Exercise Sheet 4
Due: Tuesday, June 21, 2011

Exercise 4.1 (CSPs)
The $\textit{SEND} + \textit{MORE} = \textit{MONEY}$ problem consists in finding distinct digits for the letters $D, E, M, N, O, R, S, Y$ such that $S$ and $M$ are different from zero, i.e. no leading zeros, and the equation

$$\textit{SEND} + \textit{MORE} = \textit{MONEY}$$

is satisfied.

(a) Explain in a nutshell, why it would be good to formulate the problem as a constraint satisfaction problem?

(b) Formulate the problem as a constraint satisfaction problem, i.e. what are the variables, what constraints do we have, etc.

(c) Find a solution using forward checking and arc consistency. Give the search tree.
   (Hint: consider the letters in the following order: $O, M, Y, E, N, D, R, S$.)

Exercise 4.2 (Minimax algorithm)

(a) Perform the minimax algorithm in the tree in Figure 1 using $\alpha\beta$-pruning. Traverse the tree from left to right. Annotate the nodes with their alpha and beta values.

(b) Can the nodes be ordered in such a way that $\alpha\beta$-pruning can cut off more branches? If so, give the order. Otherwise, argue why not.
**Exercise 4.3** (Generalization of the Minimax algorithm)

Consider the problem of search in a three-player game (you may assume that no alliances are allowed) without the zero-sum condition. The players are called 1, 2, and 3. Unlike in the case of two-player zero-sum games, the evaluation function now returns a triple \((x_1, x_2, x_3)\) such that \(x_i\) is the value the node has for player \(i\).

(a) Complete the game tree given below by annotating all interior nodes and the root node with the backed-up value triples.

(b) Assume that the value triple \((1, 1, 1)\) at the third leaf nodes from the left is replaced by \((0, 1, 2)\). Which problem arises now when you try to back up value triples? Suggest how to modify the back-up procedure to obtain a “robust” result at the root node.

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Sp. 1
  Sp. 2
    Sp. 3
      (2, 0, 1) (1, 1, 0) (1, 1, 1) (2, 1, 2) (2, 1, 1) (1, 2, 1) (1, 1, 1) (1, 1, 0)
```

**Exercise 4.4** (Joint Probability Distribution)

Given the joint probability distribution table

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>¬A</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>¬B</td>
<td>0.1</td>
<td>0.3</td>
</tr>
</tbody>
</table>

where cell \(A,B\) specifies the probability for \(P(A \land B)^1 = 0.4\), calculate the following probabilities:

(a) \(P(A), P(B), P(¬A), \) and \(P(¬B)\)

(b) \(P(A \lor B)\) and \(P((A \lor B) \land ¬(A \land B))\)

(c) \(P(A|B)\) and \(P(B|A)\)

The exercise sheets may and should be handed in and be worked on in groups of three (3) students. Please fill the cover sheet\(^2\) and attach it to your solution.

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\(^1\) shorthand for \(P(X_1 = A \land X_2 = B)\)

\(^2\) [http://ais.informatik.uni-freiburg.de/teaching/ss11/ki/cover-sheet.pdf](http://ais.informatik.uni-freiburg.de/teaching/ss11/ki/cover-sheet.pdf)