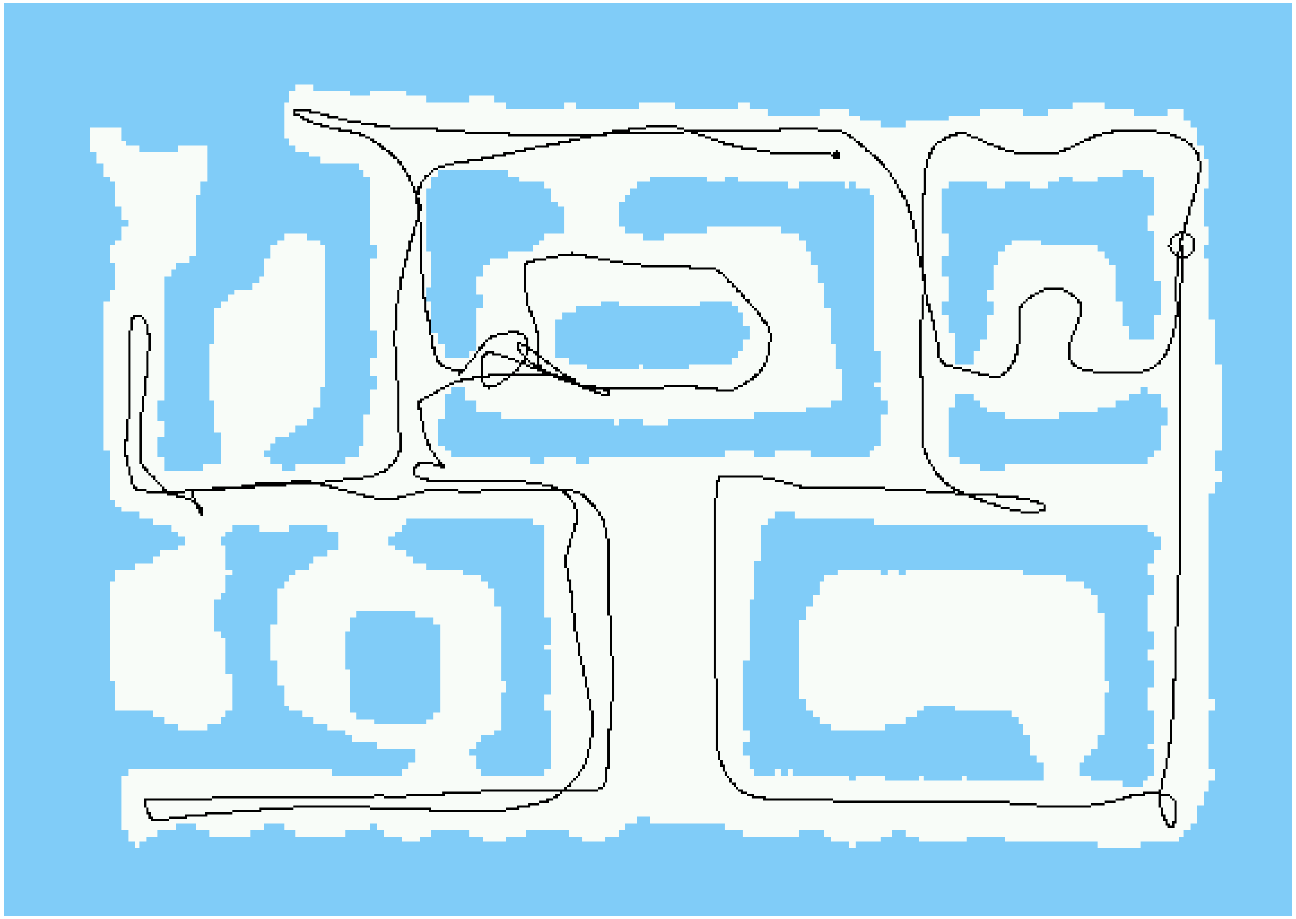
After Incorporating Ten Ultrasound Scans



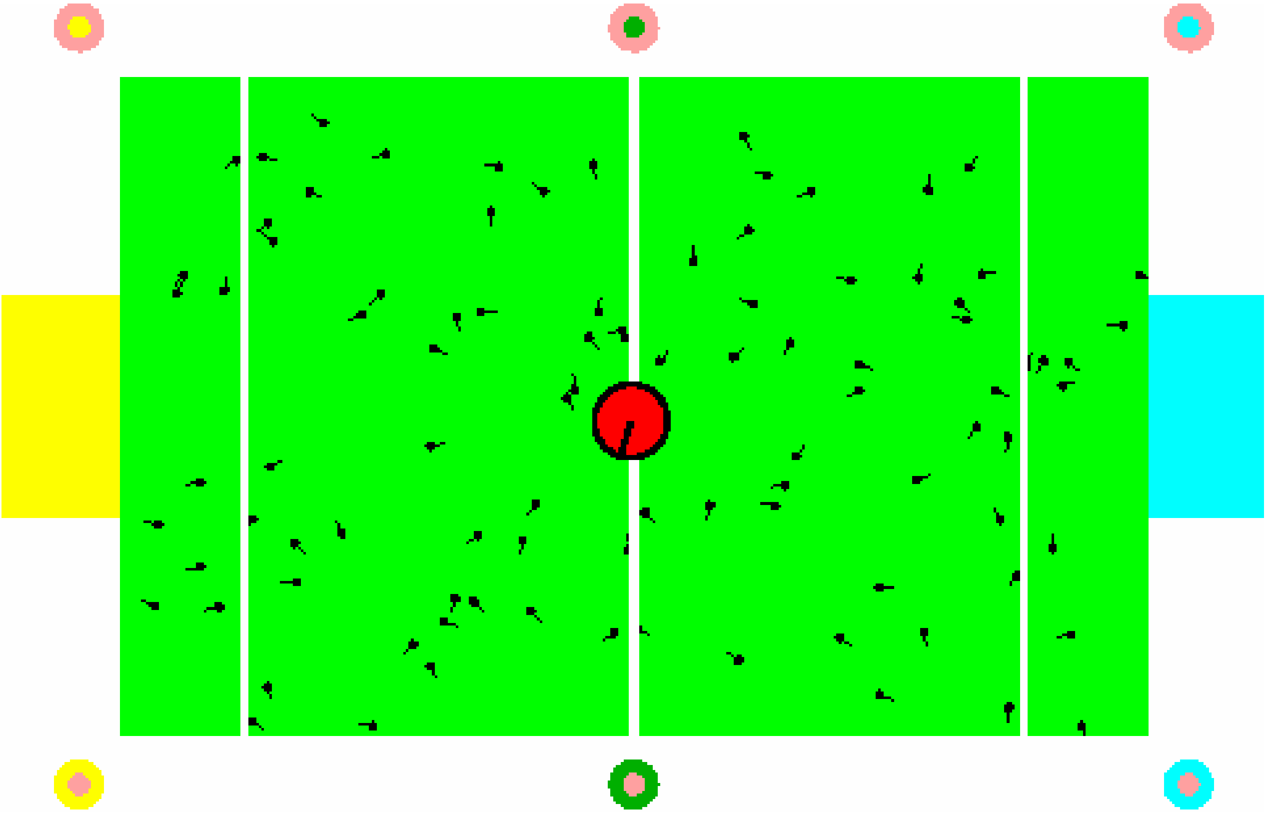
After Incorporating 65 Ultrasound Scans



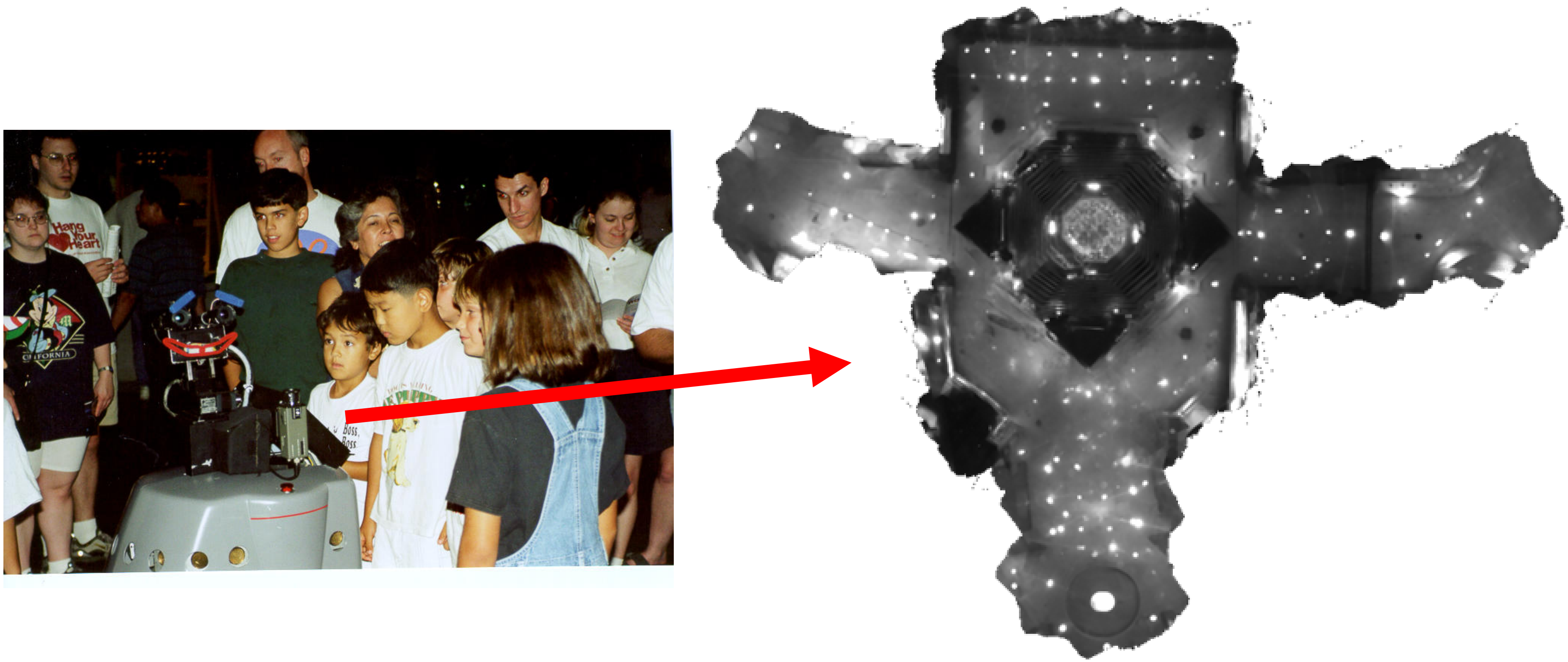
Estimated Path



Localization for AIBO robots

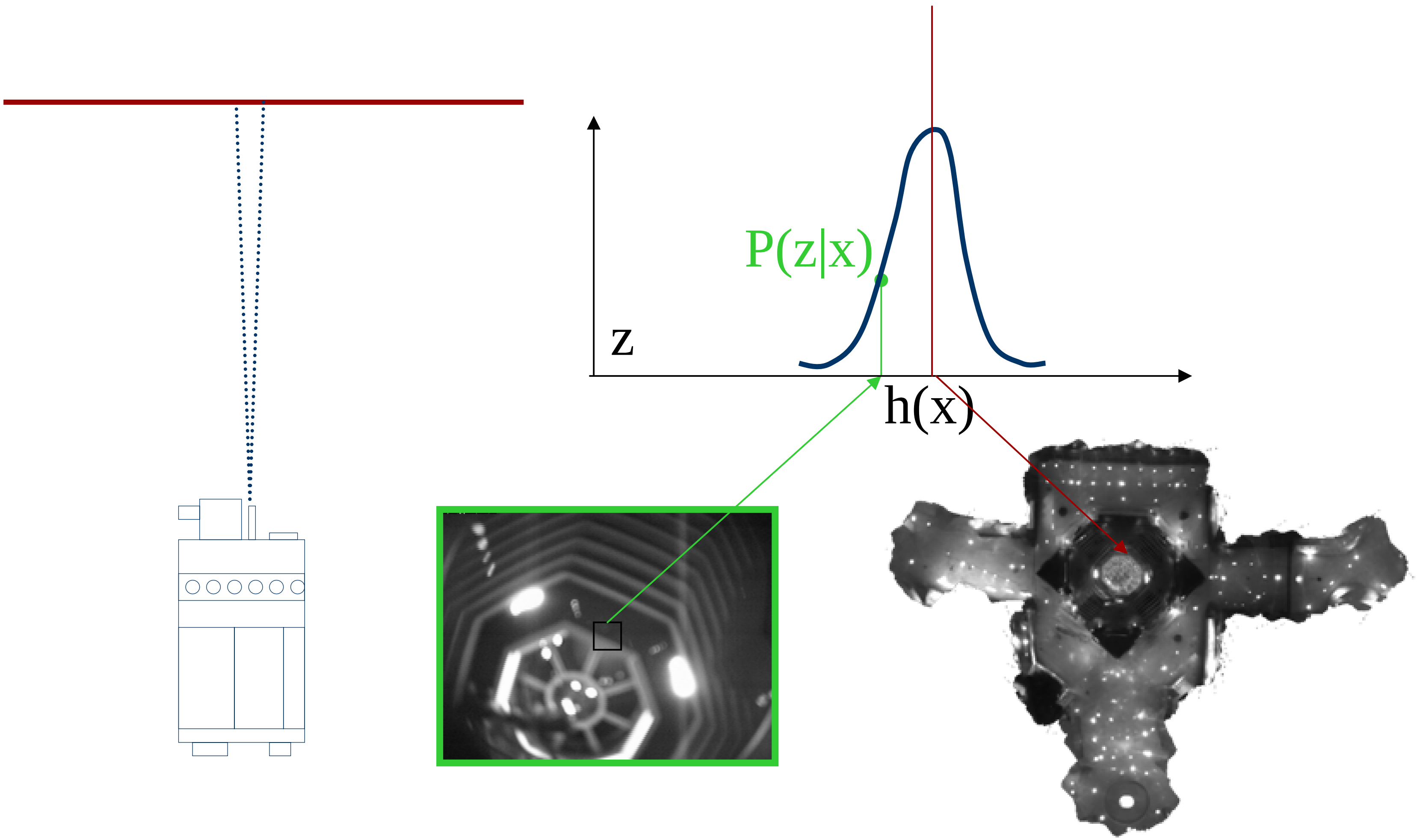


Using Ceiling Maps for Localization



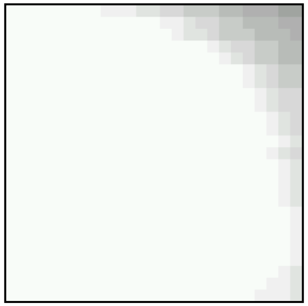
[Dellaert et al. 99]

Vision-based Localization

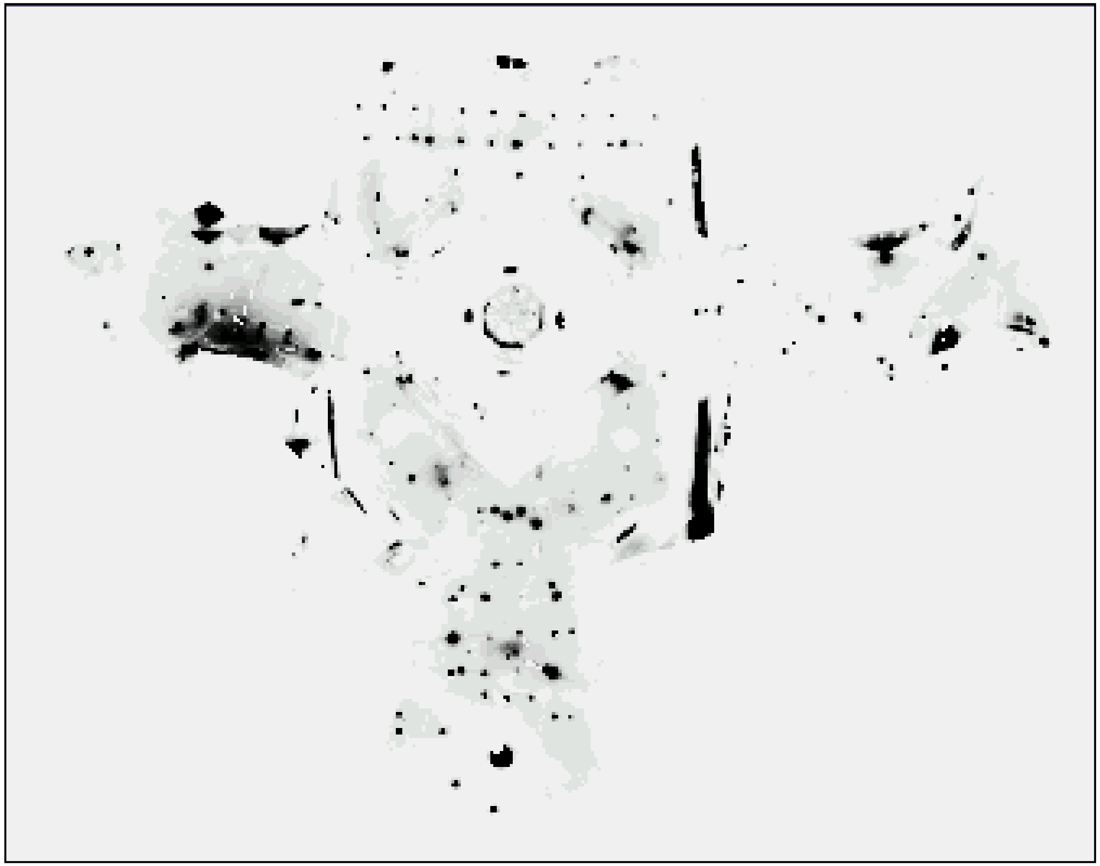


Under a Light

Measurement z:

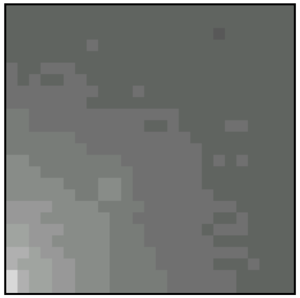


***P(z|x)*:**

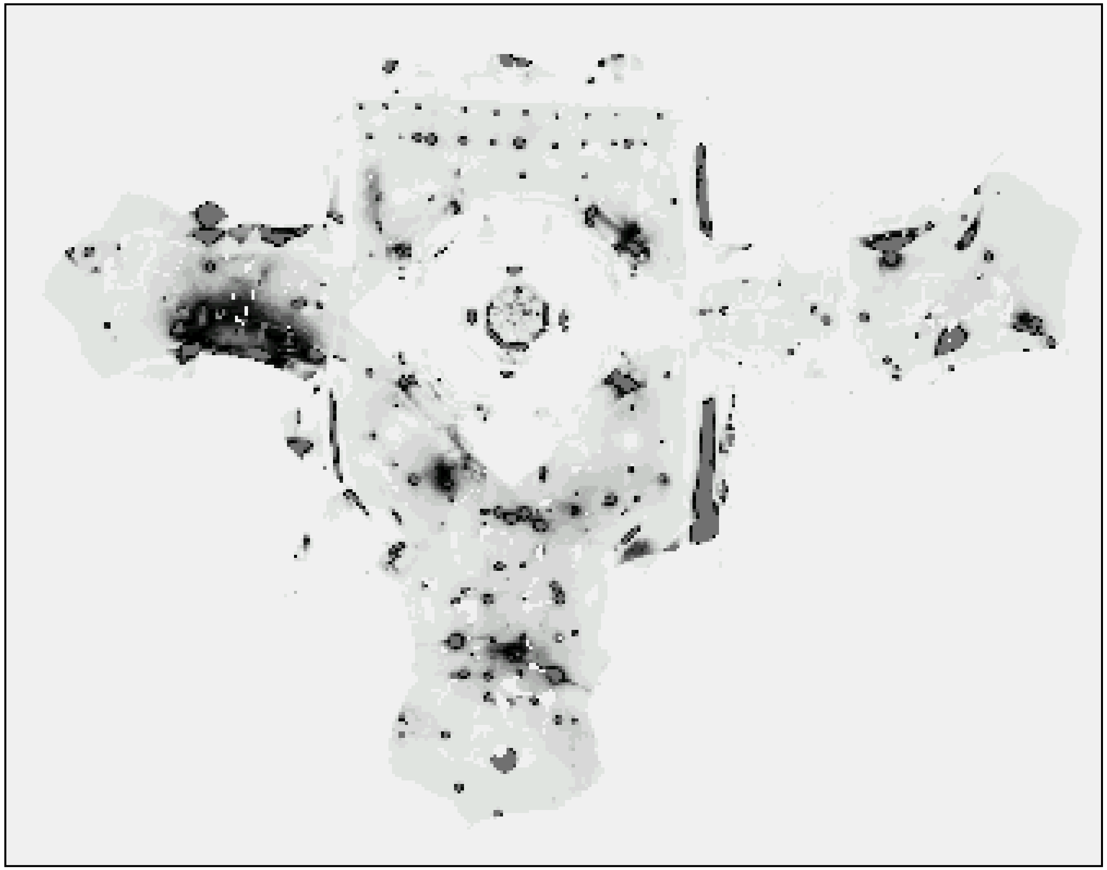


Next to a Light

Measurement z:



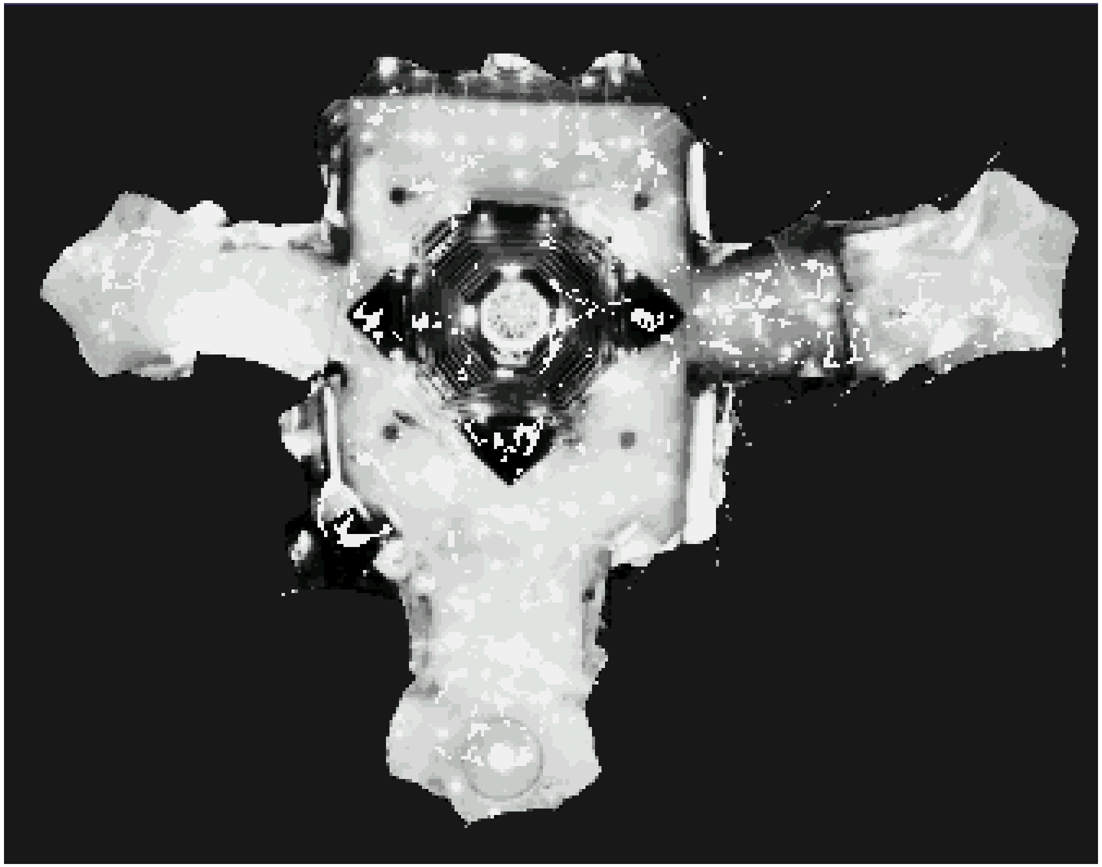
***P(z|x)*:**



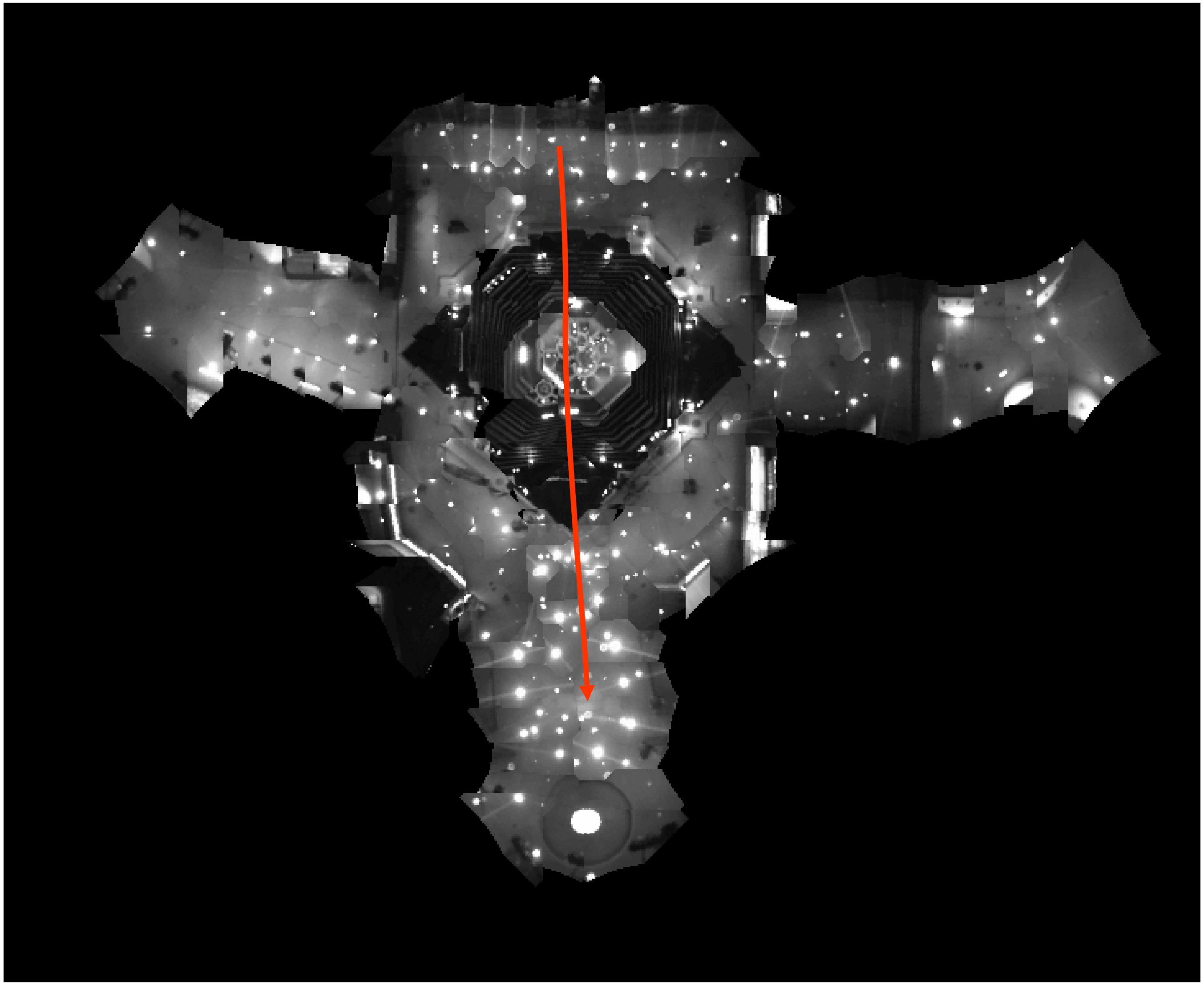
Elsewhere

Measurement z:

***P(z|x)*:**



Global Localization Using Vision



Limitations

- The approach described so far is able to
  - track the pose of a mobile robot and to
  - globally localize the robot.

- How can we deal with localization errors (i.e., the kidnapped robot problem)?

Approaches

- Randomly insert samples (the robot can be teleported at any point in time).
- Insert random samples proportional to the average likelihood of the particles (the robot has been teleported with higher probability when the likelihood of its observations drops).

Summary – Particle Filters

- Particle filters are an implementation of recursive Bayesian filtering
- They represent the posterior by a set of weighted samples
- They can model non-Gaussian distributions
- Proposal to draw new samples
- Weight to account for the differences between the proposal and the target
- Monte Carlo filter, Survival of the fittest, Condensation, Bootstrap filter

Summary – PF Localization

- In the context of localization, the particles are propagated according to the motion model.
- They are then weighted according to the likelihood of the observations.
- In a re-sampling step, new particles are drawn with a probability proportional to the likelihood of the observation.