Albert-Ludwigs-Universität Freiburg
Institut für Informatik
Lecture: Introduction to Mobile Robotics
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# Sheet 2

## Topic: Linear Algebra, Locomotion, and Sensing

Submission deadline: May 17, 2011

Submit to: `mobilerobotics@informatik.uni-freiburg.de`

### Exercise 1: Linear Algebra

(a) Consider the matrices

$$\mathbf{A} = \begin{pmatrix} 0.25 & 0.1 \\ 0.2 & 0.5 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0.25 & -0.3 \\ -0.3 & 0.5 \end{pmatrix}.$$

Are they symmetric positive definite?

(b) For

$$\mathbf{C} = \begin{pmatrix} -3 & 0 \\ 0 & 1 \end{pmatrix},$$

find the smallest value for $\lambda \in \mathbb{R}$ so that $C + \lambda I$ becomes symmetric positive definite.

(c) Write a program in Octave that determines whether a matrix is orthogonal.

(d) Use this program to investigate whether

$$\mathbf{D} = \frac{1}{3} \begin{pmatrix} 2 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{pmatrix}$$

is orthogonal.

### Exercise 2: Locomotion

A robot equipped with a differential drive starts at position $x = 1.0m$, $y = 2.0m$ and with heading $\theta = \frac{\pi}{2}$. It has to move to the position $x = 1.5m$, $y = 2.0m$, $\theta = \frac{\pi}{2}$ (all angles in radians). The movement of the vehicle is described by steering commands ($v_l$ = speed of left wheel, $v_r$ = speed of right wheel, $t$ = driving time).

(a) What is the minimal number of steering commands $(v_l, v_r, t)$ needed to guide the vehicle to the desired target location?

(b) What is the length of the shortest trajectory under this constraint?

(c) Which sequence of steering commands guides the robot on the shortest trajectory to the desired location if an arbitrary number of steering commands can be used?

(d) What is the length of this trajectory?

Note: the length of a trajectory refers to the traveled distance along the trajectory.

**Exercise 3: Sensing**

A robot is located at $x = 1.0m$, $y = 0.5m$, $\theta = \frac{\pi}{4}$. Its laser range finder is mounted on the robot at $x = 0.2m$, $y = 0.0m$, $\theta = \pi$ (with respect to the robot's frame of reference).

The distance measurements of one laser scan can be found in the file `laserscan.dat`, which is provided on the website of this lecture. The first distance measurement is taken in the angle $\alpha = -\frac{\pi}{2}$ (in the frame of reference of the laser range finder), the last distance measurement has $\alpha = \frac{\pi}{2}$ (i.e., the field of view of the sensor is $\pi$), and all neighboring measurements are in equal angular distance (all angles in radians).

Note: You can load the data file and calculate the corresponding angles in Octave using

```
scan = load("-ascii", "laserscan.dat");
angle = linspace(-pi/2, pi/2, size(scan,2));
```

(a) Use Octave to plot all laser end-points in the frame of reference of the laser range finder.

(b) Use matrices as affine transformations and homogeneous coordinates in Octave to compute and plot the center of the robot, the center of the laser range finder, and all laser end-points in world coordinates.

Note: You can equally scale the $x$ and $y$-axis of a plot using

```
axis("equal");
```