9_DynamicsIntro

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10:16 PM

$$Q = \underbrace{M(q)\ddot{q}}_{\text{Inertial torques}} + G(q) + \underbrace{C(q,\dot{q})\dot{q}}_{\text{Centripedal \& Coriolis torques}} + F(q,\dot{q}) + \underbrace{J^T(q)\,g}_{\text{Torques induced by forces applied to arm.}}$$

- Q: joint torques
- $M(q)$: symmetric P.D. mass matrix
- $G(q)$: Gravity torques
- $F(q,\dot{q})$: Friction torques

For prismatic joints, "torques" are replaced by forces

Inv. Dynamics: Given $q(t)$, compute

$Q(t)$ that would cause the arm to follow $q(t)$

Recursive Newton-Euler alg computes $Q(t)$ in

$O(N)$ time, where N is the # of links.

Forward Dynamics: Given $Q(t)$, compute $q(t)$.

i.e. Solve nonlinear, 2nd-order O.D.E.

nonsmooth if stick-slip friction is modeled.

Deriving the equations of motion.

① Write N-E equations directly

② Write NE equations through the Lagrangian approach

We will use ②.

Let T be the kinetic energy

U " " potential energy

Then $L = T - U$ is the Lagrangian.

The eqs. of motion are given by:

$$\underbrace{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i}} = Q_i - F_i - (J^T g)_i \qquad i = \{1, 2, \ldots, N\}$$

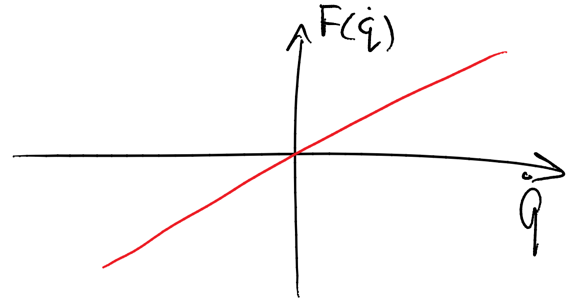
expand, noting that U is not a function of $\dot{q}$.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Q_i - F_i - (J^T g)_i$$

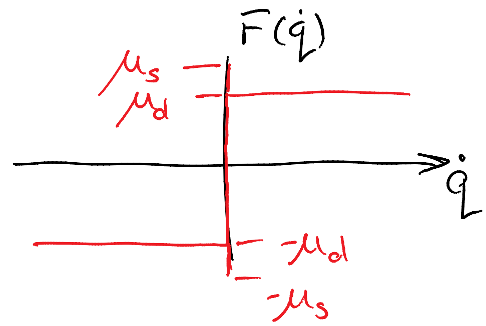
$$\sum_{j=1}^{N} \underbrace{\frac{\partial}{\partial \dot{q}_j}\left(\frac{\partial T}{\partial \dot{q}_i}\right)}_{M_{ij}(q)} \ddot{q}_j + \underbrace{\frac{\partial}{\partial q_j}\left(\frac{\partial T}{\partial \dot{q}_i}\right)\dot{q}_j - \frac{\partial T}{\partial q_i}}_{C(q,\dot{q})} + \underbrace{\frac{\partial U}{\partial q_i}}_{G(q)} = Q_i - F_i - (J^T g)_i$$

Possible joint friction models:

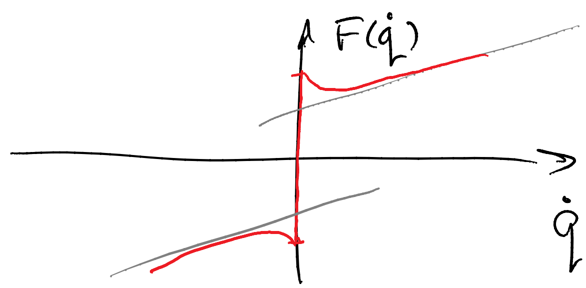
Viscous friction $F(\dot{q}) = b\,\dot{q}$
(always slipping)



Coulomb friction
(stick-slip)



Streibeck friction
stick-slip + viscous



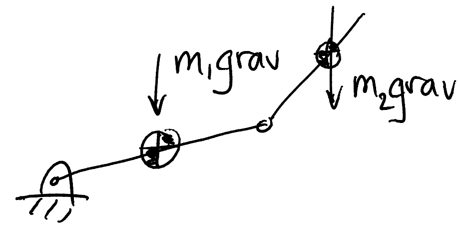
Contact is complicated.

$J_r^T(q)\,g$, where g must satisfy complex modeling constraints including stick-slip nonsmoothness and nonholonomic constraints during sticking.

All the dynamic forces affect those felt at the base.

You could map all inertia, Coriolis, contact, and gravity forces to the base using

$$g_{Base} = J_1^T \begin{bmatrix} m_1 grav \\ 0 \end{bmatrix} + \cdots$$



$$+ J_1^T \begin{bmatrix} -m\ddot{v}_1 \\ -I\dot{\omega}_1 \end{bmatrix} + \cdots$$

etc.

Or you could call Corke's rne method with the switch to return the base wrench.

9.1.6 Dynamic Manipulability

How easily can the robot accelerate its end effector in a chosen twist direction?

Consider only translational acceleration.

Assume all actuators require the same amount of energy to produce a given amount of torque

(If not true, the following expression can be waited).

In torque space (the dual of joint space),

$$Q^T Q = \text{constant}$$

defines an equal effort surface.

Ignore gravity, contact, friction, and Coriolis terms, then

$$Q = M\ddot{q}$$

Differentiate (8.2) $\longrightarrow$ $\dot{\nu} = \cancel{\dot{J}\dot{q}}^{\,0} + J\ddot{q}$

$$\dot{\nu} = J\ddot{q}$$

Assuming J^{-1} exists

$$\ddot{q} = J^{-1}\dot{\nu}$$

Now $Q = MJ^{-1}\dot{\nu}$ and $\therefore$ $Q^T Q = \dot{\nu}^T \underbrace{J^{-T} M^T M J^{-1}}_{M_x^{-1}} \dot{\nu} = 1$

$$\dot{\nu}^T M_x^{-1} \dot{\nu} = 1$$

Compare to equations on pages 178 and 179

$$\dot{q}^T \dot{q} = 1 \rightarrow \nu^T (J J^T)^{-1} \nu = 1 \longrightarrow m = \sqrt{\det(JJ^T)}$$

We see that $\underline{\underline{J M^{-1} M^{-T} J^T}}$ plays the role of $\underline{\underline{J J^T}}$

so inertia affect the measure.

The eigenvector corresponding to the smallest eigenvalue is the direction in which acceleration is most difficult.

Cartesian space dynamics

$$Q = M\ddot{q} + C + G + F + J_c^T g_{contact}$$

Assume J_E^{-1} exists.

$$J_E \dot{q} = \nu_E \quad \Longrightarrow \quad \dot{q} = J_E^{-1} \nu_E$$

$$\dot{J}_E \dot{q} + J_E \ddot{q} = \dot{\nu}_E \quad \Rightarrow \quad \ddot{q} = J_E^{-1}(\dot{\nu}_E - \dot{J}_E \dot{q})$$

$$J_E^T g_E = Q \quad \Longrightarrow \quad g_E = J_E^{-T} Q$$

Apply to dynamic equation

$$J_E^T g_E = M J_E^{-1} \dot{\nu}_E + C + F + G + J_C^T g_C \underbrace{- J_E^{-1} \dot{J}_E \dot{q}}$$

put in C

$$g_E = \underbrace{J_E^{-T} M J_E^{-1}}_{M_x} \dot{\nu}_E + \underbrace{J_E^{-T}(C - J_E^{-1} \dot{J}_E \dot{q})}_{C_x} + \underbrace{J_E^{-T}(G)}_{G_x} + \cdots$$

$$\boxed{g_E = M_x \dot{\nu}_E + C_x(x_E, \nu_E) + G_x(x_E) + F_x(\nu_E) + J_E^{-T} J_c^T g_c}$$

This is the M_x Asada used.

Under the assumptions made before: $\dot{q} = 0$, $G = 0$, $g_c = 0$

apply g_E causes $\dot{\nu}_E = M_x^{-1} g_E$.

Starting from $\nu_E = 0$, $\dot{\nu}_E \propto \nu_E$

$\therefore$ the virtual work is

$$\delta W = \nu_E^T g_E \propto \nu_E^T M_x \nu_E$$

The ellipsoid defined by $\nu_E^T M_x \nu_E = 1$ indicates how fast the arm can move in a given direction with a fixed amount of energy.