Albert-Ludwigs-Universität Freiburg
Institut für Informatik
Lecture: Introduction to Mobile Robotics
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Sheet 5

Topic: Sensor Models, Error Propagation, and Feature Extraction

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Exercise 1: Sensor Model

Remark: This exercise is to be solved without Octave.

Assume you have a robot equipped with a sensor capable of measuring the distance and bearing to landmarks. The sensor furthermore provides you with the identity of the observed landmarks.

A sensor measurement $z = (z_r, z_\theta)^T$ is composed of the measured distance z_r and the measured bearing z_θ to the landmark l. Both the range and the bearing measurements are subject to zero-mean Gaussian noise with variances σ_r^2, and σ_θ^2, respectively. The range and the bearing measurements are independent of each other.

A sensor model

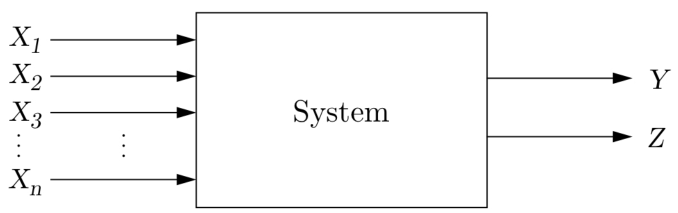
$$p(z \mid x, l)$$

models the probability of a measurement z of landmark l observed by the robot from pose x.

Design a sensor model $p(z \mid x, l)$ for this type of sensor. Furthermore, explain your sensor model.

Exercise 2: First-Order Error Propagation

Suppose the general case of a non-linear multi-input multi-output system with n correlated one dimensional input random variables $X_1, ..., X_n$ with $X_i \sim \mathcal{N}(\mu_{X_i}, \sigma_{X_i}^2)$ and (without loss of generality) two output random variables Y and Z.



We set $Y = f(X_1, ..., X_n)$ and $Z = g(X_1, ..., X_n)$ and approximate the functions $f(.)$ and $g(.)$ by a first-order Taylor series expansion:

$$Y \approx f(\mu_1, ..., \mu_n) + \sum_{i=1}^{n} \left.\frac{\partial f}{\partial X_i}\right|_{\mu_i,...,\mu_n} (X_i - \mu_i) \quad (1)$$

$$Z \approx g(\mu_1, ..., \mu_n) + \sum_{i=1}^{n} \left.\frac{\partial g}{\partial X_i}\right|_{\mu_i,...,\mu_n} (X_i - \mu_i) \quad (2)$$

Derive the expression for the covariance σ_{YZ} between Y and Z given the rules for the expected value

$$E[a] = a \quad (3)$$

$$E[aX] = aE[X] \quad (4)$$

$$E[X + Y] = E[X] + E[Y] \quad (5)$$

$$E[XY] = E[X]E[Y] \qquad \text{if X and Y are independent} \quad (6)$$

and the following definitions for mean, variance and covariance:

$$\mu_X = E[X] \quad (7)$$

$$\sigma_X^2 = E[(X - E[X])^2] \quad (8)$$

$$\sigma_{XY} = E[(X - E[X])(Y - E[Y])] \quad (9)$$

(a) Derive the expression for the covariance σ_{YZ} between Y and Z.

(b) Simplify this expression assuming stochastic independence of $X_1, ..., X_n$.

Exercise 3: Split and Merge

In this exercise, you will implement the split and merge algorithm for line extraction. Once you have completed the stubs, you can generate plots by executing `test_split` in *Octave*. You can test your implementation on two datasets. To change the dataset, modify `test_split.m` accordingly.

Complete the stub in `split_and_merge.m` for the simple threshold based split and merge. As threshold use 0.5.