8.1_ManipulatorJacobian

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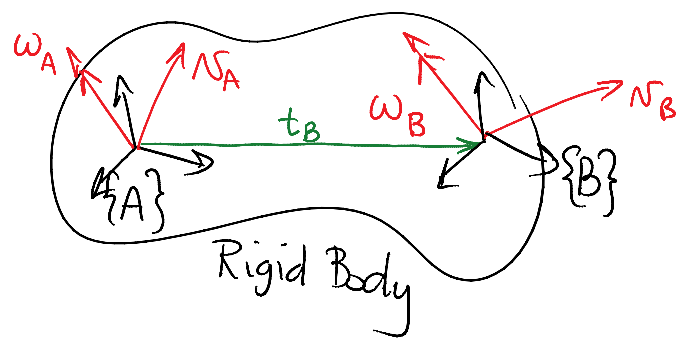
8.1.1 Transforming Velocities between Coordinate Frames

$$ {}^{A}T_{B} = \begin{bmatrix} {}^{A}R_{B} & {}^{A}t_{B} \\ 000 & 1 \end{bmatrix} $$



Let $\nu = \begin{bmatrix} v \\ \omega \end{bmatrix}$ be the spatial velocity of a point.

Choose origins as points of interest

$$ \nu_A = \begin{bmatrix} v_A \\ \omega_A \end{bmatrix} \qquad \nu_B = \begin{bmatrix} v_B \\ \omega_B \end{bmatrix} $$

$$ v_B = v_A + \omega_A \times r = \underset{(3\times3)}{I} v_A - t_B \times \omega_A = I v_A - S(t_B)\,\omega_A $$

$$ \omega_B = \omega_A $$

$$ \Longrightarrow \qquad \nu_B = \begin{bmatrix} v_B \\ \omega_B \end{bmatrix} = \begin{bmatrix} I & -S(t_B) \\ 0 & I \end{bmatrix} \begin{bmatrix} v_A \\ \omega_A \end{bmatrix} = J\,\nu_A $$

Assume we know ν_A in $\{A\}$ and want to know ν_B in $\{B\}$.

$$^A\omega_A = {}^A\omega_B \,, \quad \text{but} \quad {}^B\omega_B = {}^BR_A\,{}^A\omega_B$$

$$^BR_A\left({}^A v_B = {}^A v_A - S({}^At_B)\,{}^A\omega_A\right)$$

$$^Bv_B = {}^BR_A\,{}^Av_A - {}^BR_A\,S({}^At_B)\,{}^A\omega_A$$

$$^B\nu_B = \begin{bmatrix} {}^BR_A & -{}^BR_A\,S({}^At_B) \\ 0 & {}^BR_A \end{bmatrix} {}^A\nu_A$$

Corke's result is achieved by recalling ${}^BR_A = {}^AR_B^T$,

$S^T = -S$, and $(AB)^T = B^TA^T$

$$^B\nu = {}^BJ_A\,{}^A\nu$$

Text also uses notation:

$$^B\nu = {}^BJ_A\,{}^A\nu$$

$$\boxed{{}^B\nu = J_v({}^AT_B)\,{}^A\nu} \qquad (8.3)$$

$$\text{where } J_v({}^AT_B) = \begin{bmatrix} {}^AR_B^T & \left(S({}^At_B)\,{}^AR_B\right)^T \\ 0 & {}^AR_B^T \end{bmatrix} \qquad (8.4)$$

Matlab function: tr2jac$({}^AT_B)$

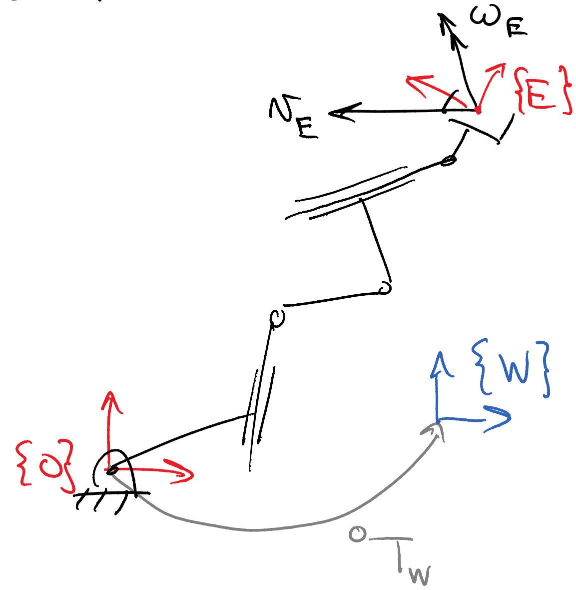
8.1.2. Jacobian in End Effector Frame

Recall in Corke's text,

$\{W\} = \{O\}$ & $\{E\} = \{N\}$

Eq.(8.2) $\nu = J(q)\,\dot{q}$

Here $\nu = {}^{W}\nu_{E}$.



The Matlab expression is:

$$\text{jacob0}(q) * \dot{q} = {}^{W}\nu_{E}$$

Now how do we map ${}^{W}\nu_{E}$ to ${}^{E}\nu_{E}$?

$$\text{jacobn}(q) * \dot{q} = {}^{E}\nu_{E}$$

Confusion in text ... last equations in 8.1.2.

$$\begin{aligned} {}^{0}\nu_{N} &= {}^{0}J_{N}\,{}^{N}\nu_{N} \\ &= J_{v}({}^{N}T_{0})\,{}^{N}J_{N}(q)\,\dot{q} \\ &= {}^{0}J(q)\,\dot{q} \end{aligned}$$

$$\Rightarrow {}^{0}J(q) = J_{v}({}^{N}T_{0})\,{}^{N}J_{N}(q)$$

$${}^{W}\nu_{E} = {}^{W}J_{E}\,{}^{E}\nu_{E}$$

But is this what is really meant?

This would compute the spatial velocity of the origin of $\{0\}$ as if $\{0\}$ were rigidly attached to $\{N\}$.

What I believe is intended is to express ${}^{N}v_N$ in $\{0\}$, ${}^{0}v_N$

$$
{}^{0}v_N = \begin{bmatrix} {}^{0}R_N & 0 \\ 0 & {}^{0}R_N \end{bmatrix} {}^{N}v_N \qquad !
$$

8.1.3. Analytical Jacobian

We have
$$
T(q) = \left[\begin{array}{ccc|c} & R(q) & & t(q) \\ \hline 0 & 0 & 0 & 1 \end{array}\right]
$$

Partition J as follows:

$$
\begin{bmatrix} v \\ \hline \omega \end{bmatrix} = \begin{bmatrix} J_p \\ \hline J_\phi \end{bmatrix} \dot{q}
$$

related to position of P

related to orientation of link containing P.

$$
v = \frac{d}{dt}(t(q)) = \frac{\partial t(q)}{\partial q} \dot{q}
$$

$$
J_p = \begin{bmatrix} \frac{\partial t_x}{\partial q_1} & \cdots & \frac{\partial t_x}{\partial q_N} \\ \frac{\partial t_y}{\partial q_1} & \cdots & \frac{\partial t_y}{\partial q_N} \\ \frac{\partial t_z}{\partial q_1} & \cdots & \frac{\partial t_z}{\partial q_N} \end{bmatrix}
$$

J_ϕ is more complicated.

We want $\omega = J_\phi(q)\,\dot{q}$

Recall $\dot{R}(q) = S(\omega)R(q)$

$$\Rightarrow \dot{R}(q)R^T(q) = S(\omega) = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

Use $\dot{R} = \frac{\partial R}{\partial q_1}\dot{q}_1 + \frac{\partial R}{\partial q_2}\dot{q}_2 + \dots + \frac{\partial R}{\partial q_N}\dot{q}_N$

Postmultiply by $R^T(q)$

Simplify to yield $\omega_x =$ function of q and linear in $\dot{q}$

$\omega_y =$ "

$\omega_z =$ "

Results can be arranged into $J_\phi\,\dot{q} = \omega$

Combining J_P and J_ϕ yields:

$\nu = J(q)\,\dot{q}$, where the functional dependence on q is known explicitly.

8.1.4. Jacobian Condition & Manipulability

$$\nu = J(q)\,\dot{q}$$

row of zeros?

$$\begin{bmatrix} \vdots \\ \nu_z \\ \vdots \end{bmatrix} = \begin{bmatrix} \text{Don't Care} \\ 0 \quad 0 \quad 0 \quad \cdots \quad 0 \\ \text{Don't Care} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \end{bmatrix}$$

$\Rightarrow$ corresp. velocity $= 0$!

Column of zeros? $\Longrightarrow$ corresp. joint motion doesn't effect end-eff. vel.

column of large values $\Longrightarrow$ Corresp joint has large effect.

The inverse relationship – $\dot{q} = J^{-1}\nu$

column of large values $\Longrightarrow$ desired values of ν can require large joint velocities

Determining singular configurations analytically.

Assume J is (6×6).

Singular configs exist when $\det(J) = 0$

These configs should be avoided when using Cartesian space moves.

One can identify singular configs analytically by solving $\det(J) = 0$.

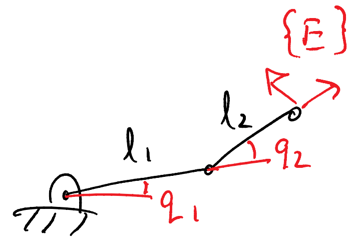
Let ${}^{0}Q_N = \{ q \mid \det({}^{0}J_N) = 0 \}$

${}^{N}Q_N = \{ q \mid \det({}^{N}J_N) = 0 \}$

Then ${}^{N}Q_N = {}^{0}Q_N$!

The singular set is determined by the kinematic structure, not the frame of reference.

Example: 2R-Planar Manipulator.



$p_x = l_1 c_1 + l_2 c_{12}$

$p_y = l_1 s_1 + l_2 s_{12}$

$\phi = q_1 + q_2$

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{\phi} \end{bmatrix} = \underbrace{\left[\begin{array}{c:c} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \\ 1 & 1 \end{array}\right]}_{{}^0J_E \ (3\times 2)} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

Every configuration is singular, because one <u>cannot</u> choose $\dot{p}_x, \dot{p}_y, \dot{\phi}$ arbitrarily and determine corresponding $\dot{q}_1, \dot{q}_2$!

Continue, ignoring ϕ.

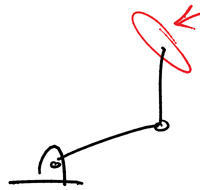
$$^{W}\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \end{bmatrix}_{E_{org}} = \underbrace{\left[\begin{array}{c:c} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{array}\right]}_{{}^WJ_E \ (2\times 2)} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\mathrm{Det}({}^{W}J_E) = J_{11}J_{22} - J_{21}J_{12} \xrightarrow{\text{simplify}} \underline{\underline{l_1 l_2 s_2}}$$

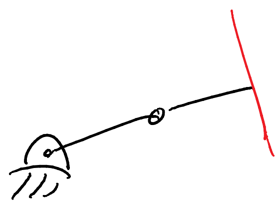
Singular configs are defined by $l_1 l_2 s_2 = 0$

$$s_2 = 0 \quad \text{iff} \quad \theta_2 = \pm n\pi, \quad n \in \mathbb{Z}$$

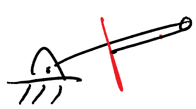
manipulability ellipse



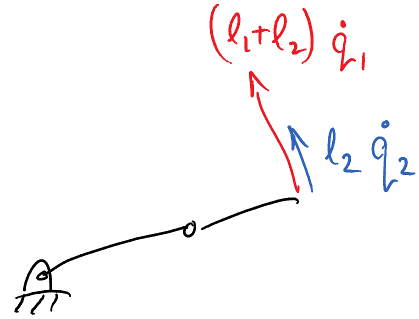
not singular



boundary singularity



$$ {}^{W}J_E(q_1, 0) = \begin{bmatrix} -(l_1+l_2)s_1 & -l_2 s_1 \\ (l_1+l_2)c_1 & l_2 c_1 \end{bmatrix}$$

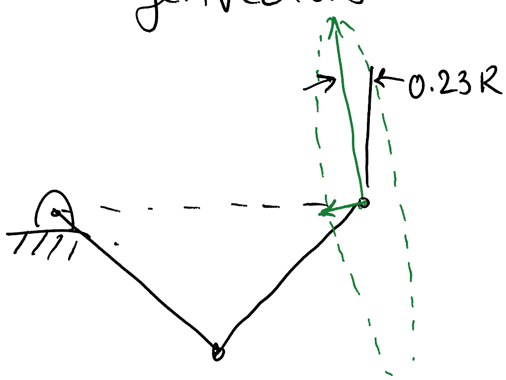


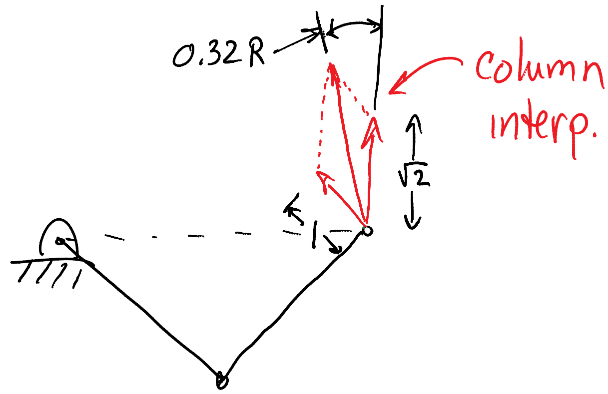
Columns are proportional

Only motion tangent to workspace boundary is possible!

See Chap 8, Sec 1, demos Matlab code

Interpretation of columns of J not same as eigenvectors of JJ^T.





Eigenvector of maximum eigen value gives the direction of greatest end-effector velocity, given that the joint velocities satisfy $\|\dot{q}\| \leq 1$.

In this direction, the robot:

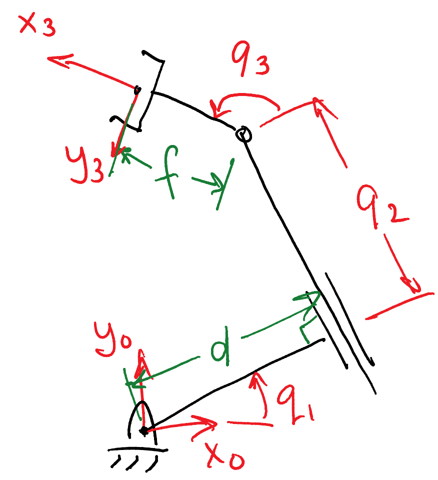
has the lowest resolution for position control.

is weakest for force application

Another planar example: RPR manipulator

Fwd. Kin Model

j	θ_j	d_j	a_j	α_j	σ_j
1	q_1	0	d	$-\pi/2$	0
2	0	q_2	0	$\pi/2$	1
3	q_3	0	f	0	0



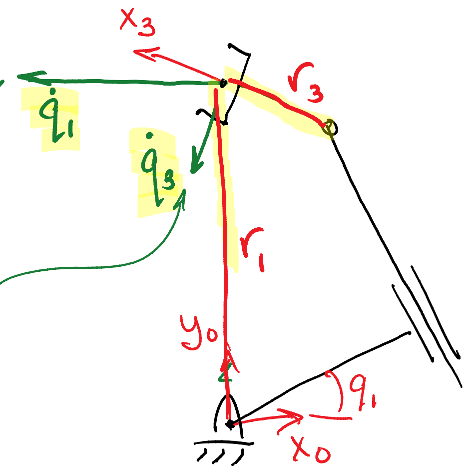
$$
{}^0T_3(q) = \begin{bmatrix} c_{13} & -s_{13} & dc_1 - q_2 s_1 + f c_{13} \\ s_{13} & c_{13} & ds_1 + q_2 c_1 - f s_{13} \\ 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0J_3 = \begin{bmatrix} -ds_1 - q_2 c_1 - f s_{13} & -s_1 & -f s_{13} \\ dc_1 - q_2 s_1 - f c_{13} & c_1 & -f c_{13} \\ 1 & 0 & 1 \end{bmatrix}
$$

length of velocity vector is:

$\|r_1(q)\|\,\dot{q}_1$

$\|r_3(q_3)\|\,\dot{q}_3$

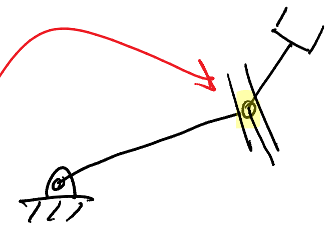


Let $\|v_3\| = 1$. What direction will yield fastest motion for $\|\dot{q}\| = 1$?

What about singularities?

Matlab's symbolic toolbox gives $\det(J) = -q_2$

$q_2 = 0 \Rightarrow$ revolute joint lies along the line of the first link.



This is an interior singularity!

Look at J

$$
{}^0J_3 = \begin{array}{c}\begin{bmatrix} -ds_1 - fs_{13} & -s_1 & -fs_{13} \\ dc_1 - fc_{13} & c_1 & -fc_{13} \\ 1 & 0 & 1 \end{bmatrix} \\ \begin{array}{ccc} \text{Col 1} & \quad\text{Col 2} & \quad\text{Col 3} \end{array}\end{array}
$$

The columns are linearly dependent!

col1 = d * col2 + col3.

Rank (J) = 2 $\therefore$ only velocities in a 2D subspace of (v_x, v_y, ω_z) can be achieved.

What velocities are impossible?

null(J) is 1 dimensional.

What motion causes no end-effector motion?

Are there any boundary singularities? No!

Kuka Jacobian?