Foundations of Artificial Intelligence

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Exercise Sheet 6

Due: Tuesday, July 19, 2011

Exercise 6.1 (Models and Logical Equivalence)

(a) Consider a vocabulary with only four atomic propositions, A, B, C, and D. How many models are there for the following formulae? Explain.
  (i) $(A \wedge B) \vee (B \wedge C)$
  (ii) $A \vee B$
  (iii) $(A \Leftrightarrow B) \wedge (B \Leftrightarrow C)$

(b) How many pairwise non-equivalent propositional formulae can be constructed from the atomic propositions $A_1, A_2, \ldots, A_n$? Justify your answer.

Exercise 6.2 (CNF Transformation, Resolution Method)

The following transformation rules hold, whereby propositional formulae can be transformed into equivalent formulae. Here, φ, ψ and χ are arbitrary propositional formulae:

$$\neg\neg\varphi \equiv \varphi \quad (1)$$

$$\neg(\varphi \vee \psi) \equiv \neg\varphi \wedge \neg\psi \quad (2)$$

$$\varphi \vee (\psi \wedge \chi) \equiv (\varphi \vee \psi) \wedge (\varphi \vee \chi) \quad (3)$$

$$\neg(\varphi \wedge \psi) \equiv \neg\varphi \vee \neg\psi \quad (4)$$

$$\varphi \wedge (\psi \vee \chi) \equiv (\varphi \wedge \psi) \vee (\varphi \wedge \chi) \quad (5)$$

Additionally, the operators $\vee$ and $\wedge$ are associative and commutative.
Consider the formula $((C \wedge \neg B) \leftrightarrow A) \wedge (\neg C \rightarrow A)$.

(a) Transform the formula into a clause set K using the CNF transformation rules. Write down the steps.

(b) Afterwards, using the resolution method, show that $K \models (\neg B \rightarrow (A \wedge C))$ holds.

Exercise 6.3 (Davis-Putnam Procedure)

Use the *Davis-Putnam* procedure to find a model for each of the following formulae, or prove that the particular formula has no model. Whenever possible, apply *unit propagation.* At each step, indicate which rule you have applied.

(a) $(P \vee \neg Q) \wedge (\neg P \vee Q) \wedge (Q \vee \neg R) \wedge S \wedge (\neg S \vee \neg Q \vee \neg R) \wedge (S \vee R)$

(b) $(P \vee Q \vee S \vee T) \wedge (P \vee S \vee \neg T) \wedge (Q \vee \neg S \vee T) \wedge (P \vee \neg S \vee \neg T) \wedge (P \vee \neg Q) \wedge (\neg R \vee \neg P) \wedge R$

Exercise 6.4 (DNF Satisfiability Testing)

Show that for propositional formulae in Disjunctive Normal Form, satisfiability can be decided in polynomial time.

The exercise sheets may and should be handed in and be worked on in groups of three (3) students. Please fill the cover sheet[1] and attach it to your solution.

[1] `http://ais.informatik.uni-freiburg.de/teaching/ss11/ki/cover-sheet.pdf`